GOODS-MARKET FRICTIONS AND INTERNATIONAL TRADE*

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Abstract

We add goods-market frictions to a general equilibrium dynamic model with heterogeneous exporting producers and identical importing retailers. Our tractable framework leads to endogenously unmatched product varieties that reduce welfare, attenuate welfare responses, increase the trade elasticity, and operate mainly through the extensive margin. Quantitative results based on U.S. and Chinese data suggest that reducing international search costs to their domestic levels raises U.S. and Chinese welfare by 5.6 and 4 percent, respectively. A model with search frictions attenuates ex-ante welfare responses by 85 percent and changes the trade elasticity from -3.2 to -5.5 relative to a model without search frictions. The trade elasticity with respect to search costs is -0.7 and search frictions make the intensive and extensive margins of trade with respect to variable costs about equally important. Our framework provides a baseline for analyzing the aggregate implications of search frictions in models of trade.

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1 Introduction

Locating and building connections with overseas buyers is a prevalent firm-level barrier to exporting.[1] Firms pursue costly activities to overcome these barriers.[2] Despite the prevalence and magnitude of these costs at the firm level, how these barriers affect aggregate welfare and trade flows in general equilibrium is not well understood.

In this paper, we formalize this barrier to exporting as a goods-market friction between importing retailers and exporting producers in a Melitz-style model. The key insight is that an endogenous fraction of producers are actively looking for retailers but are yet to match with a partner. This unmatched rate alters the *levels* of aggregate variables and the *changes* in aggregate variables in response to shocks, because when producers are unmatched their associated varieties cannot be traded. We derive analytic expressions for the the gravity equation, the welfare response to foreign shocks, the elasticity of imports with respect to variable trade (iceberg) costs, and the elasticity of imports with respect to search costs. Finally, we quantify the effects of search frictions with a calibrated version of the model.

Our analytically tractable framework combines random matching and Nash bargaining (Pissarides, 2000, Ch. 1) with a general equilibrium trade model with heterogeneous producers and identical retailers (Hopenhayn, 1992; Melitz, 2003; Chaney, 2008). Our model includes many destination-origin markets, and we assume that all retailers and producers, including those in domestic markets, face search frictions. Each search market in our model is summarized by an endogenous sufficient statistic called "market tightness," defined as the ratio of searching retailers to searching producers. Market tightness determines the fraction of unmatched producers (mass of unmatched product varieties). Unmatched varieties cannot be consumed and are therefore absent from imports, the indirect utility (welfare) function, and all other aggregates. This feature sets our work apart from standard trade models, in which every producer that chooses to export finds a buyer, but our framework nests those models when we remove the search friction.

Goods-market frictions reduce aggregate import flows relative to a model without them in three ways. First, the fraction of unmatched producers reduces aggregate imports because a fraction of foreign varieties are not matched to importing retailers in equilibrium. While aggregate imports are lower, the quantity of any variety traded within a match is the same as in a model without search because Nash bargaining maximizes the profits earned from

[1] Kneller and Pisu (2011) find that "identifying the first contact" and "establishing initial dialogue" are more common obstacles to exporting than "dealing with legal, financial and tax regulations overseas" in a survey of U.K. firms.

[2] Eaton, Eslava, Jinkins, Krizan, and Tybout (2014) report that the four most expensive costs for Colombian exporters (in order) are maintaining foreign sales offices, supporting sales representatives abroad, researching potential foreign buyers, and sustaining a web presence.

consumers. Second, the negotiated price of imports is always lower than the final sales price paid by consumers whereas these prices are the same in standard trade models. Finally, by raising the up-front costs associated with entering foreign markets, search costs deter low-productivity producers from searching for a trading partner.

Search frictions attenuate the ex-post welfare response to foreign shocks relative to a model without search frictions. For example, protecting the domestic market by raising tariffs raises the price index (hurting consumers) and the incentive for retailers to enter the domestic market. This makes it easier for domestic producers to meet domestic retailers, which raises the fraction of matched domestic varieties (helping consumers). As such, for the same change in consumption shares and parameter values, our model predicts smaller welfare losses in response to tariff increases. More generally, we extend the welfare results in Arkolakis, Costinot, and Rodríguez-Clare (2012) (henceforth ACR) to an environment that includes endogenous goods-market frictions between importers and exporters.

Search frictions magnify the response of imports to iceberg trade costs in two ways relative to a model without search. First, our trade elasticities depend on the fraction of matched producers in domestic and foreign markets. For example, raising foreign tariffs reduces producers' matched rate in that market and raises domestic producers' matched rate via protectionism. Second, the import elasticity is also affected by the endogenous markup between import and final prices. This markup magnifies the effects of tariffs on trade flows because raising tariffs on a foreign country, for example, reduces the markup for imports in the foreign market but raises the markup in the domestic market.

Search frictions affect trade flows mainly through the extensive margin because they introduce a new "matched" margin that captures how changes in search costs affect the fraction of producers that are matched.

Using an approach advanced by Su and Judd (2012) and Dubé, Fox, and Su (2012)—mathematical programming with equilibrium constraints (MPEC)—we simultaneously recover parameters of the model and solve for the accompanying equilibrium endogenous variables to match U.S. and Chinese data. These data include economic aggregates, business start-up costs, and trading partner separation rates, among other measures. To calibrate importing retailers' search costs, we use the fraction of U.S. (Chinese) firms exporting to China (the United States), similar to Armenter and Koren (2014), Eaton et al. (2014), and Eaton, Jinkins, Tybout, and Xu (2016). To calibrate domestic retailers' search costs, we use manufacturing capacity utilization rates in each country, as in Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2017) and Petrosky-Nadeau, Wasmer, and Weil (2018). We show that standard estimates of the trade elasticity inform the elasticity of the number of matches with respect to the number of searching producers. As a whole, the calibration matches the data well and delivers a realistic economic environment for the United States and China.

Search frictions play an important quantitative role in our calibration. First, reducing international retailers' search costs to their domestic levels would increase U.S. and Chinese welfare by 5.6 and 4 percent, respectively. Second, ex-ante Chinese welfare changes in response to unilateral tariff increases are 85 percent smaller in our model than in a model without search frictions. Both the domestic consumption share and the domestic producers' matched rate attenuate the response of welfare. The domestic consumption share response is smaller in the model with search because the matched rate in the foreign market, which is always less than one, mutes the response of the domestic price index. Third, ex-post welfare evaluation using the formula in ACR implies welfare changes to unilateral tariff changes that are smaller than true welfare changes because standard log-linear estimates of the trade elasticity that omit search frictions are negatively biased for the structural parameters that are necessary to evaluate welfare changes. Fourth, the trade elasticity in our baseline economy with search is -5.5 relative to -3.2 in a model without search frictions. In our baseline economy, about half of the trade elasticity is explained by changes in the unmatched rate. Finally, the trade elasticity with respect to search costs operates through the extensive-matched margin and is -0.69. This is less than the elasticity with respect to variable costs (-5.2) but more than the elasticity with respect to entry costs (-0.1). Search frictions also raise the variable cost elasticity by changing the elasticity of the extensive margin from -0.18 to -2.37, making the intensive and extensive margins of trade with respect to variable costs about equally important.

International goods-market frictions are motivated by three well-known stylized facts. First, international intermediaries play an important role moving goods from producers to final consumers (Rossman, 1984; Rauch, 1996). For example, Bernard, Jensen, Redding, and Schott (2010, table 1) document that 87 percent of U.S. imports from China in 2002 were imported by intermediate-goods firms. The important role of intermediaries has been attributed informally to search costs (Blum, Claro, and Horstmann, 2009; Ahn, Khandelwal, and Wei, 2011). Second, informal trade barriers have large effects on trade flows (Eaton and Kortum, 2002; Rauch and Trindade, 2002; Melitz and Toubal, 2014). For example, the effect of common language and colonial ties dummies on trade are larger than the effects of regional- or free-trade agreements (Head and Mayer, 2014, table 3.4). While we do not specify the source of retailers' search costs in our model, informal trade barriers could proxy for them. Third, our model is consistent with the fact that common language increases trade mainly through the extensive margin (Lawless, 2010; Egger and Lassmann, 2015).

There is a recent expanding literature on search between international trading partners. While the microfoundation of our model differs from the new consumer margin in Arkolakis (2010), the two models are similar in that they both affect the levels of aggregate variables.

However, endogenous matching in our model changes the response of aggregate variables to shocks while Arkolakis (2010) leaves them unaffected relative to a model without search or the new consumer margin. In particular, the new consumer margin does not change the trade elasticity and the ex-post welfare change remains as in ACR.

Chaney (2014) includes search-friction and network effects in a rich model of the extensive margin of trade. He leaves the intensive margin, aggregate trade flows, and aggregate welfare implications of his model to future work while closely matching the empirical distribution of the number of countries served by French exporters.

Brancaccio, Kalouptsidi, and Papageorgiou (2020a) study endogenous shipping costs in a model of the transportation sector. Our paper and that paper share a matching function that exhibits constant returns to scale, Nash bargaining, and steady-state analysis of a dynamic model. They find that endogenous transportation costs induce network effects and dampen the effect of exogenous shocks on trade flows. In contrast, we have no network effects and search frictions magnify the effect of exogenous transportation cost shocks on trade flows.

Our framework is similar to Drozd and Nosal (2012), who use search frictions between producers and retailers to explain several pricing puzzles, and Benguria (2015), who shows that goods-market frictions provide a micro foundation for the costs of entering foreign markets. Allen (2014) rationalizes search frictions as costly information acquisition about agricultural market conditions across regions in the Philippines. Eaton et al. (2014) and Eaton et al. (2016) structurally estimate complex search models with endogenous contact rates, many-to-many matches, and learning about foreign markets but are restricted to partial equilibrium frameworks. Eaton, Kortum, and Kramarz (2018) is a related general equilibrium model. Each of these papers leave aggregate welfare implications to future work. Lim (2018) endogenizes production networks between heterogeneous firms, but focuses on domestic relationships using Compustat data.

Lenoir, Mejean, and Martin (2018) focus on explaining empirical patterns in French export data using a partial equilibrium Eaton and Kortum (2002) model with search frictions. Antràs and Costinot (2011) model intermediation between traders (retailers) and farmers (producers) in a Ricardian model and present quantitative exercises while Startz (2018) includes both contracting and search frictions. Bernard and Moxnes (2018) survey research on search and matching in trade.

2 Model

Our model features many countries and is similar to Melitz (2003) and Chaney (2008). In particular, we are motivated by the facts summarized in Bartelsman and Doms (2000) and Syverson (2011): Even within similar industries, firms exhibit persistent differences in measured productivity. We index producers of goods by their productivity, φ. This permanent productivity is exogenously given and known to producers.

As is standard, each country has a representative consumer that has utility over products, including a homogeneous good and differentiated varieties from all countries. Our model, however, assumes that these consumers can access differentiated goods only via ex-ante homogenous intermediaries called retailers.[3] Moreover, as in the work by Diamond (1982), Pissarides (1985), and Mortensen (1986), a costly process of search governs how producers and retailers find one another. Aside from this goods-market friction, our model nests Melitz (2003) and Chaney (2008). We develop a continuous-time framework and focus on steady-state implications. Our framework allows for search frictions in domestic and international goods markets.

We index each differentiated-goods market using do to denote destination-origin country pairs. This market includes exporting producers in country o and importing retailers in country d. We will sometimes omit this notation to conserve space.

2.1 Consumers

We assume the representative consumer in destination market d has Cobb-Douglas utility, U_d, over a homogeneous good and a second good that is a constant elasticity of substitution (CES) aggregate of differentiated varieties, indexed by ω, from all origins, indexed by $k \in \{1, \dots, O\}$. The two goods are combined with exponents $1-\alpha$ and α, respectively. The differentiated goods are substitutable with constant elasticity, $\sigma > 1$, across varieties and destinations and we denote the value of total consumption as C_d in destination country d. Formally the consumer's problem is

$$\begin{aligned} \max_{q_d(1), q_{dk}(\omega)} \quad & q_d(1)^{1-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\ s.t.\ C_d \quad = \quad & p_d(1)\, q_d(1) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega, \end{aligned} \tag{1}$$

which results in the following demand for the homogeneous good and each differentiated variety, respectively

$$q_d(1) = \frac{(1-\alpha)\, C_d}{p_d(1)}, \qquad q_{do}(\omega) = \alpha C_d \frac{p_{do}(\omega)^{-\sigma}}{P_d^{1-\sigma}}. \tag{2}$$

[3] Although in principle producers could circumvent retailers and contact final consumers directly, we avoid this possibility by assuming that the net value of matching with a retailer is always greater than the net value of forming a relationship directly with a final consumer. This approach is similar to earlier work by Wong and Wright (2014), who assume that a middleman is necessary rather than deriving the conditions under which this is the case.

Cobb-Douglas preferences across sectors imply that the consumer allocates share $1-\alpha$ of total consumption expenditure to the homogeneous good and share α to the differentiated goods. We could easily extend our framework to any number of Cobb-Douglas sectors, as in Chaney (2008).

The homogeneous good has price $p_d(1)$. Define P_d as the price index for the bundle of differentiated varieties and P_{do} as the price index for the bundle of varieties produced in country o and consumed in country d:

$$P_d = \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)^{1-\sigma} \, d\omega\right]^{\frac{1}{1-\sigma}} = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \quad (3)$$

The ideal price index that minimizes expenditure to obtain utility level $U_d = 1$ is $\Xi_d = \left[p_d(1) / (1-\alpha)\right]^{1-\alpha} \left[P_d/\alpha\right]^{\alpha}$. To derive these equations, we solve the consumer's utility maximization and expenditure minimization problems explicitly in appendix A.1.

2.2 The matching function, producers, retailers, and bargaining

Searching or matching in one market does not affect the costs of searching across other markets. In particular, we assume that there are no economies of scale in one market for individual producers and retailers from currently being in a match or from searching in other markets. These assumptions ensure that we can study each "segmented" market independently because, although individual behavior will affect (and be affected by) aggregate variables, they are taken as given by atomistic producers and retailers.

Segmented markets could be relaxed to allow for increasing returns to search for either producers or retailers but doing so would not change the qualitative results of our paper. As long as the search costs for retailers are positive, producers' finding rate will be finite (section 3.4).[4]

In equilibrium, every matched producer fills one, and only one, product vacancy in each *do* search market. A matched producer has no incentive to pay additional search costs to look for another vacancy in the same search market because vacancies are identical and the quantity exchanged within each match already maximizes profits (section 3.2). As such, retailer vacancies and product varieties always match one-to-one.

The number of retailers in our model is immaterial. What matters is the number of product vacancies, which can originate from one retailing firm posting all vacancies, all

[4]Furthermore, recent empirical evidence is not definitive on whether there exists increasing returns in the number of trading partners. The lack of increasing returns to the number of matches is consistent with the results of Arkolakis (2010) and McCallum (2017). Using other approaches or focusing on particular industries, there is some evidence for increasing returns (Moxnes, 2010; Hanson and Xiang, 2011; Chaney, 2014; Morales, Sheu, and Zahler, 2019).

retailers posting one vacancy each, or anything in between. As such, variety-to-vacancy matches are always one-to-one but producer-to-retailer matches can be all-to-one, one-to-one, or many-to-one in each search market. Many search models, including Pissarides (2000), share this feature. Section 3.4 discusses how to pin down the number of retailers with the assumption that each retailer posts only one vacancy.

We are aware that in the data, international retailers and producers simultaneously engage with several business partners, but product-level trade is typically one-to-one. Many-to-many relationships have been highlighted in Eaton et al. (2014) and Eaton, Kortum, and Kramarz (2017). However, Sugita, Teshima, and Seira (2017) find that, while U.S. importers and Mexican exporters in textiles transact with multiple firms, the main seller and buyer account for the bulk of each firm's total trade. These authors conclude that "a one-to-one matching model is a fair approximation of product-level matching in Mexico-U.S. textile/apparel trade." Similarly, Eaton et al. (2014) find that roughly 80 percent of matches are one-to-one in Colombia-U.S. manufacturing trade. In light of this evidence, and because matching in our model can be interpreted as one retailer to one producer, we will refer to vacancies and retailers interchangeably.

2.2.1 The matching function

The matching function, denoted by $m\left(u_{do}N_o^x, v_{do}N_d^m\right)$, gives the flow number of relationships formed at any moment in time as a function of the stock number of unmatched producers, $u_{do}N_o^x$, and unmatched retailers, $v_{do}N_d^m$, in the *do* market. N_o^x and N_d^m represent the total mass of producing firms in country o and retailing firms in country d, respectively, that exist regardless of their match status. The fraction of producers in country o looking for retailers in country d is u_{do}. The fraction of retailers that are searching for producing firms in this market is v_{do}.

As in many studies of the labor market (Pissarides, 1985; Shimer, 2005), we assume that the matching function takes a Cobb-Douglas form:

$$m\left(u_{do}N_o^x, v_{do}N_d^m\right) = \xi\left(u_{do}N_o^x\right)^{\eta}\left(v_{do}N_d^m\right)^{1-\eta}, \tag{4}$$

in which ξ is the matching efficiency and η is the elasticity of matches with respect to the number of searching producers. Stevens (2007) presents microfoundations for a Cobb-Douglas matching function in a setting of heterogeneous matches when marginal search costs are approximately linear.

The matching function in equation (4) is homogeneous of degree one. Therefore, market tightness, $\kappa_{do} = v_{do}N_d^m/u_{do}N_o^x$, which is the ratio of the mass of searching retailers to the mass of producers in a given market, is sufficient to determine contact rates on both sides of that market.[5] In particular, the rate at which retailers in country d contact producers in country o, $\chi(\kappa_{do})$, is the number of matches formed each instant over the number of searching retailers:

$$\chi(\kappa_{do}) = \frac{m(u_{do}N_o^x, v_{do}N_d^m)}{v_{do}N_d^m} = \frac{\xi(u_{do}N_o^x)^\eta (v_{do}N_d^m)^{1-\eta}}{v_{do}N_d^m} = \xi\kappa_{do}^{-\eta}.$$

Notice that retailers' contact rate falls with market tightness $(d\chi(\kappa_{do})/d\kappa_{do} < 0)$ because with more retailers relative to producers, the search market becomes congested with retailers.

The rate at which producers in country o contact retailers in country d is the number of matches formed each instant over the number of searching producers, so that the producer contact rate is $\xi\kappa_{do}^{1-\eta} = \kappa_{do}\chi(\kappa_{do})$. Producers' contact rate rises with tightness $(d\kappa_{do}\chi(\kappa_{do})/d\kappa_{do} > 0)$, also called a market thickness effect. Market tightness is defined from the perspective of producers so that the market is tighter when there are relatively more retailers than producers.

2.2.2 Producers

We assume the homogeneous good is produced with one unit of labor under constant returns to scale in each country. We also assume there is free entry into the production of that good, there are no search frictions in that sector, and that this good is freely traded. Since it is costless to trade, a no-arbitrage condition implies that the price of the homogeneous good must be the same in all countries $(p_d(1) = p(1)\,\forall d)$, and because it is made with one unit of labor in each country, it must also be the case that $w_d = p(1)\,\forall d$. As in Chaney (2008), and to simplify our analysis, we only consider equilibria in which every country produces some of the numeraire. Therefore, the homogeneous good will serve as the global numeraire with $p_d(1) = 1\forall d$. We could solve the model without the homogeneous good sector and endogenize wages using market clearing conditions for labor, but the analysis would become analytically intractable and this complication would not alter our main finding that the endogenous unmatched fraction of producers is important for the levels and changes of aggregate variables.

There are two production costs for differentiated goods. First, producers face the familiar variable cost function indexed by productivity:

$$t(q_{do}, w_o, \tau_{do}, \varphi) = q_{do}w_o\tau_{do}\varphi^{-1}. \tag{5}$$

Here w_o is the competitive wage in the exporting (origin) country, $\tau_{do} \geq 1$ is a parameter

[5] We use continuous time Poisson processes to model the random matching of retailers and producers. Thus, the contact rate defines the average number of counterparty meetings during one unit of time. Appendix A.2 contains more details.

capturing one plus the iceberg transport cost between destination d and origin o, and q_{do} is the amount produced and traded between destination d and origin o. This variable cost function implies a constant-returns-to-scale production function in which labor is the only input. The firm that produces quantity $q_{do}(\omega)$ of variety ω has productivity φ and marginal cost equal to $w_o \tau_{do} \varphi^{-1}$. Following Melitz (2003), we interpret higher productivity firms as producing a symmetric variety at a lower marginal cost. Second, total production cost is $t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do}$ in which f_{do} is the fixed cost of production. We could include non-tradeable goods in our framework by increasing the number of sectors and setting the iceberg trade costs in some of these sectors to infinity.

We assume that productivity is exogenous and Pareto distributed with the same cumulative density function in all countries:

$$G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}, \tag{6}$$

in which $\varphi \in [1, +\infty)$. The probability density function is $g(\varphi) = \theta \varphi^{-\theta-1}$. We assume that $\theta > \sigma - 1$ so that aggregate variables determined by the integral $\int_{\bar{\varphi}}^{\infty} z^{\sigma-1} dG(z)$ are bounded. The Pareto distribution has been widely used in trade models and describes firms' size well (Axtell, 2001; Gabaix, 2009).

The value of a producer with productivity φ being matched to a retailer, $X_{do}(\varphi)$, can be summarized by a value function in continuous time:

$$rX_{do}(\varphi) = n_{do} q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + \lambda (U_{do}(\varphi) - X_{do}(\varphi)). \tag{7}$$

This asset equation states that the flow return at the risk-free rate, r, from the value of producing must equal the flow payoff plus the expected capital gain from operating as an exporting producer. Each producer is indexed by exogenous productivity, φ. The flow payoff consists of $n_{do} q_{do}$, the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable, $t(q_{do}, w_o, \tau_{do}, \varphi)$, and fixed cost of production, f_{do}. The negotiated price, n_{do}, and the quantity traded, q_{do}, are determined through a bargaining process that we describe in sections 2.2.4, 3.1, and 3.2. The last term in equation (7) captures the dissolution of the match, which occurs at exogenous rate λ and leads to a capital loss of $U_{do}(\varphi) - X_{do}(\varphi)$ as the producer loses value $X_{do}(\varphi)$ but gains the value of being an unmatched producer, $U_{do}(\varphi)$.

The value that an unmatched producer receives from looking for a retail partner without being in a business relationship, $U_{do}(\varphi)$, satisfies

$$rU_{do}(\varphi) = -l_{do} + \kappa_{do} \chi(\kappa_{do}) (X_{do}(\varphi) - U_{do}(\varphi) - s_{do}). \tag{8}$$

The flow search cost, l_{do}, is what the producer pays when looking for a retailer; it captures the costs we highlighted in the introduction—namely, maintaining foreign sales offices, sending sales representatives abroad, researching potential foreign buyers, and establishing a web presence. The second term captures the expected capital gain, in which $\kappa_{do}\chi(\kappa_{do})$ is the endogenous rate at which producing firms contact retailers, and s_{do} is the sunk cost of starting up the relationship.

The producing firm also has the option of remaining idle and not expending resources to look for a retailer. For producers, the value of not searching, $I_{do}(\varphi)$, satisfies

$$rI_{do}(\varphi) = h_{do}, \tag{9}$$

in which choosing to remain idle provides the flow payoff, h_{do}. The value to a producer of remaining idle can be interpreted, for example, as the value of the stream of payments after liquidation or the flow payoff from home production if these firms are viewed as entrepreneurs. We include an idle state because omitting it has the unappealing consequence that all producers search in all markets, even if they expect to reject all retailer contacts. Allowing producers to optimally choose not to search in each market is both more general and more intuitive. We further clarify the importance of the idle state in section 3.3.

Creating each producer with heterogeneous productivity, φ, requires a one-time sunk "exploration" cost, e_d^x, similar to di Giovanni and Levchenko (2012). To maintain aggregate analytical tractability, we assume that the number of differentiated-goods producers, N_d^x, is proportional to endogenous consumption expenditure, C_d, as in Chaney (2008). As such, N_d^x is endogenous but there is no entry decision by producers and we account for the resources expended to create producers in the resource constraint (equation 22). We discuss a microfoundation for the entry decision by potential producers in sections 3.4 and 4.1. In that alternative approach, an unbounded mass of potential producers chooses whether or not to take a productivity draw by comparing the cost, e_d^x, to the expected future benefits of entry. Free entry of potential producers in this setting is analytically intractable and does not affect any conclusions about search frictions and international trade.

2.2.3 Retailers

All retailers are ex-ante identical but have values that vary ex-post only because producers are heterogeneous. The value of a retailing firm in a business relationship with a producer of productivity φ, is defined by the asset equation,

$$rM_{do}(\varphi) = p_{do}q_{do} - n_{do}q_{do} + \lambda\left(V_{do} - M_{do}(\varphi)\right). \tag{10}$$

The flow payoff from being in a relationship is the revenue generated by selling q_{do} units of the product to a representative consumer at a final sales price p_{do} (determined by their inverse demand curve from equation 2) less the cost of acquiring these goods from producers at negotiated price n_{do}. Retailers do not use the product as an input in another stage of production but only facilitate the match between producers and consumers. We show in section 3 that including an additional intermediate input does not substantively affect our main conclusions. In the event that the relationship undergoes an exogenous separation, at rate λ, the retailing firm loses the capital value of being matched, $V_{do} - M_{do}(\varphi)$.

The value of being an unmatched retailer, V_{do}, satisfies

$$rV_{do} = -c_{do} + \chi(\kappa_{do}) \int [\max\{V_{do}, M_{do}(\varphi)\} - V_{do}]\, dG(\varphi)\,. \tag{11}$$

Retailers need to pay a flow cost, c_{do}, to search for a producing affiliate. At Poisson rate $\chi(\kappa_{do})$, retailing firms meet a producer of unknown productivity.

Producers' productivities are ex-ante unknown to retailers so retailers take the expectation over all productivities they might encounter when computing the expected continuation value of searching. As a result, the value, V_{do}, is not a function of a producer's productivity, φ, but rather a function of the expected payoff. We assume that upon meeting, but before consummating a match, retailers learn the productivity of the producer. Depending on the producer's productivity, φ, retailers choose between matching with that producer, which generates value $M_{do}(\varphi)$, and continuing the search, which generates V_{do}. Hence, the capital gain to retailers from meeting a producer with productivity φ can be expressed as $\max\{V_{do}, M_{do}(\varphi)\} - V_{do}$. In an equilibrium with free entry into retailing, this approach is equivalent to retailers observing producers' productivity after matches are formed.

2.2.4 Bargaining

Upon meeting, the retailer and producer bargain over the negotiated price, n_{do}, and quantity, q_{do}, simultaneously. We assume that these objects are determined by the generalized Nash bargaining solution, which, as shown by Nash (1950) and Osborne and Rubinstein (1990), is equivalent to maximizing the following Nash product:

$$\max_{q_{do}, n_{do}} [X_{do}(\varphi) - U_{do}(\varphi)]^{\beta} [M_{do}(\varphi) - V_{do}]^{1-\beta}, 0 \leq \beta < 1, \tag{12}$$

in which β is producers' bargaining power. The total surplus created by a match, which is the value of the relationship to the retailer and the producer less their outside options, is $S_{do}(\varphi) = M_{do}(\varphi) - V_{do} + X_{do}(\varphi) - U_{do}(\varphi)$. In appendix A.3, we derive an expression for the match surplus and for the value of a relationship, $R_{do}(\varphi)$, in terms of model primitives, which also provides theoretical underpinnings for results in Heise (2016) and Monarch and Schmidt-Eisenlohr (2018). Although different approaches to sharing match surplus (Burdett and Mortensen, 1998; Cahuc, Postel-Vinay, and Robin, 2006) will lead to changes in details and specific expressions, our main results about the effects of endogenous market tightness (section 5) will remain.

3 Optimal search and matching in equilibrium

The retailing and producing firms use backward induction to maximize their value. The second stage is the solution that results from bargaining over price and quantity after a retailer and producer meet and decide to match. In the first stage, retailers and producers, taking the solution to this second-stage bargaining problem as given, choose whether to search for a business partner, or to remain idle. Because producers are heterogeneous, their decision to search or not depends on their productivity resulting in a minimum productivity threshold that makes searching worthwhile. A free entry condition characterizes retailers' decisions to search and defines equilibrium market tightness. Finally, there exists a steady-state fraction of unmatched producers that are actively looking for a retail partner and unmatched retailers that are actively looking for a producer.

3.1 Bargaining over price

Bargaining over the negotiated price, n_{do}, results in a price that gives β of the total surplus to producers and $1-\beta$ of the surplus to retailers:

$$X_{do}(\varphi) - U_{do}(\varphi) = \beta S_{do}(\varphi), \qquad M_{do}(\varphi) - V_{do} = (1-\beta) S_{do}(\varphi). \tag{13}$$

We refer to this expression as the "surplus sharing rule" and include its derivation in appendix A.4.1.[6]

The negotiated price that splits the surplus according to equation (13) is

$$n_{do} = [1-\gamma_{do}]\, p_{do} + \gamma_{do} \frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}}{q_{do}}, \tag{14}$$

in which $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})} \in [0,1]$. Appendix A.4.2 has detailed derivations and appendix A.4.3 proves that $\gamma_{do} \in [0,1]$. We remind the reader that equation (14) is a function of producer's productivity, φ, but we omit this notation to conserve space.

The equilibrium negotiated price, n_{do}, is a convex combination of the final sales price and

[6] We also point out that the reasoning behind the restriction that $\beta < 1$ in equation (12) is evident in equation (13). Retailing firms have no incentive to search if $\beta = 1$, as they get none of the resulting match surplus and therefore cannot recoup search costs, $c_{do} > 0$. Any solution to the model with $c_{do} > 0$ and positive trade between retailers and producers also requires $\beta < 1$.

the average total production cost less producers' search costs. A price outside of this range would be unsustainable. The highest negotiated price, n_{do}, that retailers are willing to pay is the final sales price, p_{do}, and the lowest negotiated price that producers are willing to accept is the average total production cost, $\left(t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do}\right)/q_{do}$, net of the cost of looking for a retailer, l_{do}, and the expected sunk cost, $\kappa_{do}\chi(\kappa_{do})s_{do}$. The search costs of producers, l_{do} and s_{do}, enter negatively in equation (14) because they erode producers' bargaining position and thereby allow retailers to negotiate a lower transaction price.

The negotiated price also depends on the bargaining power and the finding rate of producers. As producers gain all the bargaining power ($\beta \to 1$), then $\gamma_{do} \to 0$ and $n_{do} \to p_{do}$, so producers take all the profits from the business relationship. Similarly, if producers find retailers immediately (no search frictions) so that the finding rate $\kappa_{do}\chi(\kappa_{do}) \to \infty$, and the sunk cost, s_{do}, is set to zero, then the negotiated price also converges to the final sales price, $n_{do} \to p_{do}$, which is the same as in Melitz (2003) and Chaney (2008). We provide details in appendix A.4.4 and we show how our model nests standard trade models, including this pricing result, in section 4.7.

Many papers document that retail prices exceed import prices, which can be rationalized in a variety of ways. Our model rationalizes this difference as a markup that compensates retailers for searching. Previous work rationalizes this difference as domestic transportation costs and wholesaler markups, among other explanations (Burstein, Neves, and Rebelo, 2003; Goldberg and Campa, 2010; Berger, Faust, Rogers, and Steverson, 2012). We extend our model to incorporate other inputs that are necessary to bring products to final consumers, including distribution costs, in the next section and show that it makes the negotiated price a function of the cost of the other input but does not alter the main results of our paper.

3.2 Bargaining over quantity

Bargaining over quantity, q_{do}, implies that the quantity exchanged *within* matches equates marginal revenue obtained by retailers from consumers with the marginal production cost, as shown in appendix A.5.1 equation (A38). This result together with our differentiated demand curve from equation (2), and our cost function from equation (5) imply that the final consumer price in market d for a good from market o is:

$$p_{do}\left(\varphi\right) = \mu w_o \tau_{do} \varphi^{-1}, \tag{15}$$

in which $\mu = \sigma / \left(\sigma - 1\right) > 1$. We present the details of this derivation in appendix A.5.2.

The quantity traded within matches in our model is the same as in a model without search frictions. The quantity depends on consumers' demand curve p_{do}, the pricing power of retailers, and the production cost function, $t\left(q_{do}, w_o, \tau_{do}, \varphi\right)$. We show in appendix A.5.3 that including an additional input in retailers' production function does not change this result.

Nevertheless, although the quantity exchanged does not depend on search frictions, these frictions do affect the mass of matches formed. We turn to this topic in the next section.

3.3 Producers' search productivity thresholds

Given the outcome of bargaining in the second stage, we derive whether retailers and producers will search for a business partner at all in the first stage. Because producers differ by productivity, this first stage leads to a productivity threshold, $\bar{\varphi}_{do}$, that makes the producer indifferent between searching and remaining idle, $U_{do}(\bar{\varphi}_{do}) - I_{do}(\bar{\varphi}_{do}) = 0$.[7] We show in appendix A.6.2 that this indifference condition leads to an implicit function that equates variable profits from the match and what we call the "effective entry cost,"

$$\pi_{do}(\bar{\varphi}_{do}) \equiv p_{do}(\bar{\varphi}_{do})\, q_{do}(\bar{\varphi}_{do}) - t(q_{do}, w_o, \tau_{do}, \bar{\varphi}_{do}) = F(\kappa_{do}). \tag{16}$$

The effective entry cost is defined as

$$F(\kappa_{do}) \equiv f_{do} + \left(\frac{r+\lambda}{\beta\kappa_{do}\chi(\kappa_{do})}\right) l_{do} + \left(1 + \frac{r+\lambda}{\beta\kappa_{do}\chi(\kappa_{do})}\right) h_{do} + \left(\frac{r+\lambda}{\beta}\right) s_{do}, \tag{17}$$

and is the sum of the fixed cost of production, f_{do}, and the (appropriately discounted) flow cost of searching for a retailer, l_{do}, the opportunity cost of remaining idle, h_{do}, and the sunk cost of starting up a business relationship, s_{do}. Equation (17) provides a novel micro-level interpretation of export entry costs. Benguria (2015) makes a closely related point.

With our functional form assumptions, equations (16) and (17) imply that the threshold productivity is

$$\bar{\varphi}_{do} = \mu\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)\left(\frac{F(\kappa_{do})}{C_d}\right)^{\frac{1}{\sigma-1}}. \tag{18}$$

We present details in appendix A.6.3.

Equation (18) shows that the threshold productivity depends endogenously on producers' finding rate $\kappa_{do}\chi(\kappa_{do})$ through the effective entry cost. Intuitively, higher κ_{do} reduces the time spent searching by producers, the effective entry cost, and the threshold productivity.

Another innovation of our model is that the opportunity cost of remaining idle, h_{do}, is an important determinant of the productivity threshold and the fraction of active producers through equation (17). As pointed out by Armenter and Koren (2014), the fraction of exporting firms is an important moment for parameter identification and one that has been exploited by Eaton et al. (2014) and Eaton et al. (2016), among others. Allowing for the

[7] There exists an alternative threshold, $\underline{\varphi}_{do}$, which makes the producer and retailer indifferent between consummating a relationship upon contact and continuing to search, $X_{do}(\underline{\varphi}_{do}) - U_{do}(\underline{\varphi}_{do}) = 0$. We show in appendix A.6.1 that the binding threshold is defined by $\bar{\varphi}_{do}$, because $\bar{\varphi}_{do} > \underline{\varphi}_{do}$ if $l_{do} + h_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do} > 0$.

possibility that producers optimally choose not to search could change the estimates of entry barriers in these important papers.

Equations (16) and (17) also nest the conditions defining the threshold productivity in many trade models. In particular, when we eliminate search frictions and set $h_{do} = -s_{do}\left(r+\lambda\right)/\beta$, we recover the same threshold productivity as Chaney (2008). We present a more complete discussion of this result in section 4.7 and relate equations (16) and (17) to expressions in other standard trade frameworks in appendix A.6.4. Appendix A.6.5 clarifies the importance of the idle state and its relationship to the threshold productivity.

3.4 Retailer entry

Here we specify the conditions under which unmatched retailers search to match with producers. As is standard in the labor literature (Pissarides, 1985; Shimer, 2005), we assume free entry into retailing so that in equilibrium, the value of being an unmatched retailer, V_{do}, is driven to zero. The ability to expand retail shelf space or post a product online until it is no longer valuable to do so provides an intuitive basis for this assumption.

Using equation (11) together with our assumption of free entry into the market of unmatched retailers, $V_{do} = 0$, implies that

$$\frac{c_{do}}{\chi\left(\kappa_{do}\right)} = \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right). \tag{19}$$

This equation defines the equilibrium market tightness, κ_{do}, that equates the expected cost of being an unmatched retailer, on the left, with the expected benefit from matching, on the right. In defining equation (19), we remove the maximum over V_{do} and $M_{do}\left(\varphi\right)$ from equation (11) and simply integrate from the threshold productivity level defined by equation (18). This simplification is possible as long as $M_{do}(\varphi)$ is strictly increasing in φ so that the ex-post value of being matched is strictly increasing in producers' productivity. In appendix A.7 we prove this result. We emphasize that equation (19) does not inform the binding productivity threshold $\bar{\varphi}_{do}$, which is solely determined by equation (18).

To get intuition from equation (19), notice that as the expected benefit (the right-hand side) from retailing rises, free entry implies that retailers enter the search market, which raises market tightness, $\kappa_{do} = v_{do}N_d^m/u_{do}N_o^x$, and, through congestion effects, reduces the rate at which searching retailers contact searching producers, $\chi\left(\kappa_{do}\right)$. This increases retailers expected cost of search (the left-hand side). Hence, free entry ensures that V_{do} is zero at all times and that κ_{do} always satisfies equation (19). With free entry into retailer search, market tightness, κ_{do}, is finite if and only if retailers' search cost, c_{do}, is positive. If searching for producers was free ($c_{do} = 0$) but matching was associated with positive expected payoff, then free entry would lead to an infinite number of retailers in the economy driving producers' finding rate to infinity. Conversely, if there were an infinite number of retailers in the search market, then the flow cost of search must be zero. We provide a formal proof for this result in appendix A.8.

This result, together with equation (19), highlights that retailers' cost of searching for producers, c_{do}, along with our assumption of free entry into retailing is at the heart of our model. As the retailer cost $c_{do} \to 0$, producers find retailers instantly, relieving the search friction.

One way we motivate goods-market frictions is from survey reports of the high cost of "identifying the first contact" and "establishing initial dialogue" reported by producers (Kneller and Pisu, 2011). Our approach focuses on the role of retailer search costs as the origin of the search friction. However, any reported producer costs, which in our model are captured by the effective entry cost in equation (17), are influenced by equilibrium variables, and in particular market tightness, κ_{do}. Therefore, retailers' flow search costs, c_{do}, will affect producers' equilibrium costs as well.

Free entry also interacts with assumptions about how firms of both types come into existence. We describe those assumptions in detail in appendix A.9, showing in appendix A.9.1 that, for retailers, free entry into search implies free entry into existence. In appendix A.9.2 we consider the alternative assumptions of free entry into production and free entry into search for producers and show that those yield additional restrictions on equilibrium market tightness. We find our baseline approach of setting $V_{do} = 0$ to be a natural starting point, but other approaches lead to similar effects of search frictions, and the major implications of our paper remain the same.

3.5 Matching in equilibrium

In the steady state, there exists a set of unmatched producers that are actively looking for a retail partner and unmatched retailers that are actively looking for a producer. These steady-state fractions of unmatched retailers and producers correspond to frictional unemployment and unfilled vacancies in the labor literature, and will be positive as long as the finding rates are finite and the separation rate is non-zero. The mass of producers that are matched to retailers and selling their products is $(1 - u_{do} - i_{do})\, N_o^x$, in which a fraction u_{do} are unmatched and actively searching for retailers and a fraction i_{do} choose not to search and therefore remain idle.

In steady state, the flows into the unmatched-producer state must equal the outflows. In any given instant, $(1 - u_{do} - i_{do})\, N_o^x$ matched producers separate exogenously at rate λ. Consequently, the inflow into the unmatched state is $\lambda\,(1 - u_{do} - i_{do})\, N_o^x$. Flows out of this state are $\kappa_{do}\chi\,(\kappa_{do})\, u_{do}N_o^x$ because $u_{do}N_o^x$ producers find matches at rate $\kappa_{do}\chi\,(\kappa_{do})$. Setting these flows equal to each other and re-arranging yields:

$$\frac{u_{do}}{1-i_{do}} = \frac{\lambda}{\lambda+\kappa_{do}\chi\left(\kappa_{do}\right)}. \tag{20}$$

The fraction of idle producers, i_{do}, that choose not to search is defined by the steady-state productivity threshold, $\bar{\varphi}_{do}$, and the exogenous distribution of productivity:

$$i_{do} = \int_{1}^{\bar{\varphi}_{do}} dG\left(\varphi\right) = G\left(\bar{\varphi}_{do}\right). \tag{21}$$

The fraction of producers that are active, $1-i_{do}$, corresponds to the labor force participation rate in the labor literature. While u_{do} is the fraction of producers that are unmatched, $u_{do}/\left(1-i_{do}\right)$ is the fraction of active producers that are unmatched and is equivalent to the labor unemployment rate, which is characterized as the fraction of the labor force that is actively searching for a job. Equation (20) implies different predictions about the extensive margin relative to standard trade models because in our model some highly productive varieties are endogenously and randomly unmatched. In this way, we provide a search theoretic explanation for what Armenter and Koren (2014) refer to as "balls-and-bins" facts about the extensive margin of trade.

4 Model aggregation and general equilibrium

In this section, we discuss the aggregate resource constraint, the ideal price index, and general equilibrium. We also present a graphical depiction of the general equilibrium, derive the gravity equation, discuss the efficiency properties of the model, and show that our model nests a standard trade model without search frictions.

4.1 Aggregate resource constraint

The aggregate resource constraint in this economy can be expressed using either the income or expenditure approach to aggregate accounting. Typically, models of international trade highlight the income perspective. We find it more natural to focus on the expenditure approach:

$$\begin{aligned} Y_d = & \underbrace{p_d\left(1\right)q_d\left(1\right) + \sum_{k=1}^{O}\left(1-\frac{u_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}} p_{dk}\left(\varphi\right)q_{dk}\left(\varphi\right)dG\left(\varphi\right)}_{\text{Aggregate consumption } (C_d)} \\ & + \underbrace{N_d^x e_d^x + \sum_{k=1}^{O}\kappa_{dk}u_{dk}N_k^x c_{dk} + u_{kd}N_d^x\left(l_{kd}+s_{kd}\kappa_{kd}\chi\left(\kappa_{kd}\right)\right) + \left(1-u_{kd}-i_{kd}\right)N_d^x f_{kd}}_{\text{Aggregate investment } (I_d)}. \end{aligned} \tag{22}$$

Consumption expenditure, C_d, is the total resources devoted to consumption of both the homogeneous good and the differentiated varieties, evaluated at final consumer prices. Investment expenditure, I_d, is the resources devoted to creating producing firms, to creating retailer-producer relationships, and to paying for the per-period fixed costs of production.[8]

To account for all resources in the economy, we assume all profits earned and costs incurred by firms for investment and production, including iceberg transport costs, are paid to labor. Therefore, we do not have iceberg costs that "melt away" in transit or that are levied and then wasted by the government. Our structure ensures that changing iceberg costs do not change total resources but instead only introduce distortions. An alternative and identical setup would be to assume that iceberg costs are not paid by firms to workers but are instead levied by the government and then rebated to consumers as lump-sum transfers, which is related to the approach in Irarrazabal, Moxnes, and Opromolla (2015). In that setting, both the expenditure and income approaches would include a government term and aggregate profits would be reduced by the amount of the government's revenue, but total payments to labor would remain the same.

We also treat total payments to idle producers, $\sum_{k=1}^{O}(1-i_{kd})\,N_d^x h_{kd}$, as balanced lump-sum transfers. They enter negatively in the expenditure approach as a lump-sum tax on consumers or firms and enter positively as an additional lump-sum expenditure by the government. As such, these cancel out on the expenditure side of the accounting identity. Finally, we impose balanced trade, so that net exports do not appear in the accounting identity (22).

Total resources paid to labor are defined by $Y_d = w_d L_d (1+\pi)$, in which L_d is the exogenous labor endowment, w_d is the equilibrium wage, and

$$\pi = \frac{\Pi}{\sum_{k=1}^{O} w_k L_k}, \tag{23}$$

is the dividend from a share of global profits, Π. Profits arise because we restrict the number of producers, N_d^x, to be proportional to aggregate consumption expenditure, C_d. We assume that each worker in country d owns w_d shares of a global mutual fund that owns all producers and redistributes profits as in Chaney (2008). This ownership structure simplifies our model allowing many of our aggregate equilibrium expressions to be solved in closed form but does not affect our main conclusions. In appendix A.9.2 we consider the alternative

[8] The mass of producers that are matched to retailers and selling their products is $(1-u_{do}-i_{do})\,N_o^x$. Producers that are idle or searching for retailers but are currently not in a business relationship do not contribute to aggregate output, consumption, or prices. The integral term times $(1-i_{do})^{-1}$ captures the conditional average sales of producers that have productivity above the cutoff necessary to match. Another way to see that all aggregate variables must be scaled in this way is to compute the mass of matched producers $[(1-u_{do}-i_{do})\,/\,(1-i_{do})]\,N_o^x \int_{\bar{\varphi}_{do}}^{\infty} dG(\varphi) = (1-u_{do}-i_{do})\,N_o^x$.

assumption of free entry into production for potential producers. Appendix A.10.1 details our assumptions about the number of producers. Additional details about the income and expenditure approaches to accounting, resources available for consumption and investment, and the global mutual fund are included in appendix A.10.2. The per-capita dividend, π, is proportional to global consumption because $\Pi = \alpha C/\sigma$.

4.2 The ideal price index for differentiated goods

We can move from indexing over the unordered set of varieties in equation (3) to the distribution of productivities using the steps in appendix A.11.1. We can then use the optimal final sales price that results from Nash bargaining over quantity given in equation (15) along with the productivity threshold from (18) to derive the price index for differentiated goods in country d:

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d, \tag{24}$$

in which $\lambda_2 \equiv \left(\theta/\left(\theta-(\sigma-1)\right)\right)^{-\frac{1}{\theta}} \left(\sigma/\alpha\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(C/\left(1+\pi\right)\right)^{-\frac{1}{\theta}}$, $C = \sum_{k=1}^{O} C_k$ is global consumption, and $\rho_d \equiv \left(\sum_{k=1}^{O} \left(C_k/C\right)\left(1-u_{dk}/\left(1-i_{dk}\right)\right)\left(w_k\tau_{dk}\right)^{-\theta} F_{dk}^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$. More details appear in appendix A.11.2 and to conserve on notation, we will sometimes refer to $F\left(\kappa_{do}\right)$ as F_{do}. Equation (24) closely resembles the price index in Chaney (2008, equation 8) and our model also includes a "multilateral resistance" term, ρ_d.

Importantly, the introduction of search frictions makes the price index a function of the consumption weighted average of the equilibrium matched rates of all producers throughout the world in addition to the usual iceberg and entry costs. Introducing search frictions increases the price level by increasing the multilateral resistance term because the fraction of producers that are matched is always less than one $1-u_{do}/\left(1-i_{do}\right) \leq 1$. Search frictions mute the effect of tariff changes on the price index because these changes only affect matched firms. We quantify the effect of search frictions on the response of the consumption share and the price index to tariff changes in section 7.2.

4.3 General equilibrium

A steady-state general equilibrium consists of threshold productivities, $\bar{\varphi}_{do}$, and market tightnesses, κ_{do}, $\forall do$, aggregate consumptions, C_d, and wages, w_d, $\forall d$, and the per-capita dividend, π, which jointly satisfy the zero-profit conditions (equation 18), the free-entry conditions (equation 19), the aggregate resource constraints (equation 22), the no-arbitrage condition for the freely traded homogeneous good (section 2.2.2), and redistribution of profits via dividend (equation 23). The exogenous parameters are β, λ, r, η, ξ, θ, σ, α, e_d^x, L_d, c_{do}, f_{do}, h_{do}, l_{do}, s_{do}, and τ_{do}, in which d and o vary by countries. We elaborate on this definition in appendix A.12 and provide graphical intuition in the next section.

4.4 A graphical depiction of the general equilibrium

We depict the general equilibrium with five graphs: three for the search market and two for the trade market. These heuristic graphs are for an arbitrary good, φ, bilateral market do, and country d, so that our discussion is without loss of generality. The equilibrium in our model exists and is unique as shown in each figure with details in appendix A.13. All endogenous variables are jointly determined.

First, figure 1a depicts the equilibrium retailers' expected negotiated cost, $\mathbb{E}_{\varphi}\left[n_{do}\left(\varphi\right) q_{do}\left(\varphi\right)\right]^{*}$, and goods-market tightness, κ_{do}^{*}, in the do search market. The retailers' expected negotiated cost curve (derived from equation 14 in appendix A.13.1) slopes up because a tighter market means that it is easier for unmatched producers to find retailers. This raises producers' outside option and allows them to negotiate a higher price. The retailers' free entry curve (equation 19) slopes down because a higher expected negotiated cost lowers the value of being a matched retailer and so leads to less retailer entry, lowering market tightness. Figure 1a is analogous to Pissarides (2000, figure 1.1), which depicts equilibrium wage and labor market tightness.

Second, figure 1b depicts the equilibrium negotiated price for one good in the do market, $n_{do}^{*}\left(\varphi\right)$, taking as given goods-market tightness in the do market, κ_{do}^{*}. The negotiated price curve (equation 14) slopes up for the same reason that the retailers' expected negotiated cost curve slopes up in figure 1a: a tighter market means that it is easier for unmatched producers to find a retailer. This raises producers' outside option and enables them to negotiate a higher price. The negotiated price of any single atomistic variety does not affect goods-market tightness so the market tightness curve is a vertical line.

Third, figure 1c depicts the equilibrium final sales price, $p_{do}^{*}\left(\varphi\right)$, and the quantity traded within a relationship, $q_{do}^{*}\left(\varphi\right)$. The demand curve (equation 2) slopes down because demand for a particular variety falls when its price rises. Monopolistic competition then implies that the marginal revenue curve slopes down as well. The marginal cost curve is not a function of quantity because producers' cost function (equation 5) is linear in quantity.

Fourth, figure 1d depicts the equilibrium per-capita dividend from a share of global profits, π^{*}, and the threshold productivity, $\bar{\varphi}_{do}^{*}$. The per-capita dividend (equation 23) does not depend on the threshold productivity because it is proportional to global consumption as mentioned in section 4.1. The producer search threshold curve (equation 18), slopes down because a higher dividend, π, implies less competition and fewer producers, $N_{d}^{x} = C_{d}/\left(1+\pi\right)$, and a higher price index, P_{d}. Because goods are imperfect substitutes, a higher price index raises the profitability of each variety making the threshold producer less productive. This threshold productivity curve is the familiar zero-cutoff profit curve in Melitz (2003, figure 1).

Finally, figure 1e depicts the equilibrium wage, w_{d}^{*}, and consumption, C_{d}^{*}. The consumption curve slopes up because the expenditure and income approaches to national accounting (equations 22 and $Y_d = w_d L_d (1 + \pi)$, respectively) imply that a higher wage generates higher income and higher consumption. As in Chaney (2008), assumptions about the homogeneous good (described in section 2.2.2) imply that $w_d^* = 1\ \forall d$ so that the wage curve is a horizontal line.

4.5 The gravity equation

The gravity structure in our model, albeit more complicated, is similar to the gravity structure common to many trade models. Total imports by destination d from origin o in the differentiated goods sector is the total value of all imported varieties evaluated at negotiated prices. In our context, these imports are given by the following proposition.

Proposition 1. *The gravity equation in our model is:*

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right)\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) \alpha \left(\frac{C_o C_d}{C}\right)\left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}, \tag{25}$$

in which the fraction of matched exporters, $1 - u_{do}/\left(1 - i_{do}\right)$*, and the import markup,* $1 - b\left(\cdot\right) = 1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - \left(\sigma - 1\right)\right)\right)$ *with* $\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right)s_{do}$*, reduce imports relative to a model without search frictions.*

Proof. See appendix A.14.1. □

The main message is clear: Search frictions reduce trade flows in three ways. First, search frictions give rise to a fraction of unmatched exporters, $u_{do}/\left(1 - i_{do}\right) \in [0, 1]$. Second, trade flows are diminished because negotiated import prices, n_{do}, are lower than final sales prices, p_{do}, and imports are computed using negotiated import prices. These lower import prices lead to the endogenous import price markup term, which reduces imports because $1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) \in \left[\left(\sigma - 2\right)/\left(\sigma - 1\right), 1\right]$. Third, search frictions reduce imports because of the negative exponent on the effective entry cost, F_{do}, which is increasing in search frictions, as shown in equation (17). We present further details in appendix A.14.2.

Even if imports are measured at final sales prices, as assumed in the typical gravity equation, search frictions have a significant effect on final consumption. By evaluating imports at final sales prices, p_{do}, instead of negotiated prices, n_{do}, we obtain

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \alpha \left(\frac{C_o C_d}{C}\right)\left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}, \tag{26}$$

as shown in appendix A.14.3. This equation defines consumption expenditure in destination d on differentiated goods produced in origin o. Even without the difference between final and import prices caused by $b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)$, search frictions lead to a mass of unmatched and searching producers $u_{do}/\left(1-i_{do}\right)$, which lowers consumption. Search frictions also affect imports through the effective entry cost, F_{do}, but with the same exponent as existing models.

Our gravity equation (25) implies that international search frictions are consistent with nations trading too much with themselves and too little with each other—sometimes called the "mystery of missing trade" (Rauch, 2001). This mystery implies that trade barriers are large (without barriers, world trade would increase more than fivefold (Eaton and Kortum, 2002, p. 1770)).

Subtracting equation (25) from equation (26) gives total period profits accruing to importers in matched relationships, Π^m_{do}, because consumption expenditure must equal imports plus the period profits of matched importers, $C_{do} = IM_{do} + \Pi^m_{do}$ (appendix A.14.4).

Total period profits to importers determines the expected benefit of retailing in equation (19) and pins down market tightness because
$\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) = \Pi^m_{do}/\left(r+\lambda\right)\left[1-u_{do}/\left(1-i_{do}\right)\right] N^x_o$. Hence, the expected value of becoming a matched retailer is equal to the discounted average flow profits to retailers. Despite the value of posting a vacancy being driven to zero by free entry, $V_{do}=0$, flow profits, Π^m_{do}, are always positive as long as retailers' search costs, c_{do}, are positive, so importers can recoup the costs expended while searching.

4.6 Efficiency

Indirect utility (welfare) in country d when preferences are homothetic, as they are in our model, is defined by real consumption expenditure, $W_d = C_d/\Xi_d$, in which C_d is consumption expenditure in country d (defined in equation 22) and Ξ_d is the ideal price index (defined in section 2.1), as shown in appendix A.15. We define global welfare as the sum of welfare over all countries: $\sum_d W_d$. The global social planner maximizes global welfare.

The decentralized equilibrium in our model is not efficient in general, so that global welfare in the decentralized equilibrium does not necessarily attain the global welfare in the social planner's solution. Our model has the standard matching externality because retailers and producers do not internalize how searching affects equilibrium matching probabilities. Our model also has participation and output externalities because the threshold producer does not internalize their effect on average match productivity, as in Albrecht, Navarro, and Vroman (2010) and Julien and Mangin (2017). Because our decentralized economy has many externalities, it differs from standard labor search models and the Hosios (1990) condition, which sets producers' bargaining power, β, equal to the matching elasticity, η, does not ensure efficiency. Additionally, with many search markets, we conjecture that adjusting one bargaining parameter cannot simultaneously internalize the externalities in all search markets. If bargaining parameters varied by *do* market, we conjecture that there exists a generalized Hosios condition that internalizes the matching, participation, and output externalities, as in Mangin and Julien (2020) and Brancaccio, Kalouptsidi, Papageorgiou, and Rosaia (2020b). Moreover, because the trade part of our model is similar to the model in Dhingra and Morrow (2019), their results suggest that, together with a generalized Hosios condition, our decentralized equilibrium would attain the social planner's solution. We leave formally characterizing the efficiency properties of our model to future work. We discuss efficiency in more detail in appendix A.16 and show that adjusting one bargaining power cannot simultaneously internalize the externalities in all search markets in appendix figure A1.

4.7 Nesting trade models without search frictions

Our model nests trade models without search frictions if and only if retailers' search costs are zero, $c_{do} = 0 \ \forall do$, as discussed in section 3.4. Our model's equilibrium definition differs from the definitions in trade models without search only in that we introduce market tightness, k_{do}. When search costs are zero, free entry into product vacancies leads to infinite market tightness and instantaneous matching for producers. Instantaneous matching implies that all producers are matched (equation 20), as in a standard trade model without search frictions.

In particular, our model exactly reproduces Chaney (2008) if and only if retailers' search costs are zero, and we make the same parameter value restrictions that he does $(s_{do} = h_{do} = e^x_d = 0, \forall d, \text{ and}, \forall o)$. We demonstrate this equivalence by showing that all equilibrium equations are the same. If retailers' search costs are zero, market tightness is infinite and the negotiated price (14) attains the final sales price, given by equation (15). There is, in effect, no intermediate retailer; producers sell their goods directly to the final consumer at price p_{do}. Instant contacts for producers imply that the effective entry cost (equation 17) equals the fixed cost of production, $F_{do} = f_{do}$, and our threshold productivity expression (equation 18) coincides with Chaney (2008, equation 7). With no search costs, the only investment expenditure in the aggregate accounting (equation 22) is the fixed cost of production and total income $Y_d = (1 + \pi)\, w_d L_d$, which also matches Chaney (2008, equation 9). Our assumptions about the homogenous good imply that $w_d = 1 \ \forall d$ and the per-capita dividend is determined by equation (23). With the same equations defining the equilibrium variables, our ideal price index (equation 24) and gravity equation (proposition 1) would coincide with equations 8 and 10, respectively, in Chaney (2008).

5 Changes to welfare, trade flows, and the margins of trade

In this section, we use our model to derive analytic expressions for welfare changes in response to foreign shocks, the consumption and trade elasticities, and the intensive and extensive margins of trade adjustment to search cost changes.

5.1 Welfare changes in response to foreign shocks

In this section, we discuss how adding search frictions changes the response of welfare to foreign shocks. We relate this to ACR, who show that, in a large class of trade models, welfare (indirect utility) changes can be summarized by two sufficient statistics: the change in the domestic consumption share in response to a shock and the elasticity of trade with respect to variable trade costs.

Definition 1. *Define a foreign shock in country d as a change from $(\mathbf{L}, \mathbf{e^x}, \mathbf{f}, \mathbf{c}, \mathbf{h}, \mathbf{l}, \mathbf{s}, \tau)$ to $(\mathbf{L}', \mathbf{e^x}', \mathbf{f}', \mathbf{c}', \mathbf{h}', \mathbf{l}', \mathbf{s}', \tau')$ such that $(L_d, e^x_d, f_{dd}, c_{dd}, h_{dd}, l_{dd}, s_{dd}, \tau_{dd}) = (L'_d, e^{x\prime}_d, f'_{dd}, c'_{dd}, h'_{dd}, l'_{dd}, s'_{dd}, \tau'_{dd})$.*

Proposition 2. *Assume that: 1) $l_{dd} = -h_{dd}$ so that $F(\kappa_{dd})$ is a parameter, 2) the number of producers in d do not change so that $d\ln(N^x_d) = 0$, and 3) productivity, φ, has a Pareto distribution given by equation (6). Then, the change in welfare associated with any foreign shock in country d in our model can be computed as*

$$\hat{W}_d = \hat{\lambda}_{dd}^{-\frac{\alpha}{\theta}} \left(\widehat{1 - \frac{u_{dd}}{1 - i_{dd}}}\right)^{\frac{\alpha}{\theta}} \hat{C}_d^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)}, \tag{27}$$

in which $\hat{x} \equiv x'/x$ denotes the change in any variable x between the initial and the new equilibrium, $\lambda_{dd} \equiv C_{dd}/C_d$ is the share of country d's total expenditure on differentiated goods produced domestically.

Proof. Appendix B.1 derives the proof with the general result in B.1.6 and proposition 2 in B.1.7. □

Equation (27) states that the change in welfare in country d, $\hat{W}_d$, is a function of the changes in the share of domestic expenditure at final prices, $\hat{\lambda}_{dd}$, changes in the rate at which domestic producers are matched in the domestic market, $1 - \widehat{u_{dd}/(1 - i_{dd})}$, and the change in consumption itself, $\hat{C}_d$. This proposition predicts welfare changes before, ex-ante, or evaluates welfare changes after, ex-post, a foreign shock. Ex-ante welfare prediction also requires predictions about the responses of endogenous variables to exogenous foreign shocks. Ex-post evaluation takes the response of endogenous variables as given.

There are a few differences between the ACR welfare expression and equation (27). First, knowing only changes in the consumption ratio, λ_{dd}, and the parameters α, θ, and σ is insufficient for ex-post welfare analysis. One would also need to know the changes in the matched rate and changes in the level of consumption. The change in consumption enters into equation (27) but not the welfare equation in ACR because search and sunk costs imply that profits are not proportional to output. Second, if the rates at which partners find one another are exogenous parameters and profits are proportional to output, equation (27) collapses to the expression in ACR and any welfare effects are the same as in the standard model. Sending the search cost, c_{do}, to zero would also result in the standard expression as long as profits are proportional to output. Third, the matched rate in equation (27) could serve to attenuate the welfare change in response to a change in variable trade costs in comparison with the standard model. Consider, for example, the effect of destination d raising tariffs on products from origin o in a model with search. Higher tariffs result in a higher price index, which makes being a retailer in the domestic market more valuable and induces more retailers to enter the domestic market. With more retailers in the market, the rate at which domestic producers find domestic partners increases, and the matched rate, $1 - u_{dd}/\left(1 - i_{dd}\right)$, increases. A higher domestic matched rate attenuates the welfare losses from higher tariffs. Fourth, the effects of tariffs could also be attenuated through attenuated changes in the domestic consumption share, $\hat{\lambda}_{dd}$, because tariff changes only affect matched varieties instead of all varieties above the exporting threshold. In section 7.2 we quantify the effects of the matched rate in response to specific foreign shocks in a calibrated version of our model showing that ex-ante welfare attenuation can be quantitatively large. In section 7.3 we show that the formula in ACR understates ex-post welfare changes in response to unilateral tariff changes.

5.2 Consumption and trade elasticities

The general form of the elasticity of trade with respect to variable trade costs is provided by ACR (equation 21) for Melitz (2003) models under slight restrictions on the number of producers and is potentially heterogeneous across do. If productivity, φ, follows equation (6), the source for heterogeneity does not vary across do and the trade elasticity is the negative of the Pareto shape parameter, $\partial \ln\left(IM_{do}/IM_{dd}\right)/\partial \ln\left(\tau_{d'o}\right) = -\theta$ if $d' = d$, and 0 if $d' \neq d$. In this class of models, the trade elasticity and the consumption elasticity are the same, in which consumption is imports evaluated at final sales prices.

It is generally not true, however, that the consumption elasticity needed to evaluate welfare and the trade elasticities are the same (Melitz and Redding, 2015). In our model, they differ because consumption is evaluated at final sales prices, while imports are evaluated at negotiated prices. As a result, consumption and import elasticities differ from each other by the effect that iceberg cost changes have on the endogenous import markups:

$$\frac{\partial \ln\left(IM_{do}/IM_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} + \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} - \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)}, \tag{28}$$

in which d' can differ from d. Because the effects of both do and dd import markup elasticities are weakly negative, the trade elasticity in our model is more negative than the consumption elasticity, meaning that trade shares respond more strongly to iceberg cost changes than consumption shares. Appendix B.2.1 through B.2.13 derive the consumption elasticity, with the general expression presented in appendix B.2.14. Appendix B.2.15

provides a comparison with the standard elasticity without search frictions. Appendixes B.2.16 and B.2.17 derive the elasticity in proposition 3. Appendix B.2.18 shows that markup responses are weakly negative and appendix B.3.1 derives equation (28).

Proposition 3. *Assume that: 1) $l_{do} = -h_{do}$ so that $F(\kappa_{dd})$ and $F(\kappa_{do})$ are parameters, 2) the number of producers in d and o do not change with iceberg trade cost changes so that $\partial \ln (N_d^x) / \partial \ln (\tau_{d'o}) = \partial \ln (N_o^x) / \partial \ln (\tau_{d'o}) = 0$, 3) the matching function elasticity is η from equation (4), and 4) productivity is Pareto with shape θ given by equation (6). Then the elasticity of trade shares to iceberg trade costs in our model with goods-market frictions is given by*

$$\frac{\partial \ln (IM_{do}/IM_{dd})}{\partial \ln (\tau_{d'o})} = \begin{cases} -\theta + \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial \ln \kappa_{do}}{\partial \ln \tau_{d'o}} - \left(\frac{u_{dd}}{1-i_{dd}}\right)(1-\eta)\frac{\partial \ln \kappa_{dd}}{\partial \ln \tau_{d'o}} & \\ +\frac{\partial \ln (1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}))}{\partial \ln (\tau_{d'o})} - \frac{\partial \ln (1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}{\partial \ln (\tau_{d'o})} & if\ d' = d \\ \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial \ln \kappa_{do}}{\partial \ln \tau_{d'o}} - \left(\frac{u_{dd}}{1-i_{dd}}\right)(1-\eta)\frac{\partial \ln \kappa_{dd}}{\partial \ln \tau_{d'o}} & \\ +\frac{\partial \ln (1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}))}{\partial \ln (\tau_{d'o})} - \frac{\partial \ln (1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}{\partial \ln (\tau_{d'o})} & if\ d' \neq d \end{cases} . \tag{29}$$

Proof. See appendixes B.2 and B.3.1. □

Our trade elasticity is heterogeneous across *do* and depends not only on the usual Pareto parameter θ, but also on terms related to the search market and markup effects. First, our trade elasticity depends on the fraction of unmatched producers, the elasticity of matches with respect to the number of searching producers, and the elasticity of market tightness with respect to iceberg costs in the *do* and *dd* product markets. Together, these terms make the trade elasticity in our model as least as negative as the analogous trade elasticity in the class of models from ACR. This results follows for three reasons. First, unmatched rates are weakly positive, the matching elasticity is between 0 and 1, $\eta \in [0, 1]$, and the *do* market-tightness elasticity with respect to iceberg cost is negative. Second, the *dd* market-tightness elasticity is positive. These two market-tightness elasticity terms reduce the trade elasticity relative to many analogous trade models that do not have search frictions. Finally, the effects of both *do* and *dd* price markups on the import elasticity are also weakly negative, which reduces further our trade elasticity. See appendix B.3.2 for details.

We consider the relative magnitude of the matched and markup effects on trade elasticities in section 7.4. Under our calibration, we find that while both these effects make trade more responsive to iceberg costs, the *do* matched rate term has the largest effect and the markup terms effects are more modest. Finally, we point out that a search model with exogenous matching rates or markups will give the same consumption and trade elasticities as a model without search frictions.

5.3 Search costs and the intensive and extensive margins of trade

We decompose the response of imports to changes in search, variable, and fixed costs into intensive and extensive margins in our model. The introduction of search frictions operate mainly through the extensive margin of trade as shown in proposition 4.

Proposition 4. *The intensive and extensive margin elasticities with respect to search costs are given by*

$$\frac{d\ln\left(IM_{do}\right)}{d\ln c_{do}} = \underbrace{\underbrace{(\sigma-1)\frac{d\ln P_d}{d\ln c_{do}} + \frac{d\ln C_d}{d\ln c_{do}}}_{\text{Final sales elasticity}} + \underbrace{\frac{d\ln\left(1-b_{do}\left(\cdot\right)\right)}{d\ln c_{do}}}_{\text{Markup elasticity}}}_{\text{Intensive margin elasticity}} \tag{30}$$
$$+ \underbrace{\underbrace{(\sigma-\theta-1)\left(\left(\frac{1}{\sigma-1}\right)\frac{d\ln F\left(\kappa_{do}\right)}{d\ln c_{do}} - \left(\frac{1}{\sigma-1}\right)\frac{d\ln C_d}{d\ln c_{do}} - \frac{d\ln P_d}{d\ln c_{do}}\right)}_{\text{Threshold elasticity}} + \underbrace{\left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{d\ln\kappa_{do}}{d\ln c_{do}}}_{\text{Matched elasticity}}}_{\text{Extensive margin elasticity}}.$$

Proof. See appendix B.4, which also derives similar decompositions in our model of the effect of iceberg costs, $d\ln\left(IM_{do}\right)/d\ln\tau_{do}$, and fixed costs, $d\ln\left(IM_{do}\right)/d\ln F_{do}$. □

Usual intensive and extensive margin decompositions from, for example, Chaney (2008) and Arkolakis (2010), include the "final sales" and "threshold" elasticities in equation (30). Our decompositions have three additional terms relative to Chaney (2008, pg. 1716). First, search frictions imply a "matched elasticity" margin that captures how changes in tightness, in response to changing trade costs, affect the fraction of matched producers. Second, search frictions imply a markup elasticity that captures how negotiated prices change in response to changes in tightness and trade costs. Finally, trade costs affect market tightness which changes the effective entry cost and threshold elasticity. Section 7.4 presents numerical results from this decomposition for our baseline calibration.

By assuming that all general equilibrium effects are small, we can approximate the effect of search costs on imports from proposition 4 as

$$\frac{d\ln\left(IM_{do}\right)}{d\ln c_{do}} \approx \underbrace{-\left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{1-\eta}{\eta}\right)}_{\text{Matched elas.=Ext. margin elas.}}, \tag{31}$$

in which $d\ln\kappa_{do}/d\ln c_{do} \approx -1/\eta$. Appendix B.4 elaborates on this approximation.

Search costs affect trade flows through the extensive margin and via the matched elasticity margin, specifically. While we do not specify the source of retailers' search costs in our model, linguistic proximity could proxy for them. As such, equation (31) is consistent with the fact that common language increases trade mainly through the extensive margin (Lawless, 2010; Egger and Lassmann, 2015).

6 U.S. and China calibration and model fit

We use data for China and the United States in 2016 to calibrate our model, but our procedure can be generalized to include more trading partners or a different time period. The calibration proceeds in two steps. First, we externally calibrate parameters that can be normalized or that are standard in the literature. Table 1 summarizes these parameters. We could find alternative values for retailers' flow cost of search such that the equilibrium is unchanged for different values of the matching efficiency, ξ, and so we normalize this efficiency to one (Shimer, 2005). We benchmark the producers' bargaining power, β, at 0.5 (Drozd and Nosal, 2012; Eaton et al., 2014, 2016). We set the elasticity of substitution between differentiated varieties, σ, to four, consistent with median estimates in Broda and Weinstein (2006), and implying a final sales price markup over marginal production cost of 33 percent. We set the Pareto parameter to be consistent with the firm-size distribution estimate from Axtell (2001) of $1.06 = \theta / (\sigma - 1)$, which implies that θ equals 3.18. We set the fraction of consumption expenditure spent on differentiated goods, α, to 0.5.

We parameterize the iceberg cost as a function of tariffs and distance, $\tau_{do} = a_1^o \times \text{tariff}_{do} \times \text{distance}_{do}^{a_2}$. Effective ad valorem tariff rates on Chinese imports from the United States are 6.3 percent and on U.S. imports from China are 2.9 percent in 2016 according to the World Integrated Trade Solution database (WB, 2019b). There are no domestic tariffs so that $\text{tariff}_{uu} = \text{tariff}_{cc} = 1$, in which subscripts u and c denote the United States and China. The symmetric distance between the U.S. and China is the population-weighted distance normalized by the U.S. internal distance from Head, Mayer, and Ries (2010). The parameters a_1^o and a_2 will be internally calibrated. Parameters of the model are at annual frequency and we set the annual interest rate to 5 percent.

The second step in our approach internally calibrates the remaining parameters by solving an MPEC following Dubé et al. (2012) and Su and Judd (2012). MPEC simultaneously recovers parameters of a model and solves for the accompanying equilibrium endogenous variables. Our parameters will minimize the distance between moments in the data and the model subject to constraints that define the model's equilibrium:

$$\begin{aligned} \left(\hat{\Omega}, \hat{\Phi}\right) &= \underset{\Omega,\Phi}{\arg\min} \left(M\left(\Omega,\Phi\right) - M\right)' W \left(M\left(\Omega,\Phi\right) - M\right) \\ \text{subject to } & \Gamma\left(\Phi;\Omega\right) = 0 \\ & \Psi\left(\Phi,\Omega\right) \leq 0, \end{aligned} \tag{32}$$

in which $\hat{\Omega}$ are internally calibrated parameters, $\hat{\Phi}$ are endogenous variables solving the model at parameter values $\hat{\Omega}$, $M\left(\Omega,\Phi\right)$ are the moments implied by the model at parameters Ω and endogenous variables Φ, M are observed moments, and W is a weighting matrix. $\Gamma(\Phi;\Omega)$ captures the equilibrium conditions defined in section 4.3, which depend on the endogenous variables, Φ, for any given set of parameters Ω, and these conditions must hold with equality. $\Psi(\Phi,\Omega)$ defines nonlinear equilibrium and parameter inequality constraints, examples of which are that the idle rate i_{do} cannot be negative in equilibrium and that the elasticity of matches with respect to the number of searching producers $\eta \in [0,1]$. Appendix C.1 describes these inequality constraints and contains numerical details related to solving equation (32).

6.1 Intuition for parameter identification

The parameters that solve equation (32) are jointly determined by all the moments, but in this section we discuss intuition for parameter identification by relating certain moments to particular parameters. Table 1 summarizes this discussion.

The search frictions in our model are governed by retailers' flow search cost, c_{do}. If the fraction of matched exporters is low, it implies that there are few searching retailers, market tightness is low, and that international search costs are high. Consequently, we use the fact that 21 percent of Chinese firms export (WB, 2018) and that six percent of U.S. firms export to China (CB, 2016a,b) to identify c_{uc} and c_{cu}, respectively. Eaton et al. (2014) and Eaton et al. (2016) also use the fraction of firms that export to identify search model parameters. We use manufacturing capacity utilization to inform the level of domestic search frictions in goods markets, as in Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2017) and Petrosky-Nadeau et al. (2018). We target 75 and 74 percent manufacturing capacity utilization in the United States and China in 2016, respectively, to inform c_{uu} and c_{cc} (FRB, 2020; NBSC, 2016a). We also assume that international search costs are simply the domestic search cost plus a symmetric international premium so that $c_{uc} = c' + c_{uu}$, $c_{cu} = c' + c_{cc}$, and $c' \geq 0$. This symmetry assumption implies, for example, that the cost a Chinese retailer faces to search for a U.S. producer is the same as the cost that that retailer would face to find a Chinese producer plus c'. We present further details about identification of the retailers' flow search costs in appendix C.2. Our quantitative results are robust to search costs that are far below our baseline calibration (section 7).

Targeting log-linear estimates of the trade elasticity informs the elasticity of matches with respect to the number of searching producers, η. This moment is informative because as the matching elasticity increases to one, producers' contact rate becomes unresponsive to changes in market tightness, as shown in appendix C.3. Without an endogenous response in the producers' matched rate, trade becomes less responsive to variable trade costs and the trade elasticity increases. We target a trade elasticity of -6 based on a range of empirical estimates, which vary between -4 and -10 (Eaton and Kortum, 2002; Anderson and van Wincoop, 2004; Romalis, 2007; Imbs and Mejean, 2015).

The average duration of a Chinese and U.S. trading relationship is about one year

(Monarch and Schmidt-Eisenlohr, 2018, Figure 9), which identifies our separation parameter, λ, because average match duration in the model is $1/\lambda$. This observed expected duration is also broadly consistent with survival probabilities among Colombian-U.S. trading relationships (Eaton et al., 2014).

The Doing Business Indicators database (WB, 2019a) informs the cost of business start ups and the fixed costs of foreign trade, f_{do} (as in di Giovanni and Levchenko, 2012, 2013). We discuss the details of this approach, along with the calibration of l_{do} and s_{do}, in appendix C.4. Because the threshold productivity in equation (18) is defined by the effective entry cost, F_{do}, the quantitative results depend on that cost and less so on its individual components.

Trade in both directions between between China and the United States together with the level of absorption of domestic production (IM_{uu} and IM_{cc}), as well as tariffs and distance between the two countries, identifies a_1^u and the elasticity on distance, a_2. We define IM_{uu} and IM_{cc} as manufacturing value added minus merchandise exports plus merchandise imports similar to Dekle, Eaton, and Kortum (2008).

Labor endowments, L_c and L_u, and the exploration costs, e_u^x and e_c^x, are informed by the levels of gross domestic products (GDPs), aggregate consumptions, and the ratio of consumption to GDP in China and the U.S., as reported in the national accounts of each country (BEA, 2016a; WB, 2016; BEA, 2016b; NBSC, 2016b).

In section 2.2.2, we assume that the minimum draw from the productivity distribution is one. The minimum draw informs the flow value associated with being idle, h_{do}, because we assume that if search costs, tariffs, and the U.S. input cost premium are zero and countries are in autarky then the exporting threshold in equation (18) is at its minimum. These steps are similar to the fixed production cost normalization in di Giovanni and Levchenko (2013). Appendix C.5 has details about this normalization.

6.2 Parameter values

The parameters that solve equation (32) are presented in table 1. We internally calibrate the elasticity of matches with respect to the number of searching producers, η, to 0.58. This value is similar to the elasticity of matches with respect to the number of unemployed workers estimated using labor data (0.5 to 0.7 in Petrongolo and Pissarides, 2001) and suggests that the matching technology might be similar in the two contexts.

Average retailer search costs in domestic (international) markets are about five (11) percent of average annual retailer revenue. In other words, domestic (international) retailers spend about five (11) percent of one year's revenue to form a match that lasts one year on average (because $1/\lambda = 1$). These domestic search costs are similar in magnitude to the labor costs of posting vacancies (Silva and Toledo, 2009). It is intuitive that average international search costs as a fraction of average firm revenue are higher than average domestic search costs. Total retailer search costs paid by all retailers are 0.08 percent and 0.04 percent of GDP in the United States and China, respectively. Average retailer search costs are about \$30 million in domestic markets (uu and cc), about \$80 million in the uc market, and about \$20 million in the cu market. Our calibrated search costs are not directly comparable to estimates from the structural literature that uses micro-level data because the models and parameter identification strategies differ substantially (Eaton et al., 2014).

As we mention in section 6.1, the quantitative results depend on the effective entry cost, F_{do}, and less so on its individual components among which are f_{do} and h_{do}. Our calibration implies that F_{do} is about 25 percent of average annual producer revenue in domestic markets. Effective entry costs are 87 percent of average annual revenue for Chinese producers exporting to the United States. For U.S. producers, effective entry costs are about 13 times higher than average revenue, reflecting that entry barriers are high so that only six percent of U.S. producers export to China. Because the opportunity cost of remaining idle, h_{do}, is the largest component of F_{do} in our calibration, changing the fixed cost, f_{do}, or relaxing our restrictions on producer search costs, l_{do}, or sunk costs, s_{do}, implies similar quantitative results as in section 7.

In our model it is more expensive to create a variety than it is to pay the effective entry cost to begin searching. For example, in the United States, exploration costs are about six times higher than the effective entry cost in the uu market. These results are similar to di Giovanni and Levchenko (2012), who find that exploration costs are 15 times higher than export costs in the United States.

The calibrated elasticity of trade costs with respect to distance, a_2, is 0.06. This implies that our trade elasticity with respect to distance is -0.4, which lies within the wide rage of estimates in a meta-analysis by Disdier and Head (2008). Our calibrated iceberg trade costs, τ_{do}, are 1.3, 1.5, 1.2, and 1.0 in the uu, cu, uc, and cc markets, respectively. These iceberg costs are broadly in line with values used in di Giovanni and Levchenko (2012) and Irarrazabal et al. (2015).

6.3 Model fit

Table 2 presents the moments from the model using the baseline calibration from table 1 and shows that the model matches the data well. The model implies domestic absorption of production in China (the United States) that is lower (higher) than in the data because we use observed data, which include many countries, to calibrate a model with only two countries and we assume balanced trade, as discussed in appendix C.6. The model provides a realistic economic environment for general equilibrium exercises, a topic we pursue in the next section.

7 Quantitative general equilibrium results

This section presents several quantitative exercises that emphasize the important role of search frictions for changes in aggregate welfare, the trade elasticity, and the margins of trade. Section 7.1 examines how welfare changes when we eliminate search frictions. Section 7.2 decomposes the response of ex-ante welfare to a unilateral tariff using our analytical result from proposition 2. Section 7.3 shows that ex-post welfare evaluation using the formula from Arkolakis et al. (2012) implies welfare changes in response to unilateral tariff changes that are smaller than true welfare changes for any positive search costs. Section 7.4 shows that search frictions, through their effect on the unmatched rate, make trade substantially more responsive to iceberg costs, quantifying our results from proposition 3. Finally, section 7.5 highlights that the trade elasticity with respect to search costs operates through the extensive-matched margin, as suggested by our analytical results in section 5.3 and proposition 4.

7.1 Welfare effects of reducing retailers' search costs

Entirely eliminating domestic and international search frictions raises U.S. welfare by 11.4 percent and Chinese welfare by 13 percent. Reducing international search frictions to domestic levels raises U.S. welfare by 5.6 percent and Chinese welfare by 4 percent. These are sizable effects and are similar in magnitude to the effect of moving to autarky in a simple Armington model with one or multiple sectors (Costinot and Rodríguez-Clare, 2014, table 4.1). Lowering search frictions increases welfare by reducing the price index through reallocating production across countries. This mechanism operates the same way as typical reductions of tariffs in trade models.

To compute the equilibrium in our model without search frictions, we set all parameters to the baseline values listed in table 1, but reduce retailers' search costs to zero in domestic and foreign markets, $c_{do} = 0,\ do = \{uu, uc, cu, cc\}$, reproducing the model of Chaney (2008). Because search is free for retailers they flood the search markets, sending market tightness in each market to infinity. As a result, producers find retailers instantly and the fraction of matched producers in each market rises to 100 percent as shown in table 3 column (2.1).

Column (2.2) of table 3 reports that U.S. imports from China increase by 255 percent, and Chinese imports from the United States rise by about 850 percent. Chinese imports rise more than US imports because eliminating search frictions increases the producer matched rate by 93 percentage points in the CH-US market and by only 81 percentage points in the US-CH market. In contrast to increasing imports, absorption of domestic production, IM_{dd}, in both countries declines by about 28 percent. The reason that domestic production falls but imports rise is that the final sales price indexes of imported goods (P_{do} in equation 3), decline more than the final sales price indexes of domestically produced goods. For example, domestic final prices fall by about 10 percent in both countries, but international final prices fall by 45 percent and 61 percent in the United States and China, respectively. Differentiated goods prices from all sources decline so the price indexes in both countries fall, by 10.2 percent in the United States and 11.5 percent in China. As a result, welfare rises by about 11 percent in the United States and 13 percent in China. Appendix table A1 provides more details.

Eliminating all search frictions is an extreme case, so in the next exercise we set all parameters to the baseline values listed in table 1, but reduce retailers' international search costs to their domestic levels, $c_{do} = c_{dd}$. Column (3.2) of table 3 reports that welfare in the United States is 5.6 percent higher and welfare in China rises by 4 percent. The mechanisms that increase welfare are the same as when search frictions are eliminated in column (2): International prices fall more than domestic prices (which actually rise in this case). Lower international prices and greater imports of those goods serve to reduce the overall price index and raise welfare. Appendix table A1 lists more variables and how they change. Appendix D.1 and appendix table A2 describe how the calibrated model fits observed moments for different levels of search frictions.

7.2 Decomposing the ex-ante welfare response to unilateral tariffs

Search frictions attenuate ex-ante welfare changes in response to a 10 percent unilateral tariff by about 85 percent relative to an environment without search frictions. This occurs because, relative to the standard model, both the domestic consumption share and the domestic producers' matched rate attenuate the response of welfare. This quantifies our analytic results from proposition 2.

At first we set all parameters to the baseline values listed in table 1, but reduce retailers' search costs to zero in domestic and foreign markets, $c_{do} = 0 \; \forall do$, as in the frictionless example of the previous section. Column (1) of table 4 shows that without search frictions, Chinese welfare falls by about 1.5 percent in response to a 10 percent tariff increase on U.S. imports. This reduction in welfare is governed by the 10.3 percent increase in the domestic consumption share, along with the parameters α, θ, and σ, and is consistent with the results in ACR. This model features no search frictions so the domestic matched rate is always one.

Column (2) of table 4 shows that in the model with search frictions, Chinese welfare falls by 0.24 percent when China raises unilateral tariffs on U.S. goods by 10 percent. Decomposing this welfare reduction using our analytic results in proposition 2 suggests that welfare changes for three reasons. First, the domestic consumption share rises by about 1.8 percent because foreign goods are more expensive after the tariff increase and this reduces welfare to 99.7 percent of the pre-tariff level. Second, the tariff raises the Chinese price index, allowing Chinese retailers to earn higher revenue from Chinese consumers. The increased value of being a matched retailer in the Chinese market leads to more retailer entry and raises the domestic matched rate for Chinese producers by 0.2 percent. This higher matched rate serves to attenuate the reduction in welfare caused by the lower domestic consumption share by 0.04 percent. Third, Chinese aggregate consumption changes in both cases but by a trivial amount.

The domestic consumption share response is smaller in the model with search because the matched rate in the CH-US market, which is always less than one, serves to mute the response of the price index to tariff changes (equation 24). For example, extremely high search costs would result in only a few matched firms being affected by tariffs, and would dramatically reduce tariffs' effects on the price index. Moreover, tariff increases endogenously reduce the matched rate in the CH-US search market, which further mutes the price index change relative to a model without search frictions.

Our welfare attenuation result is robust to search costs that are much smaller than our baseline calibration, as shown in appendix D.2. In particular, changes in welfare are still about 25 percent smaller than in the model without search frictions even when search costs are one percent of our baseline calibration (appendix table A3, column 5).

7.3 Ex-post welfare evaluation: Actual vs. ACR

Ex-post welfare evaluation using the formula in ACR implies welfare changes in response to unilateral tariff changes that are smaller than true welfare changes for any positive search costs. The main reason that the ACR formula understates the welfare change is because standard log-linear estimates of the trade elasticity that omit search frictions are negatively biased for θ (appendix C.3).

In our baseline calibration, a 10 percent unilateral tariff increase by China on U.S. imports increases the Chinese import share by 1.8 percent and reduces Chinese welfare by 0.24 percent, as shown in table 5 column (1). The ACR formula is $\hat{W}_c = (\widehat{IM_{cc}/C_c})^{-\alpha/\hat{\theta}}$, in which $-\hat{\theta}$ is taken to be the estimated ex-post log-linear import elasticity. This formula implies a welfare reduction of 0.13 percent, understating the true decline by about 45 percent. In columns (2) through (5) of table 5, we present welfare changes for different levels of search frictions and different Chinese tariff increases on U.S. imports so that the ex-post change in import shares is always the same (1.8 percent) and the ex-post log-linear import elasticity is reestimated. Even when retailers' flow search costs are 1 percent of their baseline value, the ACR formula understates the true welfare decline by about 25 percent. The two welfare changes coincide in a model without search frictions.

7.4 Search frictions increase the responsiveness of trade to tariffs

The trade elasticity in our baseline economy with search is -5.5 relative to -3.2 in a model without search frictions. Trade responds more strongly to tariffs mainly because the international producers' matched rate magnifies the effects of a tariff increase. This quantifies our analytic results from proposition 3 in which we show that the trade elasticity in our model is at least as negative as the analogous elasticity in the Chaney (2008) model.

First we set all parameters to the baseline values in table 1 but remove all search costs ($c_{do} = 0 \ \forall do$). Without search frictions, a 10 percent (9.5 log percent) unilateral tariff on Chinese imports from the United States ($\tau'_{cu} = 1.1 \times \tau_{cu}$) reduces import and consumption shares by about 30 log percent. This reduction implies that the trade and consumption elasticities are -3.18 (column 1 of table 6), which is exactly equal to the negative of the Pareto shape parameter ($-\theta$) that we derive analytically in appendix B.2.16, and matches the predictions in Chaney (2008, p. 1716).

In our model with search frictions, the trade elasticity is -5.47 (column 2 of table 6). Our trade elasticity results are robust to lower levels of search frictions, as shown in appendix D.3. In particular, even with search frictions at one percent of our baseline calibration we find that the trade elasticity is -4 compared to -3.18 in a model without search frictions (table A4, column 5).

The main difference between the trade elasticities in the models with and without search costs is the effect tariff changes have on the fraction of U.S. producers that are matched with Chinese retailers. This elasticity of the matched rate in the cu market with respect to τ_{cu} is -2.12. Higher tariffs reduce the benefit to Chinese retailers of being matched with U.S. producers, leading to less Chinese retailer entry, and fewer U.S. producers matched in the cu search market.

The decomposition shown in proposition 3 also has an indirect protectionism effect that operates through the matched rate in the domestic, cc, market. As tariffs on U.S. imports rise, the Chinese price index increases, making it more valuable to be a matched domestic Chinese retailer, which leads to entry into the domestic retailing market. Greater entry raises the Chinese domestic matched rate for Chinese producers and subtracts from the standard elasticity. (This is the same mechanism that ensures that welfare is attenuated in proposition 2). In the calibration of our model, this protectionism effect is small, as are the effect of tariffs on the number of producers and the effective entry costs. The elasticity for trade flows is slightly more negative than the consumption elasticity because the markup term also change, as shown in equation (28), but these effects are also small.

While the elasticity in our framework with search frictions is more negative than the Pareto shape parameter would imply, it remains within the range of empirical estimates (section 6.1).

7.5 Search costs and the margins of trade

The import elasticity with respect to search frictions operates through the extensive-matched margin and is -0.69. Search frictions raise the variable cost elasticity by changing the elasticity of the extensive margin from -0.18 to -2.37 and make the intensive and extensive margins about equally important for the variable cost elasticity. This elasticity lies between the elasticity with respect to variable costs (-5.2) and with respect to effective entry costs (-0.1).

We decompose the response of Chinese imports from the United States to changes in cu search, variable, and fixed costs into intensive and extensive margins in table 7. This quantifies our analytical decomposition from proposition 4. Search costs, c_{do}, mainly affect imports through the extensive-matched margin, which is much larger than their effect on the other terms in the decomposition (column 1). Equation (31) performs well and approximates that the elasticity with respect to search costs is -0.68, instead of -0.69.

In table 7, columns labelled ‘‘Baseline search costs’’ report results for our baseline calibration including general equilibrium effects. Columns labelled ‘‘Chaney exact’’ include general equilibrium effects but do not have search frictions. Columns labelled ‘‘Chaney approx.’’ report approximate results from Chaney (2008), which ignore general equilibrium effects. See appendix B.4 for derivation of the margins decomposition and discussion of general equilibrium effects.

Search frictions magnify the effect of tariff changes mainly through the extensive-matched margin (columns 2.1 through 2.3). In our model with search, the extensive margin elasticity with respect to variable costs, -2.37, is similar in magnitude to the intensive margin elasticity, -2.83 (column 2.1). In models without search frictions, the extensive margin elasticity is near zero for changes in either variable or fixed trade costs (di Giovanni and Levchenko, 2013), as shown in columns 2.2 and 3.2.

Search cost changes have important implications for changes in imports. The elasticity with respect to variable costs is about 7.5 ($-5.2/-0.7$) times larger than the elasticity with respect to search costs. But the trade elasticity with respect to search costs is about 6.9 ($-0.7/-0.1$) times larger than the elasticity with respect to fixed costs, which is small in models with and without search. Increasing search costs can mimic increasing variable or fixed costs by affecting producers’ matched rates. For example, doubling retailers’ search costs mimics reductions in trade flows and aggregate welfare of a 10 percent increase in bilateral tariffs, as shown in appendix D.4.

8 Conclusion

We propose a framework for studying how the costly formation of international trading relationships affects aggregate quantities. Our framework remains analytically tractable and implies that there exists an endogenous fraction of unmatched producers in general equilibrium. This endogenous fraction of unmatched producers lowers the level of welfare, attenuates the ex-ante response of welfare to foreign shocks, increases the magnitude of the trade elasticity, and operates mainly through the extensive margin of trade. A calibration using U.S. and Chinese data suggests that these effects are quantitatively meaningful.

We propose four directions for future research. First, our model can be extended to include multiple differentiated-goods sectors and relate to the empirical results of Rauch (1999). Second, we have focused on the steady state of the model, but the framework is dynamic and could be extended to include the transitions between steady states. Third, the model can be extended to incorporate endogenous separations, either in the spirit of Jovanovic (1979), which introduces learning about match quality, or Mortensen and Pissarides (1994), which suggests that larger, more productive firms are in more stable trading relationships. Fourth, matching and bargaining protocols other than Nash bargaining, as in Burdett and Judd (1983) and Moen (1997), may present alternative implications for the mass of unmatched varieties relative to our model.

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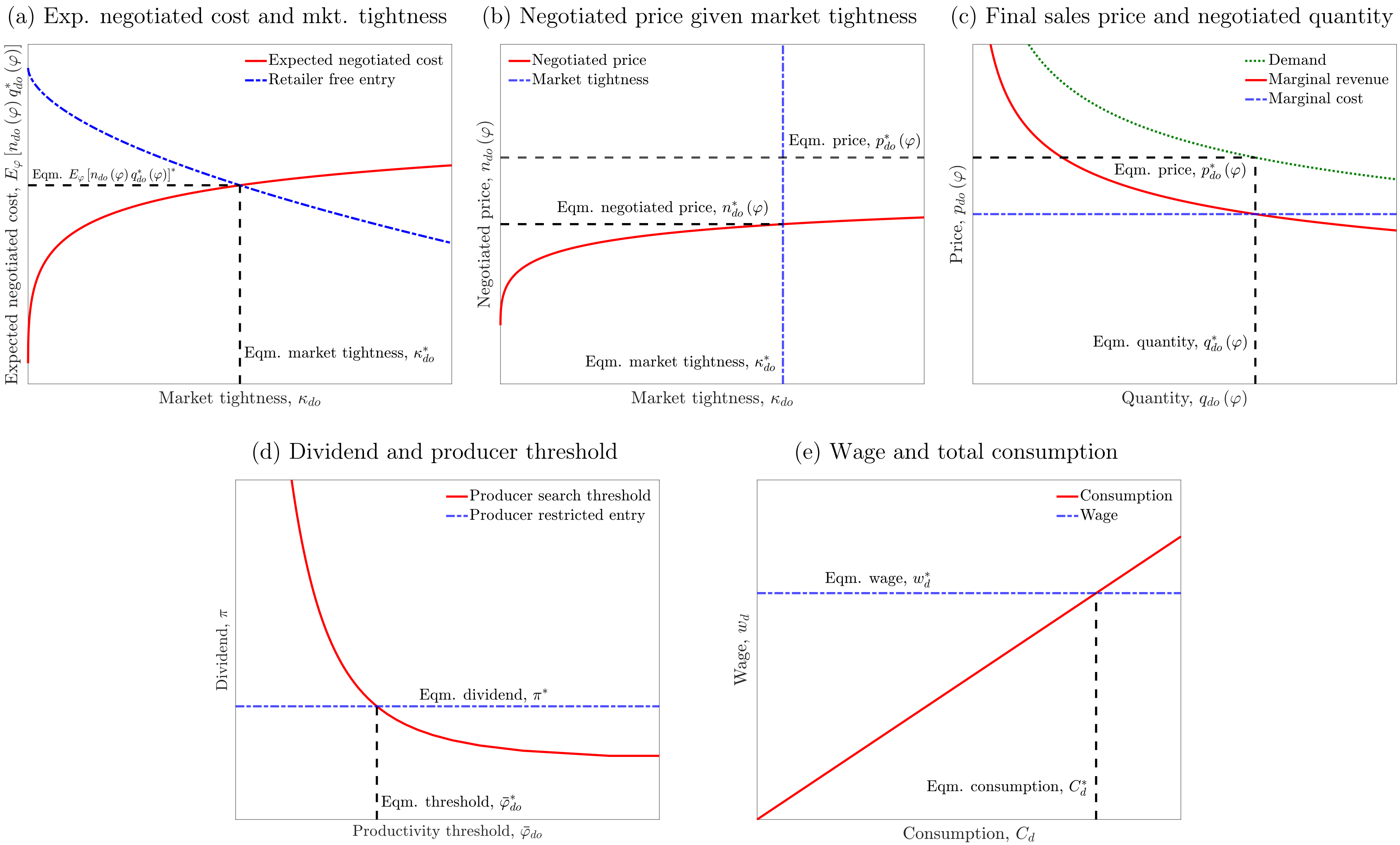
Figure 1: Graphical depiction of the model



Note: We depict the general equilibrium with five graphs: three for the search market (figures 1a, 1b, and 1c) and two for the trade market (1d and 1e). In figure 1a the expected negotiated cost and retailers' entry curves are characterized by equations (14) and (19), respectively. In figure 1b the negotiated price and market tightness curves are characterized by equations (14) and κ^*_{do} in figure 1a, respectively. In figure 1c the demand and marginal revenue curves are determined by equation (2) and the marginal cost is determined from equation (5). In figure 1d the producer restricted entry and the producer search threshold curves are characterized by equations (23) and (18), respectively. In figure 1e the consumption curve is determined by national accounting (equations 22 and $Y_d = w_d L_d (1 + \pi)$) and the wage is determined by the production structure for the homogeneous good ($w_d = 1 \ \forall d$). All endogenous variables are jointly determined. See section 4.4 for a discussion of the five graphs. The equilibrium in our model exists and is unique as shown in each figure with details in appendix A.13.

Table 1: Calibrated model parameters

Parameter	**Value**	**Reason**
Panel A. Externally calibrated parameters		
Efficiency of matching function (ξ)	1	Normalization
Producers' bargaining power (β)	0.5	Benchmark
Elasticity of substitution (σ)	4	Demand curve estimation
Pareto shape parameter (θ)	3.18	U.S. firm size distribution
Cobb-Douglas power (α)	0.5	Benchmark
Tariffs rate faced by US in US (τ_{uu})	1	Normalization
Tariffs rate faced by US in CH (tariff_{cu})	1.063	WITS database
Tariffs rate faced by CH in US (tariff_{uc})	1.029	WITS database
Tariffs rate faced by CH in CH (τ_{cc})	1	Normalization
Distance between US and CH (distance_{cu})	6.03	Relative to U.S. internal distance
Risk-free rate (r)	0.05	5% annual interest rate
Panel B. Internally calibrated parameters		
US domestic average search cost	\$28.9 mil.	US mfg. capacity utilization rate
CH importers' average search cost	\$18.1 mil.	Percent of US firms exp. to CH
US importers' average search cost	\$84.9 mil.	Percent of CH firms exp. to US
CH domestic average search cost	\$39 mil.	CH mfg. capacity utilization rate
US domestic fixed cost (f_{uu})	\$550	Cost of business start up in US
US export fixed cost (f_{cu})	\$683	Fixed foreign trade costs (CH-US)
CH export fixed cost (f_{uc})	\$664	Fixed foreign trade costs (US-CH)
CH domestic fixed cost (f_{cc})	\$28	Cost of business start up in CH
US input cost premium (a_1^u)	1.3	Abs. of dom. prod. (IM_{uu}, IM_{cc})
Effect of distance on trade costs (a_2)	0.06	Imports (IM_{cu}, IM_{uc})
Elasticity of matching function (η)	0.58	Log-linear import elasticity
Separation rate (λ)	1	Sep. rate among trading partners
Labor endowment in US (L_u)	\$18.2 tril	Consumption and GDP in US
Labor endowment in CH (L_c)	\$10.9 tril	Consumption and GDP in CH
US exploration cost (e_u^x)	\$903 mil.	US consumption to GDP share
CH exploration cost (e_c^x)	\$1.5 bil.	CH consumption to GDP share
US producers' idle flow payoff (h_{uu})	\$84 mil.	Absent barriers, no idle US-US firms
US exporters' idle flow payoff (h_{cu})	\$60 mil.	Absent barriers, no idle CH-US firms
CH exporters' idle flow payoff (h_{uc})	\$60 mil.	Absent barriers, no idle US-CH firms
CH producers' idle flow payoff (h_{cc})	\$84 mil.	Absent barriers, no idle CH-CH firms

Note: Calibrated parameters of the model are at annual frequency. The middle column of this table presents the value of the calibrated parameter. The column on the right provides the reason for externally calibrated model parameters and the main source of identification for internally calibrated parameters. The levels of the retailer search costs, c_{do} depend on the normalization of the matching efficiency, ξ, as in Shimer (2005). Instead, we report average retailer search costs, $c_{do}/\chi(\kappa_{do})$, which have intrinsic meaning, in this table and in section 6.2. We discuss the calibration methodology in section 6 and intuition for parameter identification in section 6.1. "CH" stands for China, "US" stands for the United States, and "GDP" stands for Gross Domestic Product.

Table 2: Model fit

Moment in the data	**Data**	**Model**
Log-linear import elasticity	-6	-6.9
US mfg. capacity utilization rate	75%	76%
Percent of US firms exporting to CH	6%	7%
Percent of CH firms exporting to US	21%	19%
CH mfg. capacity utilization rate	74%	71%
Cost of business start up in US	\$550	\$550
Fixed foreign trade costs (CH-US)	\$683	\$683
Fixed foreign trade costs (US-CH)	\$664	\$664
Cost of business start up in CH	\$28	\$28
US absorption of domestic prod. (IM_{uu})	\$2.8 tril.	\$4.3 tril.
CH imports from US (IM_{cu})	\$116 bil.	\$92 bil.
US imports from CH (IM_{uc})	\$463 bil.	\$595 bil.
CH absorption of domestic prod. (IM_{cc})	\$2.7 tril	\$2.2 tril
US dom. absorp. consump. ratio (IM_{uu}/C_u)	22.2%	41.4%
CH-US export consump. ratio (IM_{cu}/C_u)	0.9%	0.9%
US-CH export consump. ratio (IM_{uc}/C_c)	10.5%	12.2%
CH dom. absorp. consump. ratio (IM_{cc}/C_c)	61.5%	45.1%
Average relationship duration	1 year	1 year
GDP in US	\$18.7 tril.	\$19.4 tril.
GDP in CH	\$11.2 tril.	\$11.6 tril.
Consumption in US	\$12.8 tril.	\$10.5 tril.
Consumption in CH	\$4.4 tril.	\$4.9 tril.
US consumption to GDP share	68%	54%
CH consumption to GDP share	39%	42%

Note: The model matches the empirical targets relatively well. The middle column of this table presents the value of the moment in the data. The column on the right presents the value of the equivalent moment in the model at the calibrated parameter values in table 1. We discuss model fit in section 6.3. ‘‘CH’’ stands for China, ‘‘US’’ stands for the United States, and ‘‘GDP’’ stands for Gross Domestic Product.

Table 3: Changes in producer matched rates, imports, price indexes, and welfare when search frictions are reduced

	(1) Baseline search frictions	(2) No search frictions		(3) Reducing int'l search frictions to domestic search frictions	
	(1.1) Levels	(2.1) Levels	(2.2) Δ from baseline	(3.1) Levels	(3.2) Δ from baseline
Producer matched rate in US-US mkt.	76%	100%	24pp	73%	-3pp
Producer matched rate in CH-US mkt.	7%	100%	93pp	50%	43pp
Producer matched rate in US-CH mkt.	19%	100%	81pp	76%	58pp
Producer matched rate in CH-CH mkt.	71%	100%	29pp	69%	-3pp
US absorption of domestic prod.	$4.3 tril.	$3.1 tril.	-27.8%	$2.9 tril.	-33.4%
Chinese imports from U.S.	$92.4 bil.	$875.1 bil.	847.4%	$635.9 bil.	588.5%
US imports from China	$595.1 bil.	$2110.7 bil.	254.7%	$2069.5 bil.	247.8%
CH absorption of domestic prod.	$2.2 tril.	$1.6 tril.	-28.7%	$1.6 tril.	-25.5%
US price index for US goods	$35.5 mil./util	$32.5 mil./util	-8.5%	$36.4 mil./util	2.4%
CH price index for US goods	$102.2 mil./util	$39.5 mil./util	-61.3%	$50.4 mil./util	-50.7%
US price index for CH goods	$67.4 mil./util	$37.1 mil./util	-44.9%	$40.8 mil./util	-39.5%
CH price index for CH goods	$36.3 mil./util	$32.5 mil./util	-10.5%	$37 mil./util	1.9%
US price index	$368.5 mil./util	$331.1 mil./util	-10.2%	$348.9 mil./util	-5.3%
Chinese price index	$378.3 mil./util	$335 mil./util	-11.5%	$363.9 mil./util	-3.8%
US welfare	28.4 thous. utils	31.7 thous. utils	11.4%	30 thous. utils	5.6%
Chinese welfare	12.9 thous. utils	14.6 thous. utils	13%	13.4 thous. utils	4%

Note: Lowering search frictions increases welfare by reducing the price index through reallocating production across countries. The table presents deviations from the baseline calibration in section 6. Column (2) eliminates search frictions altogether and shows that the associated welfare gains are large. Column (3) reduces retailers' search costs in international markets to their domestic levels. For example, U.S. retailers' search cost for a partner in China are reduced to search costs for a partner in the U.S. See section 7.1 for further details. "%Δ" stands for percent change. "pp. Δ" stands for percentage point change. "CH" stands for China and "US" stands for the United States.

Table 4: Decomposing the ex-ante Chinese welfare response to a unilateral tariff increase

Determinants of welfare change	(1) No search frictions and 10% unilateral tariff	(2) Baseline search frictions and 10% unilateral tariff
Pre-tariff dom. consump. share (λ_{cc})	0.321	0.4785
Post-tariff dom. consump. share (λ'_{cc})	0.354	0.487
Ratio of dom. consump. shares $\left(\hat{\lambda}_{cc} = \lambda'_{cc}/\lambda_{cc}\right)$	1.103	1.0177
Dom. consump. shares' effect on welfare $\left(\hat{\lambda}_{cc}^{-\frac{\alpha}{\theta}}\right)$	0.985	0.9972
Pre-tariff dom. matched rate $\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)$	1	0.713
Post-tariff dom. matched rate $\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)'$	1	0.715
Ratio of dom. matched rates $\widehat{\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)}$	1	1.002
Dom. matched rates' effect on welfare $\widehat{\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)}^{\frac{\alpha}{\theta}}$	1	1.0004
Pre-tariff dom. consump. level (C_c)	\$4.9 tril.	\$4.9 tril.
Post-tariff dom. consump. level (C'_c)	\$4.9 tril.	\$4.9 tril.
Ratio of dom. consump. levels $\left(\hat{C}_c = C'_c/C_c\right)$	1	1
Dom. consump. levels' effect $\left(\hat{C}_c^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)}\right)$	1	1
Welfare as fraction of pre-tariff welfare $\left(\hat{W}_c\right)$	0.985	0.998
Welfare percent change $\left(100 \times \left[\hat{W}_c - 1\right]\right)$	-1.53	-0.24
Pre-tariff CH price index (Ξ_c)	\$335 mil./util	\$378.3 mil./util
Post-tariff CH price index (Ξ'_c)	\$340.2 mil./util	\$379.2 mil./util
Price index percent change $\left(100 \times \left[\hat{\Xi}_c - 1\right]\right)$	1.55	0.24

Note: Search frictions attenuate the ex-ante Chinese welfare response to a 10 percent tariff by about 85 percent, lowering the welfare loss from 1.5 percent to 0.2 percent. The table presents equilibrium variables in response to a 10 percent increase in unilateral tariffs on imports to China from the United States. The complete welfare response in our baseline calibration is given by proposition 2. Column (1) presents the response without search frictions, which is the same as ACR and is completely determined by the ratio of the domestic consumption shares and model parameters α, θ, and σ. Some rows in column (1) are exactly 1 because those factors do not change in a model without search frictions. Column (2) presents the decomposition of the effect in our model with search frictions. Domestic consumption rises by about 1.8 percent after the tariff increase and this reduces welfare to 99.7 percent of the pre-tariff level. Protection of the domestic market raises the domestic matched rate by 0.2 percent and serves to boost welfare by 0.04 percent, offsetting some of the tariff's negative effects. See section 7.2 for further details.

Table 5: Ex-post Chinese welfare response to a unilateral tariff increase: Actual vs. ACR

	(1) Baseline search frictions and 10% unilateral tariff	(2) 50% of baseline search frictions and 6.8% unilateral tariff	(3) 10% of baseline search frictions and 3.4% unilateral tariff	(4) 1% of baseline search frictions and 2.1% unilateral tariff	(5) No search frictions and 1.6% unilateral tariff
Log-linear import elasticity (ex-post)	-6.56	-6.4	-5.65	-4.43	-3.18
Change in import shares (%)	1.8	1.8	1.8	1.8	1.8
ACR welfare change (%)	-0.13	-0.14	-0.16	-0.2	-0.28
True welfare change (%)	-0.24	-0.25	-0.26	-0.27	-0.28

Note: Ex-post welfare evaluation using the formula in ACR implies welfare changes in response to unilateral tariffs changes that are smaller than true welfare changes for any positive search frictions. The two welfare changes coincide in a model without search frictions. To obtain the welfare change according to the ACR formula we compute $100 \times [(\widehat{IM_{cc}/C_c})^{-\alpha/\hat{\theta}} - 1]$ for each column in which $-\hat{\theta}$ is the estimated log-linear import elasticity (ex-post). See section 7.3 for further details.

Table 6: Decomposing the Chinese consumption and trade elasticities

	(1) No search frictions and 10% unilateral tariff	(2) Baseline search frictions and 10% unilateral tariff
Pareto shape parameter ($-\theta$)	-3.18	-3.18
Elasticity of CH producers	0	0
Elasticity of US producers	0	0
Elasticity of the CH-US matched rate	0	-2.12
Elasticity of the CH-CH matched rate	0	0.02
Effect of CH-CH & CH-US eff. entry costs	0	-0.13
Consumption elasticity	-3.18	-5.45
Elasticity of CH-US markup	0	-0.01
Elasticity of CH-CH markup	0	0
Trade elasticity	-3.18	-5.47

Note: Search frictions change the trade elasticity to -5.5 from -3.2 in our baseline calibration without them and about 50 percent of the overall trade elasticity is explained by the elasticity of the matched rate in the *cu* market. The table presents equilibrium variables in response to a 10 percent increase in unilateral tariffs on imports into China from the United States. The decomposition is based on proposition 3, along with (B89) and (B92) in appendix B.2. Column (1) presents the response of the consumption and trade shares to a foreign tariff shock with no search frictions, which is $-\theta$ (equation B92). Column (2) presents the decomposition of these elasticities into their components in our model with search frictions; the elasticity of the *cu* and *cc* matched rates play an important role in the decomposition even though the the effective entry cost and markup terms respond to the tariff increase. The elasticity of the CH-US and CH-CH matched rates and markups in column (1) are exactly zero because these results have no search frictions. The other zeros in the table are rounded to the second decimal point. See section 7.4 for further details. ‘‘eff’’ stands for effective.

Table 7: Intensive and extensive margins

	(1)	(2)			(3)		
	$d\ln IM_{cu}/d\ln c_{cu}$	$d\ln IM_{cu}/d\ln \tau_{cu}$			$d\ln IM_{cu}/d\ln F_{cu}$		
	(1.1)	(2.1)	(2.2)	(2.3)	(3.1)	(3.2)	(3.3)
	Baseline search costs	Baseline search costs	Chaney exact	Chaney approx.	Baseline search costs	Chaney exact	Chaney approx.
Final sales elasticity	0.02	-2.82	-1.94	-3	0	0.02	
Markup elasticity	0	-0.01			0		
Intensive margin elasticity	0.02	-2.83	-1.94	-3	0	0.02	
Threshold elasticity	-0.04	-0.3	-0.12	-0.18	-0.06	-0.06	-0.06
Matched elasticity	-0.68	-2.07			-0.04		
Extensive margin elasticity	-0.72	-2.37	-0.12	-0.18	-0.1	-0.06	-0.06
Total elasticity	-0.69	-5.2	-2.06	-3.18	-0.1	-0.04	-0.06

Note: The import elasticity with respect to search frictions operates through the extensive-matched margin and is about -0.7 in our baseline calibration. This quantifies our analytic results from proposition 4. Search frictions also change the effect of variable trade costs on the extensive margin from -0.18 to -2.37. The table reports how imports to China from the United States change in response to cu trade cost changes. Columns labelled "Baseline" report results for our baseline calibration. Columns labelled "Chaney exact" include general equilibrium effects but do not have search frictions. Columns labelled "Chaney approx." report approximate results from Chaney (2008), which ignore general equilibrium effects. Columns (1), (2), and (3) report the effect of the the search cost, trade cost, and the effective entry cost, respectively. A "0" entry in the table means that the number rounds to zero with two decimals and a missing entry means that the elasticity does not exist. See section 7.5 for further details.

A Model appendix

A.1 Utility maximization and the ideal price index

A.1.1 Utility maximization

Here we present the solution to the utility maximization problem in section 2.1. The representative consumer's maximization problem can be stated as:

$$\begin{aligned} \max_{q_d(1),q_{dk}(\omega)} \quad & q_d\left(1\right)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}\left(\omega\right)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \\ s.t. \quad & \\ C_d \quad &= \quad p_d\left(1\right)q_d\left(1\right)+\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)q_{dk}\left(\omega\right)d\omega. \end{aligned}$$

We can solve this problem by maximizing the following Lagrangian

$$\mathcal{L} = q_d\left(1\right)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\alpha 2\sigma}{\sigma-1}} - \lambda\left[p_d\left(1\right)q_d\left(1\right)+\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)q_{dk}\left(\omega\right)d\omega - C\right].$$

The first-order conditions (FOCs) for the homogeneous good and two arbitrary varieties from the same origin, ω and ω', are:

$$\mathcal{L}_{q_d(1)} = \alpha q_1^{-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\alpha\frac{\sigma}{\sigma-1}} - \lambda p_d\left(1\right) = 0$$

$$\begin{aligned} \mathcal{L}_{q_{dk}(\omega)} &= q_d\left(1\right)^{1-\alpha}\alpha\left(\frac{\sigma}{\sigma-1}\right)\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1}\left(\frac{\sigma-1}{\sigma}\right)q_{dk}\left(\omega\right)^{\frac{\sigma-1}{\sigma}-1} - \lambda p_{dk}\left(\omega\right) = 0 \\ \mathcal{L}_{q_{dk}(\omega')} &= q_d\left(1\right)^{1-\alpha}\alpha\left(\frac{\sigma}{\sigma-1}\right)\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)-1}\left(\frac{\sigma-1}{\sigma}\right)q_{dk}\left(\omega'\right)^{\frac{\sigma-1}{\sigma}-1} - \lambda p_{dk}\left(\omega'\right) = 0. \end{aligned}$$

Dividing the last two FOCs and performing some algebra yields $q_{dk}\left(\omega\right)$ in terms of $q_{dk}\left(\omega'\right)$:

$$q_{dk}\left(\omega\right) = q_{dk}\left(\omega'\right)\left[\frac{p_{dk}\left(\omega'\right)}{p_{dk}\left(\omega\right)}\right]^{\sigma}.$$

Using the ratio of the first and third FOCs delivers a relationship between $q_d\left(1\right)$ and $q_{dk}\left(\omega'\right)$:

$$q_d\left(1\right) = \frac{p_{dk}\left(\omega'\right)}{p_d\left(1\right)}\left(\frac{1-\alpha}{\alpha}\right)\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega\right] q_{dk}\left(\omega'\right)^{\frac{1}{\sigma}}.$$

Using our solution for $q_{dk}\left(\omega\right)$ for the term in brackets yields:

$$\int_{\omega\in\Omega_{dk}} q_{dk}\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega = \int_{\omega\in\Omega_{dk}} p_{dk}\left(\omega\right)^{1-\sigma} d\omega p_{dk}\left(\omega'\right)^{\sigma-1} q_{dk}\left(\omega'\right)^{\frac{\sigma-1}{\sigma}}$$

and plugging this in gives

$$q_d(1) = \left(\frac{1}{p_d(1)}\right)\left(\frac{1-\alpha}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma}\,d\omega p_{dk}(\omega')^{\sigma}\,q_{dk}(\omega').$$

Now we can write the budget constraint in terms of $q_{dk}(\omega')$ and after some algebra this gives

$$C_d = \left\{\left(\frac{1}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma}\,d\omega p_{dk}(\omega')^{\sigma}\right\} q_{dk}(\omega').$$

So demand for $q_{dk}(\omega')$ is given by

$$q_{dk}(\omega') = \alpha C_d \frac{p_{dk}(\omega')^{-\sigma}}{\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma}\,d\omega}.$$

There are a couple of things to notice here. The first is that the demand for the CES good $q_{dk}(\omega')$ is not a function of the price of the good $q_d(1)$. Also notice that we can interpret αC_d as the consumer using the fraction of total expenditure from the Cobb-Douglas level of the utility function to define the fraction of total consumption resources that are devoted to this particular variety of the differentiated good.

As we show in appendix A.1.2 the price index for the differentiated goods from origin k to destination d is

$$P_{dk} = \left\{\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma}\,d\omega\right\}^{\frac{1}{1-\sigma}}$$

and the overall price index for the differentiated goods in country d is

$$P_d = \left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma}\,d\omega\right]^{\frac{1}{1-\sigma}}.$$

This means the demand for each CES variety is the function

$$q_{dk}(\omega) = \alpha C_d \frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}$$

The demand for the homogeneous good $q_d(1)$:

$$\begin{aligned} q_d(1) &= \left(\frac{1}{p_1}\right)\left(\frac{1-\alpha}{\alpha}\right)\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma}\,d\omega p_{dk}(\omega')^{\sigma}\,q_{dk}(\omega') \\ &= (1-\alpha)\frac{C_d}{p_d(1)} \end{aligned}$$

which is just the amount $(1-\alpha)C_d$ (Cobb-Douglas) spent on the good that has price $p_d(1)$. These are the demand functions in equation (2) of the main text.

A.1.2 Expenditure minimization and the price index

Here we derive, in full, the price index associated with our utility function. First we deal with the price index for the differentiated goods. The problem is separable because at the Cobbs-Douglas level, the utility function is log additive. Then we obtain the overall price index for the homogeneous and the differentiated goods.

The expenditure minimization problem for the differentiated goods looks as follows:

$$\begin{aligned} \min_{q_{dk}(\omega)} \quad & \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega \\ s.t. \quad & \\ U_d^{\rho} \quad = \quad & \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\rho}\, d\omega \end{aligned}$$

in which, for ease of notation, we have temporarily defined $\rho \equiv \dfrac{\sigma - 1}{\sigma}$. The following steps resemble the steps taken in Varian (1992) pg. 55.

The Lagrangian is:

$$\mathcal{L} = \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega + \lambda \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\rho}\, d\omega - U_d^{\rho} \right].$$

The first-order conditions (FOCs) are therefore:

$$\mathcal{L}_{q_{dk}(\omega)} = p_{dk}(\omega) - \lambda \rho q_{dk}(\omega)^{\rho - 1} = 0$$

$$\mathcal{L}_{\lambda} : \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\rho}\, d\omega = U_d^{\rho}.$$

Rearrange the first FOC to get:

$$q_{dk}(\omega)^{\rho} = p_{dk}(\omega)^{\frac{\rho}{\rho-1}} (\lambda\rho)^{-\frac{\rho}{\rho-1}}.$$

Put this back into the utility function to get:

$$(\lambda\rho)^{-\frac{\rho}{\rho-1}} = \frac{U_d^{\rho}}{\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)^{\frac{\rho}{\rho-1}}\, d\omega}.$$

Substitute this back into the equation above to get:

$$\begin{aligned} q_{dk}(\omega)^{\rho} &= p_{dk}(\omega)^{\frac{\rho}{\rho-1}} (\lambda\rho)^{-\frac{\rho}{\rho-1}} \\ q_{dk}(\omega)^{\rho} &= p_{dk}(\omega)^{\frac{1}{\rho-1}} \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{\frac{\rho}{\rho-1}} d\omega \right]^{-\frac{1}{\rho}} U_d. \end{aligned}$$

Now we have the demand functions in terms of prices and utility (Hicksian). Substitute this back into the objective function and collect terms to obtain the expenditure function:

$$\begin{aligned} e\left(p_{dk}(\omega), U_d\right) &= \sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}}^{\infty} p_{dk}(\omega)\, q_{dk}(\omega)\, d\omega \\ &= \sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)\, p_{dk}(\omega)^{\frac{1}{\rho-1}} \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{\frac{\rho}{\rho-1}} d\omega \right]^{-\frac{1}{\rho}} U_d d\omega \\ &= U_d \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{\frac{\rho}{\rho-1}} d\omega \right]^{\frac{\rho-1}{\rho}}. \end{aligned}$$

Substitute $\rho \equiv \dfrac{\sigma-1}{\sigma}$ back into this expression to get that

$$e\left(p_{dk}(\omega), U_d\right) = U_d \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.$$

And the ideal price index for the differentiated good from country k to country d is

$$P_{dk} = e\left(p_{dk}(\omega), 1\right) = \left[\sum_{k=1}^{O} \int_{\omega\in\Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.$$

Note that this is consistent with

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

in which $P_{dk} = \left[\int_{\omega\in\Omega_{dk}}^{\infty} p_{dk}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$, which is equation (3) in the main text.

Now we move on to deriving the overall price index, including the homogeneous good. From our previous work in appendix A.1.1, we know that the optimal quantity demanded of the differentiated good is

$$q_{dk}(\omega) = \alpha C_d \frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}.$$

We also know that the optimal quantity demanded of the homogeneous good is:

$$q_d(1) = (1-\alpha)\frac{C_d}{p_d(1)}.$$

Using these, we can derive the indirect utility function with some algebra:

$$\begin{aligned}
W_d\left(p_d(1), p_{dk}(\omega), C_d\right) &= q_d(1)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}} q_{dk}(\omega)^{\frac{\sigma-1}{\sigma}}\,d\omega\right]^{\alpha\frac{\sigma}{\sigma-1}} \\
&= \left((1-\alpha)\frac{C_d}{p_d(1)}\right)^{1-\alpha}\left[\sum_{k=1}^{O}\int_{\omega\in\Omega_{dk}}\left(\alpha C_d\frac{p_{dk}(\omega)^{-\sigma}}{P_d^{1-\sigma}}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\alpha\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{1-\alpha}{p_d(1)}\right)^{1-\alpha}\left(\frac{\alpha}{P_d}\right)^{\alpha} C_d.
\end{aligned}$$

Now, we know that our utility function is HOD 1 so our welfare expression can also be written as

$$W_d\left(\Xi_d, C_d\right) = \frac{C_d}{\Xi_d},$$

in which Ξ_d is the overall price index. Setting these two welfare expressions equal to each other gives us:

$$\begin{aligned}
\frac{C_d}{\Xi_d} &= \left(\frac{1-\alpha}{p_d(1)}\right)^{1-\alpha}\left(\frac{\alpha}{P_d}\right)^{\alpha} C_d \\
\Xi_d &= \left(\frac{p_d(1)}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha}.
\end{aligned}$$

A.2 Poisson process

Consider a continuous time Poisson process in which the number of events, n, in any time interval of length t is Poisson distributed according to

$$P\left\{N(t+s) - N(s) = n\right\} = e^{-\lambda t}\frac{(\lambda t)^n}{n!} \qquad n = 0, 1, \ldots$$

in which $s, t \geq 0$, $N(0) = 0$, and the process has independent increments. The mean number of events that occur by time t is

$$E\left[N(t)\right] = \lambda t.$$

Notice that λ is defined in units of time as λ events per t. For example, if producers in our model contact nine retailers every six months, on average, then we could recast our model measured in years with $t = 1$ and $\lambda = 4.5$ because $\lambda t = 9 \times 1/2 = 4.5$.

Define t_1 as the time at which the first event occurs. Using the Poisson process, the probability that the first event occurs after time t equals the probability no event has happened before. Thus, the arrival time of the first event is an exponential random variable with parameter λ given by

$$P\{t_1 > t\} = P[N(t) = 0] = e^{-\lambda t}.$$

Conversely, the probability the first event occurs between time 0 and time t is $P\{t_1 \leq t\} = 1 - e^{-\lambda t}$. Let t_n denote the time between the $(n-1)$st and nth events, which is also consistent with the definition of t_1 as the time of the first event. Because the Poisson process has independent increments, the distribution of time between any two events, t_n, for $n = 1, 2, \ldots$ will also be an exponential random variable with parameter λ. The sequence of times between all events, $\{t_n, n \geq 1\}$, also known as the sequence of inter-arrival times, will be a sequence of $i.i.d.$ exponential random variables with parameter λ. Given this distribution, the mean time between events is

$$E[t_n] = \frac{1}{\lambda}$$

For example, if producers in our model contact nine retailers every six months, on average, so that $\lambda t = 9/2$, then the average time between contacts is $1/\lambda = 2/9$ years (or about $365.25 \times 2/9 = 81.17$ days). The arrival time of the nth event, S_n, also called the waiting time, is the sum of the time between preceding events

$$S_n = \sum_{i=1}^{n} t_i.$$

Because S_n is the sum of n $i.i.d.$ exponential random variables in which each has parameter λ and the number of events n is an integer, S_n has an Erlang distribution with cumulative density function

$$P\{S_n \leq t\} = P[N(t) \geq n] = \sum_{i=n}^{\infty} e^{-\lambda t} \frac{(\lambda t)^i}{i!},$$

and probability density function

$$f(t) = \lambda e^{-\lambda t} \frac{(\lambda t)^{n-1}}{(n-1)!}.$$

which has mean $E[S_n] = \dfrac{n}{\lambda}$. The Erlang distribution is a special case of the gamma distribution in which the gamma allows the number of events n to be any positive real number, while the Erlang distribution restricts n to be an integer. The above discussion relies heavily on (Ross, 1995, Chapter 2).

A.3 The surplus, value, and expected duration of a relationship

Denote the joint surplus accruing to both sides of a match as $S_{do}(\varphi)$. The bargain will divide this surplus such that the value of being a retailer equals $M_{do}(\varphi) - V_{do} = (1-\beta) S_{do}(\varphi)$ and the value of being a producer is $X_{do}(\varphi) - U_{do}(\varphi) = \beta S_{do}(\varphi)$, in which β is the producer's bargaining power. Using the value functions presented in the main text (7), (8), (10), and (11), we can write the surplus equation as

$$S_{do}(\varphi) = \frac{p_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + s_{do}\kappa_{do}\chi(\kappa_{do})}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}. \tag{A33}$$

The surplus created by a match is the appropriately discounted flow profit, with the search cost l_{do} and the sunk cost s_{do} also entering the surplus equation because being matched avoids paying these costs. There are three things to notice here. First, the surplus from a match is a function of productivity. We show in appendix A.7 that matches that include a more productive exporting firm lead to greater surplus, that is, $S'_{do}(\varphi) > 0$. Second, the value of the relationship will fluctuate over the business cycle as shocks hit the economy and change the finding rate $\kappa_{do}\chi(\kappa_{do})$. Finally, surplus is greater than or equal to zero when

$$p_{do}q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + s_{do}\kappa_{do}\chi(\kappa_{do}) \geq 0.$$

Specifically, at the binding productivity cutoff we can use equation (A44) and the surplus sharing rule to write

$$\beta S_{do}(\bar{\varphi}_{do}) = \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do},$$

which, in order for surplus to be positive, puts a restriction on the parameter choices and the equilibrium value of market tightness, κ_{do}.

With the definition of surplus in hand, the value of a matched relationship, $R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi)$, can be expressed as $R_{do}(\varphi) = S_{do}(\varphi)\left(\frac{r + \kappa_{do}\chi(\kappa_{do})\beta}{r}\right) - \frac{l_{do}}{r}$. The value of the relationship to the producer is, of course, $X_{do}(\varphi)$ and to the retailer $M_{do}(\varphi)$. The value of a relationship in product markets has been of recent interest in Monarch and Schmidt-Eisenlohr (2015) and Heise (2016).

Relationships are destroyed at Poisson rate λ in the model, which implies the average duration of each match is $1/\lambda$. Because the destruction rate is exogenous and does not vary in our model, the average duration of each match is constant.

A.4 Bargaining over the negotiated price

A.4.1 Surplus sharing rule

Take equation (12), log and differentiate with respect to the price n_{do} and rearrange to get

$$\beta\frac{q_{do}}{X_{do}(\varphi) - U_{do}(\varphi)} + (1-\beta)\frac{-q_{do}}{M_{do}(\varphi) - V_{do}} = 0, \tag{A34}$$

which implies the simple surplus sharing rule, equation (13): The retailer receives β of the total surplus from the trading relationship, $S_{do}(\varphi) = M_{do}(\varphi) - V_{do} + X_{do}(\varphi) - U_{do}(\varphi)$. The producer receives the rest of the surplus, $(1-\beta)S_{do}(\varphi)$.

In section 3.1 of the main text, we point out the restriction that $\beta < 1$ in equation (12) is evident in equation (13), which results from equation (A34). Retailing firms have no incentive to search if $\beta = 1$ because they get none of the resulting match surplus and therefore cannot recoup search costs $c_{do} > 0$. Any solution to the model with $c_{do} > 0$ and positive trade between retailers and producers also requires $\beta < 1$. This result can be shown explicitly by using equations (10), (11), and (14) together with $\beta = 1$ to show that for productivity, φ, levels above the reservation productivity, $\bar{\varphi}_{do}$, (defined in section 3.3), the retailing firm has no incentive to search.

Finally, we do not need to calculate the partial derivative with respect to $U_{do}(\varphi)$ or $V_{do}(\varphi)$ because the individual firms are too small to influence aggregate values. Hence, when they meet, the firms bargain over the negotiated price-taking behavior in the rest of the economy as given. In particular, the outside option of the firms does not vary with the individual's bargaining problem.

A.4.2 Solving for the equilibrium negotiated price

Equations (7), (8), (10), and the equilibrium free entry condition $V_{do} = 0$ imply that

$$M_{do}(\varphi) - V_{do} = \frac{p_{do}(q_{do})\, q_{do} - n_{do} q_{do}}{r + \lambda} \tag{A35}$$

and

$$X_{do}(\varphi) - U_{do}(\varphi) = \frac{n_{do} q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})}. \tag{A36}$$

Bargaining over price results in equation (A34) and delivers the surplus sharing rule given by equation (13), which we can rewrite as $\beta(M_{do}(\varphi) - V_{do}) = (1-\beta)(X_{do}(\varphi) - U_{do}(\varphi))$. Using this transformation of equation (13) and the definitions given by equations (A35) and (A36) we can write

$$\begin{aligned}
\beta\frac{p_{do}(q_{do})\, q_{do} - n_{do} q_{do}}{r+\lambda} &= (1-\beta)\frac{n_{do} q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} \\
\Rightarrow n_{do} q_{do} &= p_{do}(q_{do})\, q_{do}(1-\gamma_{do}) + \gamma_{do}\left[t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}\right] \\
\Rightarrow n_{do} &= [1-\gamma_{do}]\, p_{do} + \gamma_{do}\frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}}{q_{do}}
\end{aligned}$$

in which $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}$.

A.4.3 Bounding the search friction

Recall the definition

$$\gamma_{do} \equiv \frac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}.$$

Here we show that $\gamma_{do} \in [0,1]$. First, because all parameters are positive, $\gamma_{do} \geq 0$. The lower bound, $\gamma_{do} = 0$, is reached only when $\beta = 1$ and $c_{do} = 0$ simultaneously. Second, prove that $\gamma_{do} \leq 1$ by contradiction. Assuming $\gamma_{do} > 1$ implies that $0 > \beta\kappa_{do}\chi(\kappa_{do})$, which is a contradiction, as $\beta \geq 0$ and $\kappa_{do}\chi(\kappa_{do}) \geq 0$.

A.4.4 Negotiated price when producers' finding rate goes to infinity

The limit of γ_{do} when the finding rate $\kappa_{do}\chi(\kappa_{do}) \to \infty$ is simply $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})} \to 0$. More complicated is the limit of $\gamma_{do}\kappa_{do}\chi(\kappa_{do})$ as $\kappa_{do}\chi(\kappa_{do}) \to \infty$. First rewrite the expression as

$$\gamma_{do}\kappa_{do}\chi(\kappa_{do}) = \frac{(r+\lambda)(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}\kappa_{do}\chi(\kappa_{do}).$$

Dividing the top and bottom of this expression by $\kappa_{do}\chi\left(\kappa_{do}\right)$ yields

$$\gamma_{do}\kappa_{do}\chi\left(\kappa_{do}\right) = \frac{\left(r+\lambda\right)\left(1-\beta\right)}{\dfrac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}+\beta}.$$

Now use this to derive the limit

$$\begin{aligned}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\gamma_{do}\kappa_{do}\chi\left(\kappa_{do}\right) &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\frac{\left(r+\lambda\right)\left(1-\beta\right)}{\dfrac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}+\beta} \\ &= \frac{\left(r+\lambda\right)\left(1-\beta\right)}{\beta}.\end{aligned}$$

This can be used to derive the limit of the negotiated price, n_{do}, as $\kappa_{do}\chi\left(\kappa_{do}\right)\to\infty$:

$$\begin{aligned}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty} n_{do} &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\left[\left[1-\gamma_{do}\right]p_{do}+\gamma_{do}\frac{t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}-\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}}{q_{do}}\right] \\ &= \lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\left[p_{do}-\gamma_{do}p_{do}+\gamma_{do}\left[\frac{t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}}{q_{do}}\right]-\frac{\gamma_{do}\kappa_{do}\chi\left(\kappa_{do}\right)s_{do}}{q_{do}}\right] \\ &= p_{do}-p_{do}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\gamma_{do} \\ &+ \left[\frac{t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}}{q_{do}}\right]\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\gamma_{do}-\frac{s_{do}}{q_{do}}\lim_{\kappa_{do}\chi(\kappa_{do})\to\infty}\gamma_{do}\kappa_{do}\chi\left(\kappa_{do}\right) \\ &= p_{do}-p_{do}\cdot 0+\left[\frac{t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}}{q_{do}}\right]\cdot 0-\frac{s_{do}\left(r+\lambda\right)\left(1-\beta\right)}{q_{do}\beta} \\ &= p_{do}-\frac{s_{do}\left(r+\lambda\right)\left(1-\beta\right)}{q_{do}\beta}.\end{aligned}$$

The negotiated price is the final sales price, less the amount required to compensate the producer for the sunk cost to start up the business relationship. Notice that if $s_{do}=0$, then the negotiated price would be the final sales price as in standard trade models.

A.5 Bargaining over the quantity

A.5.1 Maximizing surplus

Take equation (12), log and differentiate with respect to the quantity q_{do} to get

$$\beta\frac{1}{X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)}\left(n_{do}-\frac{\partial t\left(q_{do},w_o,\tau_{do},\varphi\right)}{\partial q_{do}}\right)+\left(1-\beta\right)\frac{1}{M_{do}\left(\varphi\right)-V_{do}}\left(p_{do}\left(q_{do}\right)+\frac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}}q_{do}-n_{do}\right)=0, \tag{A37}$$

in which we compute the partials of $X_{do}\left(\varphi\right)$ and $M_{do}\left(\varphi\right)$ using equations (A36) and (A35). Now, notice that equation (13) implies that $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)=\frac{\beta}{1-\beta}\left(M_{do}\left(\varphi\right)-V_{do}\right)$, and plugging this in to equation (A37) and rearranging slightly gives

$$p_{do}\left(q_{do}\right)+\frac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}}q_{do}=\frac{\partial t\left(q_{do},w_o,\tau_{do},\varphi\right)}{\partial q_{do}}. \tag{A38}$$

This expression says that the quantity produced and traded is pinned down by equating marginal revenue in the domestic market with marginal production cost in the foreign country. This restriction is the same as what we get from a model without search and therefore implies that adding search does not change the quantity traded within each match. The profit maximization implied by this equation is crucial: Despite being separate entities, the retailer and the producer decide to set marginal revenue equal to marginal cost. The result follows because of the simple sharing rule, the maximization of joint surplus, and the trivial role of the retailer. To maximize surplus, the parties choose to equate marginal revenue and marginal cost.

A.5.2 Profit maximization

Conditional on the consumer's inverse demand (equation 2), the quantity traded between producer and retailer, $q_{do}(\omega)$, equates marginal revenue obtained by the retailer with the marginal production cost, as in equation (A38). In other words, the retailer and producer solve a profit maximization problem. In particular, they seek to maximize profits for a given variety, ω, given that the producer has productivity φ, i.e., the cost function for producing q_{do} units of variety ω for the producer is given by $w_o \tau_{do} \frac{q_{do}(\omega)}{\varphi} + w_o f_{do}$, and the retailer faces a downward sloping demand curve. Variable profits can be written as:

$$\pi_{do}(\omega) \quad = \quad r_{do}(\omega) - w_o \tau_{do} \frac{q_{do}(\omega)}{\varphi}.$$

From the utility maximization solution we know that

$$r_{do}(\omega) \quad = \quad \alpha C_d \left(\frac{p_{do}(\omega)}{P_d} \right)^{1-\sigma}.$$

Since the CES aggregator is HOD 1, we know that welfare from the differentiated goods must be $\tilde{W}_d = \frac{\alpha C_d}{P_d}$ or $\alpha C_d = P_d \tilde{W}_d$ (appendix A.15). Further we know, again from our utility maximization solution, that

$$q_{do}(\omega) \quad = \quad \tilde{W}_d \left(\frac{p_{do}(\omega)}{P_d} \right)^{-\sigma}.$$

Plugging these into our profit expression from the top yields:

$$\begin{aligned} \pi_{do}(\omega) &= \alpha C_d \left[\frac{p_{do}(\omega)}{P_d} \right]^{1-\sigma} - \frac{w_o \tau_{do}}{\varphi} \tilde{W}_d \left[\frac{p_{do}(\omega)}{P_d} \right]^{-\sigma} \\ &= P_d^{\sigma} \tilde{W}_d p_{do}(\omega)^{1-\sigma} - \frac{w_o \tau_{do}}{\varphi} P_d^{\sigma} \tilde{W}_d p_{do}(\omega)^{-\sigma}. \end{aligned}$$

Differentiating this expression with respect to the price for this particular variety, $p_{do}(\omega)$, and setting this derivative equal to zero we get:

$$\frac{\partial \pi_{do}(\omega)}{\partial p_{do}(\omega)} = 0 \quad = \quad (1-\sigma) P_d^{\sigma} \tilde{W}_d p_{do}(\omega)^{1-\sigma-1} + \sigma \frac{w_o \tau_{do}}{\varphi} P_d^{\sigma} \tilde{W}_d p_{do}(\omega)^{-\sigma-1}.$$

Solving this for $p_{do}(\omega)$ yields:

$$p_{do}(\omega) \quad = \quad \mu \frac{w_o \tau_{do}}{\varphi},$$

in which $\mu = \dfrac{\sigma}{\sigma - 1}$. Notice that since the right-hand side is not a function of ω (the index), but is a function of the productivity φ, we write

$$p_{do}(\varphi) = \mu \frac{w_o \tau_{do}}{\varphi}$$

throughout the text.

As discussed in section 2.2, our equilibrium ensures that each product variety only matches with one retail vacancy so that matched retailers have a monopoly in the variety that they import.

A.5.3 Retailer production function

In this section, we show that the conclusions of this paper are the same if the retailer produces the final good using both the good facing search frictions and another input. The value of being in a relationship for a retailer in this case is

$$rM_{do}(\varphi) = p_{do}\left(f\left(q_{do}, m_{do}\right)\right) f\left(q_{do}, m_{do}\right) - n_{do} q_{do} - e_{do} m_{do} - \lambda\left(M_{do}(\varphi) - V_{do}\right), \qquad \text{(A39)}$$

in which the retailer combines the additional input, for example materials or distribution costs, denoted by m_{do}, with the input subject to search frictions, q_{do}, according to production function $f\left(q_{do}, m_{do}\right)$ to produce the final good sold to consumers. The price of the additional input, e_{do}, is determined outside of the search model and is taken as given by the retailer.

With this new Bellman equation, logging and differentiating the Nash product in equation (12) with respect to p_{do} gives the same surplus sharing (13) rule as before. The first-order condition of equation (12) with respect to q_{do}, however, becomes

$$\begin{aligned} 0 &= \beta\left(\frac{n_{do} - \partial t\left(q_{do}, w_o, \tau_{do}, \varphi\right)/\partial q_{do}}{X_{do}(\varphi) - U_{do}(\varphi)}\right) \qquad \text{(A40)} \\ &+ (1-\beta)\frac{\left(\partial p_{do}/\partial f_{do}\right)\left(\partial f_{do}/\partial q_{do}\right) f\left(q_{do}, m_{do}\right) + p_{do}\left(q_{do}, m_{do}\right)\partial f_{do}/\partial q_{do} - n_{do}}{M_{do}(\varphi) - V_{do}}. \end{aligned}$$

Combining this with the surplus sharing rule (13) yields an expression similar to equation (A38):

$$p_{do}\left(q_{do}, m_{do}\right)\frac{\partial f_{do}}{\partial q_{do}} + \frac{\partial p_{do}}{\partial f_{do}}\frac{\partial f_{do}}{\partial q_{do}} f\left(q_{do}, m_{do}\right) = \frac{\partial t\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}. \qquad \text{(A41)}$$

This equation states that retailers and producers will negotiate to trade a quantity, q_{do}, that ensures that the marginal revenue of q_{do} equals the marginal cost of q_{do}. Because the price e_{do} of the input m_{do} is taken as given, the retailer chooses the optimal level of the input, m^*_{do}, so that the first order condition, $f_{m_{do}}\left(q_{do}, m^*_{do}\right) = e_{do}$, holds. Strict concavity of the function $f(q_{do}, m_{do})$ is sufficient to ensure that the partial derivative $f_{m_{do}}\left(q_{do}, m^*_{do}\right)$ is invertible so that $f^{-1}_{m_{do}}\left(q_{do}, e_{do}\right) = m^*_{do}$ which can be substituted into equation (A41) to get one equation in one unknown, q_{do}. The quantity traded within each match depends on the price of the other input, $q_{do}\left(e_{do}\right)$, but search frictions still do not enter equation (A41). The result in the main text—that optimal q_{do} is determined by the condition that ensures that marginal revenue from q_{do} equals the marginal cost of producing q_{do}—remains intact. Furthermore, the negotiated import price n_{do} implied by the surplus sharing rule and given in (14) will now also be a function of the input price e_{do} through $q_{do}\left(e_{do}\right)$ but it will remain a convex combination of the final sales price and average production cost and deliver the same aggregation results as the model without an additional input.

A.6 Solving for the productivity thresholds

A.6.1 Solving for the lowest productivity threshold

First, let's solve for an expression for $X_{do}(\varphi) - U_{do}(\varphi)$ by plugging in equations (7) and (8):

$$\begin{aligned} rX_{do}(\varphi) - rU_{do}(\varphi) &= n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} - \lambda\left(X_{do}(\varphi) - U_{do}(\varphi)\right) + l_{do} - \kappa_{do}\chi(\kappa_{do}(X_{do}(\varphi) - U_{do}(\varphi) - s_{do}) \\ &= n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} - (\lambda + \kappa_{do}\chi(\kappa_{do}))(X_{do}(\varphi) - U_{do}(\varphi)) \\ &\Rightarrow (r + \lambda + \kappa_{do}\chi(\kappa_{do}))(X_{do}(\varphi) - U_{do}(\varphi)) = n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} \\ &\Rightarrow X_{do}(\varphi) - U_{do}(\varphi) = \frac{n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})}. \end{aligned} \tag{A42}$$

Now plug this expression into the definition of $\underline{\varphi}_{do}$ from the main text to get

$$\begin{aligned} &\frac{n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} = 0 \\ &\Rightarrow n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} = 0. \end{aligned} \tag{A43}$$

By using the fact that $X'_{do}(\varphi) - U'_{do}(\varphi) > 0$ from above we can state that this threshold is unique.

We can be sure that for any positive cost of forming a relationship, $\dfrac{l_{do} + h_{do}}{\kappa_{do}\chi\left(\kappa_{do}\right)} + s_{do}$, if and only if $l_{do} + h_{do} + \kappa_{do}\chi\left(\kappa_{do}\right)s_{do} > 0$, the expression $X_{do}\left(\bar{\varphi}_{do}\right) - U_{do}\left(\bar{\varphi}_{do}\right)$ exceeds $X_{do}\left(\underline{\varphi}_{do}\right) - U_{do}\left(\underline{\varphi}_{do}\right)$. This result implies that as long as $X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)$ is increasing in φ, then $\bar{\varphi}_{do} > \underline{\varphi}_{do}$. In appendix A.7, we show the very general conditions under which $X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right)$ is increasing in φ. The binding productivity threshold defining the mass of producers that have retail partners is the greater of these two and hence $\bar{\varphi}_{do}$. In other words, the productivity necessary to induce a producer to search for a retail partner is greater than the productivity necessary to consummate a match after meeting a retailer due to the costs that are incurred while searching. Similarly, the productivity necessary to form a match is greater than the productivity to maintain one already in place.

A.6.2 Solving for the binding productivity threshold in equation (16)

Our threshold productivity, $\bar{\varphi}_{do}$, is given by $U_{do}\left(\bar{\varphi}\right) - I_{do}\left(\bar{\varphi}_{do}\right) = 0$. Plugging equations (8) and (9) into this definition yields

$$X_{do}\left(\bar{\varphi}_{do}\right) - U_{do}\left(\bar{\varphi}_{do}\right) = \frac{l_{do} + h_{do}}{\kappa_{do}\chi\left(\kappa_{do}\right)} + s_{do} \tag{A44}$$

Using equation (A42) in equation (A44) yields

$$\frac{n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} = \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do}$$

$$\Rightarrow n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} = (r + \lambda + \kappa_{do}\chi(\kappa_{do}))\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})}$$

$$\Rightarrow n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} = (r + \lambda)\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}$$

$$\Rightarrow n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} = (r + \lambda)\frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + h_{do}$$

$$\Rightarrow n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} - (r + \lambda)\frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} - h_{do} = (r + \lambda)s_{do}$$

$$\Rightarrow n_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} = (r + \lambda)s_{do} + (r + \lambda)\frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + h_{do}.$$

Now, plug in for the equilibrium import price, n_{do}, from equation (14), to get

$$\begin{aligned}(1 - \gamma_{do})\, p_{do}\left(q_{do}\right)q_{do} + \gamma_{do}\left(t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})s_{do}\right) - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do}\\ = (r + \lambda)\, s_{do} + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}l_{do} + \left(1 + \frac{(r + \lambda)}{\kappa_{do}\chi(\kappa_{do})}\right)h_{do}.\end{aligned}$$

which can be rearranged to obtain

$$\begin{aligned}&p_{do}\left(q_{do}\right)q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do}\\ &= (1 - \gamma_{do})^{-1}\left[(r + \lambda + \gamma_{do}\kappa_{do}\chi\left(\kappa_{do}\right))\, s_{do} + \left(\gamma_{do} + \frac{(r + \lambda)}{\kappa_{do}\chi\left(\kappa_{do}\right)}\right)l_{do} + \left(1 + \frac{(r + \lambda)}{\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do}\right].\end{aligned}$$

Further simplification of the terms with γ_{do} implies that

$$p_{do}\left(q_{do}\right)q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) = f_{do} + \left(\frac{(r + \lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)l_{do} + \left(1 + \frac{(r + \lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do} + \frac{(r + \lambda)}{\beta}s_{do}.$$

which is the expression in the main text.

A.6.3 Productivity cutoff and flow profits

Because the equilibrium price for each variety is a constant markup over marginal cost we can write the firms' variable cost function as a proportional function of revenue

$$t_{do}\left(\varphi\right) = p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)\mu^{-1}.$$

Combine the definition of flow variable profits

$$\pi_{do}\left(\varphi\right) = p_{do}\left(\varphi\right)q_{do}\left(\varphi\right) - t_{do}\left(\varphi\right)$$

with the relationship between variable costs and revenue to get that

$$\pi_{do}\left(\varphi\right) = p_{do}\left(\varphi\right)q_{do}\left(\varphi\right) - p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)\mu^{-1},$$

which simplifies to

$$\pi_{do}\left(\varphi\right) = p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)\sigma^{-1}$$

because $1-\mu^{-1}=\sigma^{-1}$. Using demand from equation (2) and the pricing rule provides revenue in this model

$$p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)=\alpha C_d P_d^{\sigma-1}\left(\mu w_o \tau_{do}\right)^{1-\sigma}\varphi^{\sigma-1}.$$

We can use revenue and the profit expression combined with (16) to derive threshold productivity in our search model. We start with the expression

$$\pi_{do}\left(\bar{\varphi}_{do}\right)=F_{do}\left(\kappa_{do}\right).$$

Then use the functional forms and the relationship between revenues and profits to write

$$\frac{\alpha}{\sigma}C_d P_d^{\sigma-1}\left(\mu w_o \tau_{do}\right)^{1-\sigma}\bar{\varphi}_{do}^{\sigma-1}=F_{do}\left(\kappa_{do}\right)$$

before arriving at

$$\bar{\varphi}_{do}=\mu\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)C_d^{\frac{1}{1-\sigma}}F_{do}\left(\kappa_{do}\right)^{\frac{1}{\sigma-1}},$$

which is presented in equation (18) in the main text.

A.6.4 Comparing our productivity threshold to previous models

Equations (16) and (17) nest the conditions defining the threshold productivity in many trade models. We consider a few interesting cases here.

When we eliminate search frictions, setting $h_{do}=-s_{do}\left(r+\lambda\right)/\beta$ recovers the same threshold productivity as Chaney (2008). Remove the search friction so that the finding rate $\kappa_{do}\chi\left(\kappa_{do}\right)\to\infty$ and $\dfrac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\to 0$ in equation (16):

$$\begin{aligned} p_{do}\left(q_{do}\right)q_{do}-t\left(q_{do},w_o,\tau_{do},\varphi\right) &= f_{do}+\left(\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)l_{do}+\left(1+\frac{(r+\lambda)}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)}\right)h_{do}+\frac{(r+\lambda)}{\beta}s_{do} \\ &= f_{do}+h_{do}+\frac{(r+\lambda)}{\beta}s_{do}. \end{aligned} \tag{A45}$$

This implies that the effective entry cost includes includes the fixed cost, the value from the outside option, the sunk cost, the bargaining power of the producer, and the discounting parameters, even if one finds a partner immediately. With $h_{do}=0$, and $s_{do}=0$, we also recover the productivity threshold in Chaney (2008).

Notice that estimating a search-frictionless Melitz-style model would yield estimates of the fixed costs of exporting, f_{do}, that are biased up because they do not account for the other terms in the effective entry cost in equation (A45). In particular, the effective entry cost is the sum of the fixed cost of production and several terms that are weakly positive, so that $F_{do}\geq f_{do}$ in (A45). A method that estimates F_{do}, therefore, would yield estimates of the effective entry cost that are weakly higher than only the fixed cost of production.

Another interesting comparison is to Eaton et al. (2014). That framework includes a flow search cost, l_{do}, but does not have a sunk cost s_{do} or any idle state and treats the effective entry cost as a parameter. To reproduce their threshold productivity condition, first set $h_{do}=-l_{do}$, which makes the effective entry cost independent of market tightness. Making this assumption together with $s_{do} = 0$ in equations (16) and (17) provides:

$$\begin{aligned} p_{do}(q_{do})\, q_{do} - t(q_{do}, w_o, \tau_{do}, \varphi) &= f_{do} + \left(\frac{(r+\lambda)}{\beta \kappa_{do} \chi(\kappa_{do})}\right) l_{do} + \left(1 + \frac{(r+\lambda)}{\beta \kappa_{do} \chi(\kappa_{do})}\right) h_{do} + \frac{(r+\lambda)}{\beta} s_{do} \\ &= f_{do} - l_{do}. \end{aligned}$$

This result is the very reason why Eaton et al. (2014) must have that $f_{do} > l_{do}$.

A.6.5 The idle state and threshold productivity

This section clarifies the importance of the idle state and its relationship to the threshold productivity. Like in Chaney (2008), producers below the threshold productivity, equation (18), do not produce in that market. We include the idle state for three reasons. First, we find it intuitively appealing to include the outside option to remain idle because that allows producers to optimally choose not to search. Without this idle state, all producers are forced to search in all markets, which is an unnatural restriction. Second, we want to highlight in equation (17) that one of the components of producers' entry costs is the opportunity cost of remaining idle. Third, setting the value of the idle state to zero does not remove the idle state and we clarify the restriction that does. We address each of these reasons below.

First, we find it intuitively appealing to include the outside option to remain idle because that allows producers to optimally choose not to search. Without an idle state all producers must either be matched or searching in all markets, which is a strong restriction. In particular, without an idle state, the binding productivity threshold changes from the one defined by the producer being indifferent between searching and not searching, $\bar{\varphi}_{do}$ (equation 18), to the one defined by the producer being indifferent between consummating or not consummating a match after meeting a retailer, $\underline{\varphi}_{do}$ defined in equation (A43). With a binding threshold of $\underline{\varphi}_{do} > 1$, all producers search but those with $\varphi < \underline{\varphi}_{do}$ reject all retailers they meet. We find this to be an unnatural consequence of eliminating the idle state. Allowing producers to choose to search in each market is both more general and more intuitive.

Second, we want to highlight in equation (17) that one of the components of producers' entry costs is the opportunity cost of remaining idle. An innovation of our model is that we provide a micro foundation for the effective cost of entering foreign markets. Including the idle state highlights that the opportunity cost of remaining idle, h_{do}, is an important determinant of the productivity threshold and the fraction of active producers through equation (17). The value to a producer of remaining idle can be interpreted, for example, as the value of the stream of payments after liquidation or the flow payoff from home production if these firms are viewed as entrepreneurs. In many static models, the effective entry cost (equation 17) is an important parameter but all of these barriers are typically attributed to the fixed cost of production, f_{do}. Allowing for the possibility that producers optimally choose not to search could change the estimates of fixed production costs.

Third, setting the idle flow payoff to zero does not necessarily eliminate the idle state in our model. Choosing an h_{do}, which could be negative, that ensures $\bar{\varphi}_{do} \leq 1$ is sufficient to eliminate the idle state from our model. This condition is sufficient because the lower bound of the productivity distribution equals 1, so all producers prefer searching to remaining idle if and only if $\bar{\varphi}_{do} \leq 1$. An idle flow payoff equal to zero, $h_{do} = 0$, does not eliminate the idle state because that restriction does not guarantee that $\bar{\varphi}_{do} \leq 1$.

A.7 The value of importing is strictly increasing in productivity

Here we show that the value of importing, $M_{do}(\varphi)$, is strictly increasing with the producer's productivity level, φ. This fact allows us to replace the integral of the max over V_{do} and $M_{do}(\varphi)$ (equation 11) with the integral of $M_{do}(\varphi)$ from the productivity threshold, $\bar{\varphi}_{do}$ (equation 19).

Starting with equation (10) and $V_{do} = 0$ we obtain

$$
\begin{aligned}
(r+\lambda) M_{do}(\varphi) &= p_{do}q_{do} - n_{do}q_{do} \\
&= p_{do}q_{do} - [1-\gamma_{do}] p_{do}q_{do} - \gamma_{do} \left(t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right) s_{do} \right) \\
&= \gamma_{do}p_{do}q_{do} - \gamma_{do} \left(t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do} \right) + \gamma_{do} \left(l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) s_{do} \right) \\
&= \gamma_{do} \left(p_{do}q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do} \right) + \gamma_{do} \left(l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) s_{do} \right).
\end{aligned}
$$

Remember that $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi\left(\kappa_{do}\right)}$. It is clear from the integral in the import relationship creation equation (19) that neither the finding rate for retailers, $\chi\left(\kappa_{do}\right)$, nor the tightness, κ_{do}, is a function of the productivity, φ. Given this, $M_{do}'\left(\varphi\right)$ and $\dfrac{\partial [p_{do}\left(q_{do}\right) q_{do} - t\left(q_{do}, w_o, \tau_{do}, \varphi\right) - f_{do}]}{\partial \varphi}$ will have the same sign. As long as flow profits without search frictions are strictly increasing in productivity, $M_{do}'\left(\varphi\right) > 0$. Using the specific functional forms for $t\left(q_{do}, w_o, \tau_{do}, \varphi\right) + f_{do}$ used above, as well as the equilibrium values for n_{do}, p_{do}, and q_{do}, we can derive this result explicitly. In this case,

$$
M_{do}(\varphi) = \alpha\gamma_{do} \left(\frac{1}{r+\lambda} \right) \left(\frac{\mu^{-\sigma}}{\sigma-1} \right) \left(w_o\tau_{do}\right)^{1-\sigma} \alpha C_d P_d^{\sigma-1} \varphi^{\sigma-1} - \gamma_{do}f_{do} + \gamma_{do} \left(l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) s_{do} \right).
$$

Therefore the derivative is

$$
\frac{\partial M_{do}\left(\varphi\right)}{\partial \varphi} = \alpha\gamma_{do} \left(\frac{1}{r+\lambda} \right) \mu^{-\sigma} \left(w_o\tau_{do}\right)^{1-\sigma} \alpha C_d P_d^{\sigma-1} \varphi^{\sigma-2}.
$$

which is always positive.

As long as $M_{do}'(\varphi) > 0$, we can demonstrate the way in which many other important quantities depend on the producer's productivity level, φ. From the surplus sharing rule (A34) can be rewritten as

$$
\beta M_{do}\left(\varphi\right) = (1-\beta) \left(X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) \right), \tag{A46}
$$

We know that in equilibrium, because $M_{do}'(\varphi) > 0$, it must be that $X_{do}'\left(\varphi\right) - U_{do}'\left(\varphi\right) > 0$. Differentiating both sides of equation (8) gives $rU_{do}'\left(\varphi\right) = \kappa_{do}\chi\left(\kappa_{do}\right) \left(X_{do}'\left(\varphi\right) - U_{do}'\left(\varphi\right) \right) > 0$. We can combine these facts to show $X_{do}'\left(\varphi\right) > U_{do}'\left(\varphi\right) > 0$. Using the definition of the joint surplus of a match $S_{do}\left(\varphi\right) = X_{do}\left(\varphi\right) + M_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) - V_{do}$ we get $S_{do}'(\varphi) > 0$. Likewise, the value of a relationship, $R_{do}\left(\varphi\right) = X_{do}\left(\varphi\right) + M_{do}\left(\varphi\right)$, has $R_{do}'(\varphi) > 0$.

A.8 Market tightness and the cost of search

Let's first prove that $\kappa_{do} < \infty$ if $c_{do} > 0$. To do this, let's prove the contrapositive: assume that $c_{do} = 0$ and show that $\kappa_{do} = \infty$. Rearrange equation (19) slightly to get

$$
0 = c_{do} = \chi\left(\kappa_{do}\right) \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right).
$$

We have shown that $M_{do}(\bar{\varphi}_{do}) \geq 0$ for any consummated match in equilibrium (Nash bargaining together with appendix A.6) and $M'_{do}(\varphi) > 0$ (appendix A.7). Therefore we know that $\int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi) > 0$. Thus, $\chi(\kappa_{do})$ must be zero. Because $\chi'(\kappa_{do}) < 0$ this is true if and only if $\kappa_{do} = \infty$.

To prove that if $c_{do} > 0$ then $\kappa_{do} < \infty$, let's use equation (19) again. In particular, because $c_{do} > 0$ it must mean that $\chi(\kappa_{do}) \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi) > 0$. As before, we know that $\int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi) > 0$ so it must be that $\chi(\kappa_{do}) > 0$ as well, which is true if and only if $\kappa_{do} < \infty$.

A.9 Producer and retailer existence

A.9.1 Retailing firms

Free entry implies that the ex-ante expected value from entering for a potential retailer equals the expected cost of entering. Assume for a moment that the potential retailers consider the value of becoming a retailer as defined by E^m_{do}. This value is characterized by the following Bellman equation

$$rE^m_{do} = -e^m_o + (V_{do} - E^m_{do}). \tag{A47}$$

The potential retailer could sell the value E^m_{do} and invest the proceeds at the interest rate r getting flow payoff rE^m_{do} forever after. Alternatively, they could pay a cost e^m_o to become a retailer, at which point they will begin in the state of having a vacancy with value V_{do} (with certainty) and give up the value of being a potential retailer E^m_{do}. Free entry into becoming a retailer implies that $E^m_{do} = 0$ in equilibrium so that

$$\begin{aligned} 0 &= -e^m_o + V_{do} \\ e^m_o &= V_{do}. \end{aligned}$$

Hence, free entry into vacancies $V_{do} = 0$ implies $e^m_o = 0$ and we cannot have a sunk cost for entry into retailing. In other words, free entry into the search market along with assuming that one must post a vacancy before matching implies free entry into retailing.

Free entry into the search market subsumes free entry into retailing and so we only have one condition defined by free entry on the retailing side given by equation (19) and restated here

$$\frac{c_{do}}{\chi(\kappa_{do})} = \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi).$$

Remember this states that product vacancies continue being created until the expected cost of being an unmatched retailer, $c_{do}/\chi(\kappa_{do})$, equals the expected benefit $\int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi)$. Because each potential retailer must post a product vacancy before forming a match, the expected cost of becoming a retailer (entering as a retailer) is the same as the expected cost of being an unmatched retailer. Likewise, the expected benefit of posting a vacancy and the expected benefit of becoming a retailer are also the same because we assume retailers must post a vacancy before matching.

Free entry into retailing and assuming that each retailer can post a certain number of vacancies determine the number of retailers in equilibrium. We show how to recover the number of retailers in the special case when each retailer posts one vacancy. This assumption implies that the mass of matched producers and retailers must be equal in the steady state:

$$\left(1-u_{do}-i_{do}\right)N_{o}^{x}=\left(1-v_{do}\right)N_{d}^{m}. \tag{A48}$$

The free-entry condition (equation 19) determines the number of retailers with vacancies, $v_{do}N_d^m$, conditional on the number of producers, N_d^x, and the fraction of searching producers, u_{do}. The former is determined as in Chaney (2008) so that the number differentiated-goods producers is proportional to aggregate consumption expenditure, $N_d^x = C_d\ (1+\pi)$. The latter is obtained from the steady-state fraction of unmatched producers (equation 20) and the fraction of idle producers (equation 21). One retailer matching to one producer pins down the retailer unmatched rate, v_{do}, which implies the number of retailers from the free-entry condition.

A.9.2 Producing firms

Similar to the entry decision of retailers, the value of entry for producers, E_{do}^x, is defined by

$$\begin{aligned} rE_{do}^{x} &= -e_{d}^{x}+\int \max\left\{I_{do}\left(\varphi\right),U_{do}\left(\varphi\right)\right\}dG\left(\varphi\right)-E_{do}^{x} \\ &= -e_{d}^{x}+\int_{1}^{\bar{\varphi}_{do}} I_{do}\left(\varphi\right)dG\left(\varphi\right)+\int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right)dG\left(\varphi\right)-E_{do}^{x}. \end{aligned}$$

We assume that the potential producer must transit through the unmatched state before forming a match. After paying e_d^x and taking a productivity draw φ, the potential producer loses the value E_{do}^x with certainty and, depending on the drawn productivity, chooses between searching for a retailer and getting value $U_{do}(\varphi)$ or remaining idle and getting value $I_{do}(\varphi)$. If we assumed free entry into production, we would get $E_{do}^x = 0$ and that

$$e_{d}^{x}=\int_{1}^{\bar{\varphi}_{do}} I_{do}\left(\varphi\right)dG\left(\varphi\right)+\int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right)dG\left(\varphi\right). \tag{A49}$$

which ensures the expected value of taking a productivity draw equals the expected cost.

Free entry into production, therefore, imposes another restriction on the equilibrium. We can use the facts that $X_{do}(\varphi)-U_{do}(\varphi)=(1-\beta)\,S_{do}(\varphi)$ and that $M_{do}(\varphi)=\beta S_{do}(\varphi)$ to write $X_{do}(\varphi)-U_{do}(\varphi)=\left(\frac{1-\beta}{\beta}\right)M_{do}(\varphi)$. Applying this to equation (8) gives

$$rU_{do}\left(\varphi\right)=-l_{do}+\kappa_{do}\chi\left(\kappa_{do}\right)\left(\left(\frac{1-\beta}{\beta}\right)M_{do}\left(\varphi\right)-s_{do}\right).$$

Computing the relevant integrals in equation (A49) gives

$$\int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right)dG\left(\varphi\right)=-\left(\frac{l_{do}+s_{do}\kappa_{do}\chi\left(\kappa_{do}\right)}{r}\right)\left(1-G\left(\bar{\varphi}_{do}\right)\right)+\frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{r}\left(\frac{1-\beta}{\beta}\right)\int_{\bar{\varphi}_{do}}^{\infty} M_{do}\left(\varphi\right)dG\left(\varphi\right).$$

Likewise, from (9) we have

$$\int_{1}^{\bar{\varphi}_{do}} I_{do}\left(\varphi\right)dG\left(\varphi\right)=\frac{h_{do}}{r}G\left(\bar{\varphi}_{do}\right).$$

Combining these with equation (A49) gives

$$e_d^x = \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right) - \left(\frac{l_{do} + s_{do}\kappa_{do}\chi\left(\kappa_{do}\right)}{r}\right)\left(1 - G\left(\bar{\varphi}_{do}\right)\right) + \frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{r}\left(\frac{1-\beta}{\beta}\right)\int_{\bar{\varphi}_{do}}^{\infty} M_{do}\left(\varphi\right) dG\left(\bar{\varphi}_{do}\right), \tag{A50}$$

which is the restriction that free entry into production for producers would place on equilibrium market tightness, κ_{do}. Because (A50) is based on the PDV of future profits and must include an assumption about the state in which producers' start, we cannot combine it with labor market clearing to derive a simple closed form mapping for the number of producers as in Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008).

From equation (A49), we can see that free entry into search for producers would require $\int_{\bar{\varphi}_{do}}^{\infty} U_{do}\left(\varphi\right) dG\left(\varphi\right) = 0$, in which case we would be left with

$$e_d^x = \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right), \tag{A51}$$

which implies that if the exploration cost is higher than the right hand side of this equation there would be no producer entry in equilibrium.

Finally, we note that simultaneous combinations of free entry on both sides of the market are possible. Combining free entry into both existence and search for retailers from equation (19) with free entry into existence for producers from equation (A50) gives

$$e_d^x = \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right) - \left(\frac{l_{do} + s_{do}\kappa_{do}\chi\left(\kappa\right)}{r}\right)\left(1 - G\left(\bar{\varphi}_{do}\right)\right) + \frac{c_{do}\kappa_{do}}{r}\left(\frac{1-\beta}{\beta}\right),$$

which is one equation in the unknown market tightness. Likewise, allowing for free entry into both existence and search for retailers and producers would give the following system that governs κ_{do} and the existence of producers:

$$\begin{aligned}
\frac{c_{do}}{\chi\left(\kappa_{do}\right)} &= \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right) dG\left(\varphi\right) \\
e_d^x &= \frac{h_{do}}{r} G\left(\bar{\varphi}_{do}\right).
\end{aligned}$$

An endogenous producer entry decision would also require small changes to aggregate accounting. For example, we could follow Melitz (2003) and assume that all profits are used to create new firms, $\Pi = N_d^x e_d^x$, instead of our global mutual fund assumption. In this approach we would remove the expenditure on the creation of new firms from investment in our aggregate resource constraint (equation 22).

A.10 Aggregate resources

A.10.1 Number of producers

Similar to Chaney (2008), we assume that the number of producers in the origin market that take a draw from the productivity distribution is proportional to consumption expenditure in the economy, C_o. The basic intuition behind this is that larger economies have a larger stock of potential entrepreneurs. To make this explicit, we denote the total mass of potential entrants as $N_o^x = \xi_o C_o$, in which the proportionality constant $\xi_o \in [0, \infty)$ captures exogenous structural factors that affect the number of potential entrants in country

k. Among others, these could include such factors as literacy levels and attitudes toward entrepreneurship. As discussed in section A.10.2, because the number of producers is fixed, the economy has profits. We assume that a global mutual fund collects worldwide profits and redistributes them as π dividends per share to each worker who owns w_o shares. We assume that $\xi_o = \frac{1}{1+\pi}$ so that

$$N_o^x = \frac{C}{(1+\pi)} \frac{C_o}{C} \tag{A52}$$

in which we have multiplied and divided by global consumption, C.

A.10.2 Aggregate accounting and the global mutual fund

Our economy has profits because we restrict producer entry and the model features monopolistic competition. We define a global mutual fund that collects all profits in the economy and rebates them back to consumers. In order to calculate total profits, we first define variable profits earned in each market pair as

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} \pi_{do}(\varphi)\, dG(\varphi)$$

in which $\pi_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi)$. Our functional form assumptions and the pricing rule in (15) ensure that profits are proportional to sales:
$p_{do}(\varphi)\, q_{do}(\varphi) - t_{do} = p_{do}(\varphi)\, q_{do}(\varphi)\, \sigma^{-1}$. Aggregating profits from each variety provides

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi)\, \sigma^{-1} dG(\varphi) = \frac{C_{do}}{\sigma} \tag{A53}$$

in which we define the value of total consumption in destination d of the differentiated good from origin o as

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi).$$

This definition for the value of consumption is consistent with equation (1) in the main text.

The income that consumers in o earn and can spend on consumption, C_o, comes from three sources. The first two sources are labor income in the production and investment sectors of the economy, $w_d L_d$. The third source is dividends from the global mutual fund, which we assume owns all firms in all countries. Each country gets a share, π, of total global profits proportional to labor income in the economy. Explicitly GDP can be written as

$$Y_o = w_o L_o (1+\pi),$$

in which

$$\pi = \Pi / \sum w_k L_k. \tag{A54}$$

Notice that the dividend per unit value of labor π is proportional to the value of the global labor endowment and so also matches Chaney (2008) equation (6) in our model. Wage income is derived providing the fixed cost of production, the formation of relationships, creating new retailers and producers, and the variable cost of production:

$$w_o L_o = \sum_{k=1}^{D} \Phi^i_{ko} + \sum_{k=1}^{D} \Phi^p_{ko} + w_o q_o (1) + \Phi^e_o$$

in which

$$\begin{aligned}
\Phi^i_{do} &= \kappa_{od} u_{od} N^x_d c_{od} + u_{do} N^x_o \left(l_{do} + s_{do}\kappa_{do}\chi\left(\kappa_{do}\right)\right) + \left(1 - u_{do} - i_{do}\right) N^x_o f_{do} \\
\Phi^p_{do} &= \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_o \int_{\bar{\varphi}_{do}} t_{do}(\varphi)\, dG(\varphi) \\
\Phi^e_o &= N^x_o e^x_o
\end{aligned}$$

The production structures for the homogeneous good is undetermined because it is freely traded and has constant returns to scale production. Like Chaney (2008), we only consider equilibrium in which every country produces some of that good. In order to simplify and make accounting for resources in every country symmetric, we also assume that each country produces what it would like to consume itself, namely $q_o(1)$. While the good can be freely traded, in equilibrium there is no international trade of the homogeneous good. Despite no trade in this good, its price is the same in all countries because of a no-arbitrage condition. Each unit of the homogeneous good requires one unit of labor to produce so the cost of producing $q_o(1)$ units of the homogeneous good is given by $w_o q_o(1)$. Importantly, our definition of the income earned from labor used in producing the differentiated good, Φ^p_{do}, includes the iceberg transport costs so that labor is compensated for transporting goods.

Summing payments to labor across all countries of the world gives

$$\Phi^h = \sum_{k=1}^{O} w_k q_k (1), \qquad \Phi^i = \sum_{k=1}^{O}\sum_{j=1}^{D} \Phi^i_{jk}, \qquad \Phi^p = \sum_{k=1}^{O}\sum_{j=1}^{D} \Phi^p_{jk}, \qquad \Phi^e = \sum_{k=1}^{O} \Phi^e_k. \tag{A55}$$

Similarly, we can define global variable profits from operation in each market either as equation (A53) or as

$$\Pi_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_o \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi) - t_{do}(\varphi)\, dG(\varphi) = C_{do} - \Phi^p_{do} \tag{A56}$$

Summing variable profits throughout the world provides global profits

$$\Pi = \sum_{k=1}^{O}\sum_{j=1}^{D} \Pi_{jk} = \sum_{k=1}^{O}\sum_{j=1}^{D} C_{jk} - \Phi^p_{jk} = \sum_{k=1}^{O}\sum_{j=1}^{D} \frac{C_{jk}}{\sigma} = \frac{\alpha}{\sigma} C \tag{A57}$$

The last two equalities come from our functional form assumptions. We can check that we have treated the global mutual fund correctly by ensuring that global income equals global expenditure. Start by defining investment in each market

$$I_{do} = \kappa_{od} u_{od} N^x_d c_{od} + u_{do} N^x_o \left(l_{do} + s_{do}\kappa_{do}\chi\left(\kappa_{do}\right)\right) + \left(1 - u_{do} - i_{do}\right) N^x_o f_{do} + N^x_o e^x_o$$

in which it is also clear that $I_{do} = \Phi^i_{do} + \Phi^e_{do}$ and global investment is

$$I = \sum_{k=1}^{O} \sum_{j=1}^{D} I_{jk}.$$

Global consumption of both homogeneous and differentiated goods is

$$C = \sum_{k=1}^{O} C_k = \sum_{k=1}^{O} p_k (1) q_k (1) + \sum_{k=1}^{O} \sum_{j=1}^{D} C_{jk}$$

To check that we have everything correct, start with total resources available in the economy $Y_o = w_o L_o (1 + \pi)$ and sum across economies

$$\begin{aligned}
\sum_{k=1}^{O} Y_k &= \sum_{k=1}^{O} w_k L_k (1 + \pi) \\
Y &= (1 + \pi) \sum_{k=1}^{O} w_k L_k \\
Y &= \left(1 + \frac{\Pi}{\sum_{k=1}^{O} w_k L_k}\right) \sum_{k=1}^{O} w_k L_k \\
Y &= \sum_{k=1}^{O} w_k L_k + \Pi \\
Y &= \Pi + \sum_{o=1}^{O} \left(\sum_{k=1}^{D} \Phi^i_{ko} + \sum_{k=1}^{D} \Phi^p_{ko} + w_o q_o (1) + \Phi^e_o \right) \\
Y &= \Pi + \Phi^i + \Phi^e + \Phi^p + \Phi^h
\end{aligned}$$

We can finish the proof by starting with the last line, which is the income approach to accounting, and showing that this expression also gives the expenditure approach

$$\begin{aligned}
Y &= \Pi + \Phi^h + \Phi^i + \Phi^e + \Phi^p \\
Y &= \Pi + \Phi^h + I + \Phi^p \\
Y &= \sum_{k=1}^{O} \sum_{j=1}^{D} \left(C_{jk} - \Phi^p_{jk}\right) + \Phi^h + I + \Phi^p \\
Y &= \sum_{k=1}^{O} \sum_{j=1}^{D} C_{jk} + \Phi^h + I \\
Y &= C + I
\end{aligned}$$

so that

$$Y = C + I, \tag{A58}$$

and

$$C = Y - I. \tag{A59}$$

Notice that we used

$$\Phi^h = \sum_{k=1}^{O} w_k q_k(1) = \sum_{k=1}^{O} p_k(1) q_k(1)$$

in the last line. Costless trading of the homogeneous good delivers a "no arbitrage condition," implying that its price must be the same in all countries, $p_k(1) = p(1)$. Because the homogeneous good is made with one unit of labor in each country, it must also be that $w_k = p_k(1) = p(1)$ pinning down the equilibrium wage in every country.

Finally, we point out that total resources in each economy are given by $Y_o = w_o L_o (1 + \pi)$. Total resources are larger than the labor endowment because the definitions of payments to labor do not account for an existing mass of firms. With an existing mass of firms, the global economy is endowed not only with labor but also with that mass. This non-labor endowment is reflected in profits made by those firms. If there is a pre-existing mass of firms that does not make profits, the additional resources are paid to labor in the form of production costs. Without a pre-existing mass of firms, the cost of creating new firms is captured in the payments to labor, Φ^e, when creating those firms.

Notice that for one country, equation (A58) can be written as

$$\begin{aligned} Y_d &= C_d + I_d \\ &= p_d(1) q_d(1) + \sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi) q_{dk}(\varphi) dG(\varphi) \\ &+ N_d^x e_d^x + \sum_{k=1}^{O} \kappa_{dk} u_{dk} N_k^x c_{dk} + u_{kd} N_d^x (l_{kd} + s_{kd} \kappa_{kd} \chi(\kappa_{kd})) + (1 - u_{kd} - i_{kd}) N_d^x f_{kd}, \end{aligned}$$

which is equation (22) in the main text.

A.11 The ideal price index with our productivity distribution

A.11.1 Moving from an index to a distribution of goods

Melitz uses the following steps to move from index ω_{do} over a continuum of goods available to consume, Ω, which we assume has measure $M_{do} = |\Omega_{do}|$, to the cumulative distribution of productivity $G(\varphi)$ and the measure of goods available for consumption $(1 - i_{do}) M_{do}$.

The following steps keep the notation in Melitz's original work. Begin with the definition for the change of variables, also known as integration by substitution, which states

$$\int_a^b f(h(\varphi)) h'(\varphi) d\varphi = \int_{i(a)}^{i(b)} f(\omega) d\omega$$

Choose to index the goods ω with the indexing number $G(\varphi) M_{do}$ which is differentiable in φ such that $\omega = h(\varphi) = G(\varphi) M_{do}$. Then we can apply the rule from left to right to get

$$\int_0^\infty f(G(\varphi) M_{do}) \frac{\partial G(\varphi) M_{do}}{\partial \varphi} d\varphi = \int_{G(0) M_{do}}^{G(\infty) M_{do}} f(\omega) d\omega = \int_0^{M_{do}} f(\omega) d\omega = \int_{\omega \in \Omega_{do}} f(\omega) d\omega.$$

We choose $G(0) = 0$ and $G(\infty) = 1$ in our context because $G(\varphi)$ is a cumulative distribution function and it allows us to start the continuous indexing such that the upper bound of the integral is the measure of Ω_{do}. More generally, change of variables allows for any $G(\varphi)$ as long as $G(\varphi)$ is differentiable in φ.

Remember that in our context $f(\omega)$ is a function that simply indexes the continuum of goods ω so that $f(\omega)$ does not vary with ω even though $f(\varphi)$ will vary with φ. Therefore, we can reassign the indexing number $G(\varphi)M_{do}$ to φ to get

$$\int_0^\infty f(G(\varphi)M_{do})\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = \int_0^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi,$$

We often integrate over $[\bar{\varphi}_{do},\infty)$ and not $[0,\infty)$ because some goods are not available in equilibrium. As long as $f(\varphi)=0$ when $\varphi<\bar{\varphi}_{do}$, we can ignore those goods and

$$\int_0^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = \int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi$$

In order to relate this expression to economically meaningful concepts, it is helpful to rewrite this as

$$\int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{\partial G(\varphi)M_{do}}{\partial\varphi}d\varphi = (1-i_{do})\,M_{do}\int_{\bar{\varphi}_{do}}^\infty f(\varphi)\frac{g(\varphi)}{(1-i_{do})}d\varphi$$

in which $i_{do}=G(\bar{\varphi}_{do})$, $g(\varphi)=\partial G(\varphi)/\partial\varphi$, and $g(\varphi)$ is a proper density because $1=\int_{\bar{\varphi}_{do}}^\infty g(\varphi)\,(1-i_{do})^{-1}\,d\varphi$. This implies that the measure of goods available to consume is $(1-i_{do})\,M_{do}$ and the density of goods available to consume is given by $g(\varphi)\,(1-i_{do})^{-1}$. The analogous measure of goods available to consume in our model is $(1-u_{do}-i_{do})\,N_o^x$ and we have the same density of goods as Melitz because the unmatched fraction of products, u_{do}, is still available to be matched and consumed.

A.11.2 Differentiated goods price index

We are able to map from the price index defined using varieties, ω, in equation (3) to a price index in terms of firm productivities, φ, using the approach in appendix A.11.1 to obtain:

$$P_d = \left[\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}^\infty p_{dk}(\varphi)^{1-\sigma}\,dG(\varphi)\right]^{\frac{1}{1-\sigma}},$$

in which $G(\cdot)$ is a cumulative density function that is defined as Pareto distributed in section 2.2.2. With our assumptions about demand and the production structure in sections 2.1 and 2.2.2 we get equation (15), which is $p_{do}(\varphi)=\mu w_o\tau_{do}\varphi^{-1}$. Plugging this into the price index gives

$$P_d = \left[\sum_{k=1}^{O}\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}^\infty\left(\frac{\mu w_k\tau_{dk}}{\varphi}\right)^{1-\sigma}dG(\varphi)\right]^{\frac{1}{1-\sigma}}$$

then we can use the moment $\int_{\bar{\varphi}_{dk}}^{\infty} z^{\sigma-1} dG(z) = \dfrac{\theta \bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta-\sigma+1}$ to get

$$P_d = \left[\sum_{k=1}^{O} \left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right) N_k^x \left(\mu w_k \tau_{dk}\right)^{1-\sigma} \frac{\theta \bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta-\sigma+1}\right]^{\frac{1}{1-\sigma}}$$

The threshold productivity is given in equation (18) in the main text, which is

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d}\right) \left(\frac{F_{do}\left(\kappa_{do}\right)}{C_d}\right)^{\frac{1}{\sigma-1}}$$

By substituting the threshold into the price index we get

$$P_d = \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \left(\sum_{k=1}^{O} \left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right) N_k^x \left(w_k \tau_{dk}\right)^{-\theta} F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$$

Then we can employ our definition for the number of producers from section A.10.1 to derive

$$P_d = \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \left(\sum_{k=1}^{O} \left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right) \left(\frac{C}{1+\pi}\right) \frac{C_k}{C} \left(w_k \tau_{dk}\right)^{-\theta} F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}}$$

Slightly rearranging terms and using the fact that $\left(1-\dfrac{u_{dk}}{1-i_{dk}}\right) = \dfrac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda+\kappa_{dk}\chi\left(\kappa_{dk}\right)}$ the price index becomes

$$\begin{aligned} P_d = & \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}} \\ \times & \; C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \\ \times & \left(\sum_{k=1}^{O} \frac{C_k}{C} \left(\frac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda+\kappa_{dk}\chi\left(\kappa_{do}\right)}\right) \left(w_k \tau_{dk}\right)^{-\theta} F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}} \end{aligned}$$

The final expression of the differentiated goods price index is a simple function of three terms

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d \tag{A60}$$

in which

$$\rho_d \equiv \left(\sum_{k=1}^{O} \frac{C_k}{C} \left(\frac{\kappa_{dk}\chi\left(\kappa_{dk}\right)}{\lambda+\kappa_{dk}\chi\left(\kappa_{dk}\right)}\right) \left(w_k \tau_{dk}\right)^{-\theta} F_{dk}\left(\kappa_{dk}\right)^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}},$$

and

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}}.$$

The expression in (A60) resembles the price index in Chaney (2008), equation (8) closely.

We note that ρ_d, the "multilateral resistance term," in that model is an equilibrium object in wages and GDP, whereas now it's an equilibrium object in wages, total consumption expenditure, and market tightness.

A.12 Defining the equilibrium

The equilibrium reduces to these equations in the equilibrium variables:

1. The free entry condition for retailers, which pins down κ_{do}:

$$\frac{c_{do}}{\chi(\kappa_{do})} = \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi)$$

Notice that now there are d times o markets and each market has an associated tightness. With our functional form assumptions, this equation can be simplified. Remember that with $V_{do} = 0$

$$M_{do}(\varphi) = \frac{p_{do}q_{do} - n_{do}q_{do}}{r + \lambda}$$

so that

$$\begin{aligned}\int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi) &= \left(\frac{1}{r+\lambda}\right)\int_{\bar{\varphi}_{do}} p_{do}q_{do} - n_{do}q_{do} dG(\varphi) \\ \Rightarrow &= \left(\frac{1}{r+\lambda}\right)\left(1 - \frac{u_{do}}{1-i_{do}}\right)^{-1}\left(\frac{1}{N_o^x}\right)\Pi_{do}^m\end{aligned}$$

in which Π_{do}^m is defined in equation (A72) and we know that

$$\Pi_{do}^m = b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})\, C_{do}$$

Using the equilibrium retailer entry condition gives

$$\frac{c_{do}}{\chi(\kappa_{do})} = \left(\frac{1}{r+\lambda}\right)\left(1 - \frac{u_{do}}{1-i_{do}}\right)^{-1}\left(\frac{1}{N_o^x}\right)\Pi_{do}^m \tag{A61}$$

so that

$$\kappa_{do} = \left(\frac{1}{r+\lambda}\right)(\lambda + \kappa_{do}\chi(\kappa_{do}))\frac{b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})(1+\pi)}{c_{do}C_o}C_{do},$$

in which we used $N_o^x = \dfrac{1}{1+\pi}C_o$ and C_{do} can be obtained by using equation (26), which is a function of equilibrium variables. In sum, this equilibrium condition can be written neatly as

$$\kappa_{do} = \left(\frac{1}{r+\lambda}\right)(\lambda + \kappa_{do}\chi(\kappa_{do}))\frac{b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})(1+\pi)}{c_{do}C_o}C_{do}.$$

2. The expression that equates variable profits with the effective entry cost, which pins down $\bar{\varphi}_{do}$:

$$\bar{\varphi}_{do} = \mu\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_o\tau_{do}}{P_d}\right)C_d^{\frac{1}{1-\sigma}}F_{do}(\kappa_{do})^{\frac{1}{\sigma-1}}.$$

in which

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d$$

and

$$\rho_d \equiv \left(\sum_{k=1}^{O} \frac{C_k}{C} \left(\frac{\kappa_{dk}\chi(\kappa_{dk})}{\lambda + \kappa_{dk}\chi(\kappa_{dk})} \right) (w_k \tau_{dk})^{-\theta} F_{dk}(\kappa_{dk})^{-\left[\frac{\theta}{\sigma-1}-1\right]} \right)^{-\frac{1}{\theta}}$$

and

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)} \right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi} \right)^{-\frac{1}{\theta}}.$$

In this simplification we have used the assumption that

$$N_o^x = \frac{1}{1+\pi} C_o$$

3. National accounting/consumer's budget constraint pins down consumption C_d:

$$C_d = Y_d - I_d$$

in which

$$I_d = \sum_{k=1}^{O} I_{dk} = N_d^x e_d^x + \sum_{k=1}^{O} \kappa_{dk} u_{dk} N_k^x c_{dk} + u_{kd} N_d^x \left(l_{kd} + s_{kd}\kappa_{kd}\chi(\kappa_{kd}) \right) + \left(1 - u_{kd} - i_{kd}\right) N_d^x f_{kd}$$

and

$$N_d^x = \frac{1}{1+\pi} C_d$$

and

$$Y_d = w_d L_d (1+\pi).$$

4. The global mutual fund pins down π:

$$\pi = \frac{\Pi}{\sum_k w_k L_k}$$

in which Π are the profits from the differentiated goods sold in all countries

$$\begin{aligned} \Pi &= \sum_k \sum_j \Pi_{jk} = \sum_k \sum_j \left(1 - \frac{u_{jk}}{1-i_{jk}} \right) N_k^x \int_{\bar{\varphi}_{jk}} p_{jk}(\varphi) q_{jk}(\varphi) - t_{jk}(\varphi) dG(\varphi) \\ &= \alpha \frac{C}{\sigma} \end{aligned}$$

A.13 A graphical depiction of the model

We detail the shape of each line in figure 1. For each sub-figure, the equilibrium values of other variables are taken as given.

A.13.1 Retailers' expected negotiated cost and market tightness

First, we characterize the retailers' expected negotiated cost in figure 1a. Begin with the negotiated price (equation 14), multiply by equilibrium quantity q_{do}, and take an expectation to obtain:

$$\mathbb{E}_{\varphi}\left[n_{do}q_{do}\right]=\int_{\bar{\varphi}_{do}}p_{do}q_{do}+\frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\xi\kappa_{do}^{1-\eta}}\left[t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}-p_{do}q_{do}\right]dG\left(\varphi\right), \tag{A62}$$

in which we have assumed that $s_{do}=0$ and that the retailer contact rate comes from the matching function (equation 4) and the expectation is taken over productivity. Figure 1a uses our baseline calibration and does not require that $s_{do}=0$ but these proofs do. We differentiate this expression with respect to κ_{do},

$$\frac{\partial\mathbb{E}_{\varphi}\left[n_{do}q_{do}\right]}{\partial\kappa_{do}}=\int_{\bar{\varphi}_{do}}-\frac{(r+\lambda)(1-\beta)}{\left(r+\lambda+\beta\xi\kappa_{do}^{1-\eta}\right)^{2}}\beta\xi\left(1-\eta\right)\kappa_{do}^{-\eta}\left[t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}-p_{do}q_{do}\right]dG\left(\varphi\right),$$

which is positive. As $\kappa_{do}\to 0$ this derivative tends to ∞ and as $\kappa_{do}\to\infty$ this derivative tends to 0. The second derivative is negative so the retailers' expected negotiated cost curve is concave.

The intercept of equation (A62) when $\kappa_{do}=0$ is

$$\mathbb{E}_{\varphi}\left[n_{do}q_{do}\right]=\int_{\bar{\varphi}_{do}}\beta p_{do}q_{do}+(1-\beta)\left[t\left(q_{do},w_o,\tau_{do},\varphi\right)+f_{do}-l_{do}\right]dG\left(\varphi\right).$$

As $\kappa_{do}\to\infty$ this equation converges to

$$\mathbb{E}_{\varphi}\left[n_{do}q_{do}\right]=\int_{\bar{\varphi}_{do}}p_{do}q_{do}dG\left(\varphi\right),$$

which recovers the Melitz (2003) model. Computing the relevant integrals implies that equation (A62) can be written as

$$\mathbb{E}_{\varphi}\left[n_{do}q_{do}\right]=\frac{C_{do}}{\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x}+\frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\xi\kappa_{do}^{1-\eta}}\left[f_{do}-l_{do}-\frac{C_{do}}{\sigma\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x}\right]$$

in which all terms but κ_{do} are taken as given for the graph in $(\kappa_{do},\mathbb{E}_{\varphi}\left[n_{do}q_{do}\right])$ space.

Second, we characterize the retailers' entry curve in figure 1a. Begin with the entry condition (equation 19), plug in for $M_{do}(\varphi)$ from appendix A.12, the retailer contact rate from the matching function (equation 4), and solve for the expected negotiated cost to get:

$$\mathbb{E}_{\varphi}\left[n_{do}q_{do}\right]=\int_{\bar{\varphi}_{do}}p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)dG\left(\varphi\right)-(r+\lambda)\frac{\kappa_{do}^{\eta}c_{do}}{\xi}=\frac{C_{do}}{\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x}-(r+\lambda)\frac{\kappa_{do}^{\eta}c_{do}}{\xi}, \tag{A63}$$

in which all terms but κ_{do} are the equilibrium values for the graph in $(\kappa_{do},\mathbb{E}_{\varphi}\left[n_{do}q_{do}\right])$ space.

Equation (A63) is negatively sloped with the derivative tending to $-\infty$ as $\kappa_{do}\to 0$ and the derivative tending to 0 as $\kappa_{do}\to\infty$. The second derivative it positive so that the line is convex and bowed in toward the origin. The intercept of equation (A63) when $\kappa_{do}=0$ is $\int_{\bar{\varphi}_{do}}p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)dG\left(\varphi\right)$, which is taken as given in this figure. The intercept when

$\mathbb{E}[n_{do}q_{do}] = 0$ is

$$\kappa_{do} = \left[\frac{\xi \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi)}{c(r+\lambda)}\right]^{\frac{1}{\eta}}.$$

Notice that we are guaranteed existence and uniqueness of $\mathbb{E}[n_{do}q_{do}]$ and κ_{do} in this market. This is because the intercept of the entry condition is $\int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi)$ and the intercept of the expected cost curve is $\int_{\bar{\varphi}_{do}} \beta p_{do}q_{do} + (1-\beta)\left[t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do}\right] dG(\varphi)$ and we know that the former is larger than the latter and we know that the entry condition slopes down and the expected cost curve slopes up.

A.13.2 Negotiated price for one good

Third, we characterize the negotiated price curve in figure 1b, which follows similar steps to above. Begin with the negotiated price (equation 14) and re-arrange to obtain:

$$\begin{aligned} n_{do}(\varphi) &= p_{do}(\varphi) \\ &+ (r+\lambda)(1-\beta)\left[r+\lambda+\beta\xi\kappa_{do}^{1-\eta}\right]^{-1}\left[\frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - p_{do}(\varphi)\, q_{do}(\varphi)}{q_{do}(\varphi)}\right], \end{aligned} \tag{A64}$$

in which we have assumed that $s_{do} = 0$ and that the retailer contact rate comes from the matching function (equation 4). We differentiate this expression with respect to κ_{do},

$$\frac{\partial n_{do}(\varphi)}{\partial \kappa_{do}} = -\frac{(r+\lambda)(1-\beta)}{\left(r+\lambda+\beta\xi\kappa_{do}^{1-\eta}\right)^2}\beta\xi(1-\eta)\,\kappa_{do}^{-\eta}\left[\frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - p_{do}(\varphi)\, q_{do}(\varphi)}{q_{do}(\varphi)}\right],$$

which is positive. We need not employ the Leibniz integral rule because the threshold productivity is assumed constant for the figure and we take as given the negotiated prices for other varieties. This derivative shows that the negotiated price slopes up in $(\kappa_{do}, n_{do}(\varphi))$ space. Notice that as $\kappa_{do} \to 0$ the derivative tends to ∞ and as $\kappa_{do} \to \infty$ the derivative tends to 0. The second derivative of the expected cost curve is negative which means that the function is concave.

The intercept of equation (A64) when $\kappa_{do} = 0$ is:

$$n_{do}(\varphi) = \beta p_{do}(\varphi) + (1-\beta)\left[\frac{t(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do}}{q_{do}(\varphi)}\right].$$

The value of equation (14) as $\kappa_{do} \to \infty$ is:

$$n_{do}(\varphi) = p_{do}(\varphi),$$

and so the negotiated price curve asymptotes to the final sales price in the Melitz (2003) model.

Fourth, we characterize the vertical market tightness line in figure 1b. Inspecting the free entry condition for retailers (equation 19) suggests that the negotiated price of a single infinitesimal producer does not affect goods-market tightness. As such, the market tightness a vertical line in $(\kappa_{do}, n_{do}(\varphi))$ space. The market tightness is taken from figure 1a.

A.13.3 Final sales price and negotiated quantity

Fifth, we characterize the marginal cost curve in figure 1c. First, recall the variable cost equation:

$$t\left(q_{do}, w_o, \tau_{do}, \varphi\right) = q_{do} w_o \tau_{do} \varphi^{-1}.$$

Differentiate to get marginal cost:

$$\frac{\partial t\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}} = w_o \tau_{do} \varphi^{-1}, \tag{A65}$$

which is independent of q_{do} so is a horizontal line in $\left(q_{do}\left(\varphi\right), p_{do}\left(\varphi\right)\right)$ space.

Sixth, we characterize the demand curve and the marginal revenue curves in figure 1c. Being with the demand curve for a differentiated variety (equation 2) and solve for inverse demand, $p_{do}\left(\varphi\right) = q_{do}\left(\varphi\right)^{-1/\sigma}\left(\alpha C_d / P_d^{1-\sigma}\right)^{1/\sigma}$, which is downward sloping because

$$\frac{\partial p_{do}\left(\varphi\right)}{\partial q_{do}\left(\varphi\right)} = -\frac{1}{\sigma} q_{do}\left(\varphi\right)^{-\frac{1}{\sigma}-1}\left(\frac{\alpha C_d}{P_d^{1-\sigma}}\right)^{1/\sigma} < 0.$$

The second derivative is positive, which means the curve is convex and bowed into the origin. Marginal revenue is given by

$$p_{do}\left(q_{do}\right) + \frac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}} q_{do} = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\alpha C_d}{P_d^{1-\sigma}}\right)^{\frac{1}{\sigma}} q_{do}\left(\varphi\right)^{-\frac{1}{\sigma}}, \tag{A66}$$

which is also downward sloping. Furthermore, marginal revenue curve always lies under the demand curve because $\left(\sigma-1\right)/\sigma < 1$. Bargaining over quantity implies that the equilibrium quantity exchanged *within* matches equates the marginal production cost (equation A65) with retailers' marginal revenue from consumers (equation A66), as shown in appendix A.5.1 equation (A38). The resulting equilibrium price is equation (15) in the main text. Notice that we are guaranteed existence and uniqueness of $p^*_{do}\left(\varphi\right)$ and $q^*_{do}\left(\varphi\right)$ in this market. This result follows because the marginal revenue curve is downward sloping with values ranging from ∞ to zero and the marginal cost curve is horizontal. Furthermore, for any variety above the threshold productivity, $\bar{\varphi}_{do}$, this figure, along with parameter restrictions such as $\sigma > 1$ and $\tau_{do} \geq 1$, and the equilibrium values for other endogenous variables, like w_o, ensures that $p^*_{do}\left(\varphi\right) > 0$ and $q^*_{do}\left(\varphi\right) > 0$.

A.13.4 Dividend and producer threshold

Seventh, we characterize the dividend curve in figure 1d. Recall that

$$\pi = \frac{\alpha C}{\sigma \sum_k w_k L_k}, \tag{A67}$$

which does not vary with the threshold productivity because consumption and wages are determined in another equilibrium figure and the economy's labor is exogenously endowed.

Sixth, we characterize the threshold productivity curve in figure 1d. For ease, we will work in $\left(\pi, \bar{\varphi}_{do}\right)$ space and then interpret our results in $\left(\bar{\varphi}_{do}, \pi\right)$ space. Recall the threshold productivity from equation (18)

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha}\right) \left(\frac{w_o \tau_{do}}{\lambda_2 \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d}\right) C_d^{\frac{1}{1-\sigma}} F_d^{\frac{1}{\sigma-1}},$$

in which

$$\rho_d \equiv \left(\sum_{k=1}^{O} \frac{C_k}{C} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) (w_k \tau_{dk})^{-\theta} F_{dk}^{-\left[\frac{\theta}{\sigma-1}-1\right]}\right)^{-\frac{1}{\theta}},$$

and

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi}\right)^{-\frac{1}{\theta}}.$$

Plugging in these definitions and re-arranging, yields the threshold productivity curve:

$$\bar{\varphi}_{do} = \left(\frac{\sigma}{\alpha}\right) \left(\frac{w_o \tau_{do}}{C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d}\right) C_d^{\dfrac{1}{1-\sigma}} F_d^{\dfrac{1}{\sigma-1}} \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\theta}-\frac{1}{\sigma-1}} C^{\frac{1}{\theta}} (1+\pi)^{-\frac{1}{\theta}}. \tag{A68}$$

This line is downward sloping because

$$\frac{\partial \bar{\varphi}_{do}}{\partial \pi} = -\frac{1}{\theta} \left(\frac{\sigma}{\alpha}\right) \left(\frac{w_o \tau_{do}}{C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \rho_d}\right) C_d^{\dfrac{1}{1-\sigma}} F_d^{\dfrac{1}{\sigma-1}} \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{\frac{1}{\theta}} \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\theta}-\frac{1}{\sigma-1}} C^{\frac{1}{\theta}} (1+\pi)^{-\frac{1}{\theta}-1} < 0.$$

The second derivative is positive so this line is convex and bowed in toward the origin. As such, the line is negatively sloped and convex in $(\bar{\varphi}_{do}, \pi)$ space. The intercept of this line as $\pi \to 0$ is:

$$\bar{\varphi}_{do} = \left(\frac{\sigma}{\alpha}\right)^{1+\frac{1}{\theta}-\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{\rho_d}\right) C_d^{\dfrac{1}{1-\sigma}} F_d^{\dfrac{1}{\sigma-1}} \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{\frac{1}{\theta}} C_d^{\frac{1}{\sigma-1}} > 0.$$

And, when $\bar{\varphi}_{do} = 1$ the per-capita dividend is positive $\pi > 0$.

Notice that we are guaranteed existence and uniqueness of $\bar{\varphi}_{do}$ and π in this market. This result follows because the threshold productivity curve is downward sloping and the per-capita dividend curve is horizontal. As long as the value of π when $\bar{\varphi} = 1$ is above the value of the per-capita dividend, the two lines cross. This condition puts a restriction on the size of the effective entry cost for some normalizations of the equilibrium wage, w_o, which is important for our calibration. We discuss the producers' flow idle benefit, the effective entry cost, and the minimum productivity draw in the context of our calibration in appendix C.5.

A.13.5 Wage and total consumption

Eighth, we characterize the consumption curve in figure 1e. Using the expenditure and income approaches to national accounting we obtain

$$\begin{aligned} w_d &= \frac{\sum_{k\neq d}^{O} \kappa_{dk} u_{dk} N_k^x c_{dk}}{L_d\left(1+\pi\right)} \\ &+ \frac{C_d\left(\frac{1}{1+\pi}\right)\left[\left(1+\pi\right)+e_d^x+\kappa_{dd}u_{dd}c_{dd}+\sum_{k=1}^{O} u_{kd}\left(l_{kd}+s_{kd}\kappa_{kd}\chi\left(\kappa_{kd}\right)\right)+\left(1-u_{kd}-i_{kd}\right)f_{kd}\right]}{L_d\left(1+\pi\right)}, \end{aligned} \tag{A69}$$

which implies a linear relationship in (C_d, w_d) space with positive intercept and positive slope.

Finally, we discuss in section 2.2.2 the production structure of the homogeneous good and that it implies that $w_d^* = 1 \forall d$, as in Chaney (2008). This line is horizontal in (C_d, w_d) space because it does not depend on consumption.

Notice that we are guaranteed existence and uniqueness of w_d and C_d in this market. This result follows because consumption is increasing in wages and wages are fixed at 1. As long as the intercept of the upward sloping line is lower than 1:

$$\sum_{k\neq d}^{O} \kappa_{dk} u_{dk} N_k^x c_{dk} < L_d\left(1+\pi\right),$$

the two lines cross.

A.14 The gravity equation with search frictions

A.14.1 Proof of proposition 1: Deriving the gravity equation

The value of total imports will be

$$IM_{do} = \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}\left(\varphi\right) q_{do}\left(\varphi\right) dG\left(\varphi\right).$$

We need to integrate over the varieties to get the total value of imports going into the domestic market. Demand for a variety, φ, in the differentiated goods sector is given in equation (2): $q_{do}\left(\varphi\right) = p_{do}\left(\varphi\right)^{-\sigma} \dfrac{\alpha C_d}{P_d^{1-\sigma}}$. Given this demand, monopolistic competition, and constant returns-to-scale production imply that producers set optimal prices according to equation (15): $p_{do}\left(\varphi\right) = \mu w_o \tau_{do} \varphi^{-1}$. For notational simplicity, define $B_{do} \equiv \alpha\left(\mu w_o \tau_{do}\right)^{-\sigma} C_d P_d^{\sigma-1}$ and combine the optimal price with the demand curve to get $q_{do}\left(\varphi\right) = B_{do}\varphi^{\sigma}$. Evaluated at final prices, the value of sales of each variety is $p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) = \mu w_o \tau_{do} B_{do} \varphi^{\sigma-1}$ and the cost to produce $q_{do}\left(\varphi\right)$ units of this variety is $t_{do}\left(\varphi\right) + f_{do} = w_o \tau_{do} B_{do} \varphi^{\sigma-1} + f_{do}$. These expressions imply that total profits generated by each variety are $p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) - t_{do}\left(\varphi\right) = A_{do}\varphi^{\sigma-1}$, in which it is also useful to define $A_{do} = w_o \tau_{do} B_{do}\left[\mu - 1\right]$. Using this profits expression, the productivity cutoff is $\bar{\varphi}_{do} = \left(\dfrac{F_{do}}{A_{do}}\right)^{\frac{1}{\sigma-1}}$, in which F_{do} is given in equation (18). The value of total imports from the negotiated price curve in equation (14) is

$$n\left(\varphi\right) q_{do}\left(\varphi\right) = \left[1-\gamma_{do}\right] p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) + \gamma_{do}\left[t_{do}\left(\varphi\right) + f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right) s_{do}\right].$$

Using the functional forms assumptions from above this becomes

$$n(\varphi)\, q_{do}(\varphi) = (\sigma - \gamma_{do})\, A_{do}\varphi^{\sigma-1} - \gamma_{do}\left[-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}\right].$$

Substituting the value of imports for a particular variety into the integral defining the value of total imports gives

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} (\sigma - \gamma_{do})\, A_{do}\varphi^{\sigma-1} - \gamma_{do}\left[-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}\right] dG(\varphi).$$

We assume productivity, φ, has a Pareto distribution over $[1, +\infty)$ with cumulative density function $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and probability density function $g(\varphi) = \theta\varphi^{-\theta-1}$. The Pareto parameter and the elasticity of substitution are such that $\theta > \sigma - 1$, which ensures that the integral $\int_{\bar{\varphi}}^{\infty} z^{\sigma-1} dG(z)$ is bounded. Using these assumptions we can compute the integral to get

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[(\sigma - \gamma_{do})\, A_{do}\frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1} - \gamma_{do}\left[-f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})\, s_{do}\right]\bar{\varphi}_{do}^{-\theta}\right],$$

in which we use the relevant moment of the productivity distribution $\int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG(z) = \dfrac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1}$. Define $\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})\, s_{do}$ to conserve on notation, substitute the export productivity threshold into this expression, and simplify to get

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[(\sigma - \gamma_{do})\frac{\theta}{\theta - \sigma + 1} + \gamma_{do}\frac{\delta_{do}}{F_{do}}\right] F_{do}^{-\left(\frac{\theta}{\sigma-1} - 1\right)} A_{do}^{\frac{\theta}{\sigma-1}}. \tag{A70}$$

Next, utilize the assumption that the number of producers in the origin market is proportional to output in that market $N_o^x = \left(\dfrac{C}{1+\pi}\right)\dfrac{C_o}{C}$ and the definition for $A_{do} = \mu^{-\sigma}\alpha\,(w_o\tau_{do})^{1-\sigma}\, C_d P_d^{\sigma-1}[\mu - 1]$ to write

$$\begin{aligned} IM_{do} &= \left(1 - \frac{u_{do}}{1 - i_{do}}\right)\left(\frac{C}{1+\pi}\right)\frac{C_o}{C}\left[(\sigma - \gamma_{do})\frac{\theta}{\theta - \sigma + 1} + \gamma_{do}\frac{\delta_{do}}{F_{do}}\right] \\ &\times F_{do}^{-\left(\frac{\theta}{\sigma-1} - 1\right)}\left(\mu^{-\sigma}\alpha\,(w_o\tau_{do})^{1-\sigma}\, C_d P_d^{\sigma-1}[\mu - 1]\right)^{\frac{\theta}{\sigma-1}}. \end{aligned}$$

We presented the price index earlier as

$$P_d = \lambda_2 \times C_d^{\frac{1}{\theta} - \frac{1}{\sigma-1}} \times \rho_d.$$

Substituting that into our value of imports and simplifying gives

$$\begin{aligned} IM_{do} &= \left(1 - \frac{u_{do}}{1 - i_{do}}\right)\left[(\sigma - \gamma_{do})\frac{\theta}{\theta - \sigma + 1} + \gamma_{do}\frac{\delta_{do}}{F_{do}}\right] \\ &\times \left(\mu^{-\sigma}\alpha[\mu - 1]\right)^{\frac{\theta}{\sigma-1}}\left(\frac{C}{1+\pi}\right)\lambda_2^{\theta}\left(\frac{C_o C_d}{C}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1} - 1\right)}. \end{aligned}$$

In the price index section earlier we also define λ_2, which we can now substitute in here and then simplify to get

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right)\left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)\right)\alpha\left(\frac{C_o C_d}{C}\right)\left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}. \tag{A71}$$

Define the bundle of search parameters

$$b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) = \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)$$

and substitute it into (A71) in order to write the gravity equation more compactly as

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right)\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)\alpha\left(\frac{C_o C_d}{C}\right)\left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}.$$

which is equation (25) in proposition 1.

A.14.2 Search frictions reduce imports

Search costs reduce imports through the unmatched rate and difference between final and negotiated prices. In order to show this result, we show that the matched rate and the aggregate markup terms are weakly in the unit interval.

First, it is easy to see that

$$\left(1 - \frac{u_{do}}{1 - i_{do}}\right) = \left(\frac{\kappa_{do}\chi\left(\kappa_{do}\right)}{\lambda + \kappa_{do}\chi\left(\kappa_{do}\right)}\right) \in [0, 1],$$

because the finding and destruction rates must be finite in any model with positive search costs, $c_{do} > 0 \forall do$.

Second, proving the bundle of search parameters

$$1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) \in [0, 1].$$

takes a few steps. Begin by proving that $\left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)\right) \leq 1$. We can prove this by noting that $\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi\left(\kappa_{do}\right)s_{do}$ so it must be that $\delta_{do} \leq F_{do}$ and therefore $\delta_{do}F_{do}^{-1} \leq 1$. Also, the restriction that $\sigma > 1$, ensures $\theta - (\sigma - 1) < \theta$. Together, these ensure $\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right) \geq 0$. Combining this with the fact that $\gamma_{do} \in [0, 1]$ ensures that $1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right) \leq 1$.

Next, show that $\left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)\right) \geq 0$ by showing that $1 \geq \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)$. Because $\gamma_{do} \in [0, 1]$ and $\sigma > 1$ we know that $\frac{\gamma_{do}}{\sigma} < 1$. Likewise, $\sigma > 1$ ensures $\theta - (\sigma - 1) < \theta$ so that $\frac{(\theta - (\sigma - 1))}{\theta} < 1$. We assume above that $\theta - (\sigma - 1) > 0$ in order to close the model. Together these imply that $\frac{(\theta - (\sigma - 1))}{\theta} \in [0, 1]$.

Finally, because $\delta_{do}F_{do}^{-1} \leq 1$ we have that $\frac{\delta_{do}}{F_{do}}\frac{(\theta-(\sigma-1))}{\theta} \leq 1$ and we have proved the result.

We can also show that

$$1+b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right) \leq \mu = \frac{\sigma}{\sigma-1}$$

Using proof by contradiction, begin by assuming that $1+b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right) > \frac{\sigma}{\sigma-1}$. Applying the definition of $b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)$, this assumption implies that

$$\gamma_{do}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-(\sigma-1)\right)\right) > \frac{\sigma\theta}{\sigma-1}$$

To close the model, we assume that $\theta-(\sigma-1)>0$ so we also know that $\theta>\sigma-1$ and that $\sigma>1$. Together these imply that

$$\theta\left(\frac{\sigma}{\sigma-1}\right) > (\sigma-1)\left(\frac{\sigma}{\sigma-1}\right) = \sigma > 1$$

Hence, our initial assumption implies that

$$\gamma_{do}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-(\sigma-1)\right)\right) > 1$$

but we showed that $\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-(\sigma-1)\right)\right) \in [0,1]$ and in section A.4.3 that $\gamma_{do}\in[0,1]$ so we have derived a contradiction and proved our desired result. Note that these steps also imply that $b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\in[0,\mu-1]$ and $1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\in\left[(\sigma-2)/(\sigma-1),1\right]$ so that the difference between negotiated and final sales prices could reduce imports by, at most, a factor of $1-(\mu-1)=(\sigma-2)/(\sigma-1)$. Finally, it is possible to relate this condition to the final sales price markup over marginal cost as $1+b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\in[1,\mu]$.

A.14.3 Consumption is imports evaluated at final sales prices

We could have also evaluated the quantity of goods imported at final sales prices $p_{do}(\varphi)$ instead of negotiated prices $n_{do}(\varphi)$. From equation (14), we can see $p_{do}(\varphi)=n_{do}(\varphi)$ if $\gamma_{do}=0$. Setting $\gamma_{do}=0$ in equation (A71) then gives

$$C_{do}=\left(1-\frac{u_{do}}{1-i_{do}}\right)\alpha\left(\frac{C_oC_d}{C}\right)\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta}F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}.$$

We can obtain the same result by integrating

$$C_{do}=\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x\int_{\bar{\varphi}_{do}}^{\infty}p_{do}(\varphi)\,q_{do}(\varphi)\,dG(\varphi).$$

A.14.4 Total period profits for importers in matched relationships

Total period profits accruing to importers in matched relationships are given by $\Pi_{do}^{m} = C_{do} - IM_{do}$. Subtracting (25) from equation (26) implies that:

$$\Pi_{do}^{m} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}. \tag{A72}$$

We could also obtain this quantity if we integrate profits to each variety over all imported varieties.

A.15 Deriving aggregate welfare

Here we outline the steps to show that the indirect utility function (welfare) is C_d/Ξ_d, in which C_d is total consumption expenditure, p is the vector of prices for each good, and Ξ_d is the ideal price index. Assume that preferences are homothetic, which is defined in Mas-Colell, Whinston, and Green (1995), section 3.B.6, page 45. This means that they can be represented by a utility function that is homogeneous of degree one in quantities and that the corresponding indirect utility function is linear in total consumption expenditure. We can begin with the indirect utility function and then manipulate it as follows

$$\begin{aligned}
W_d\left(p, C_d\right) &= W_d\left(p, 1\right) C_d \\
W_d\left(p, e\left(p, u\right)\right) &= W_d\left(p, 1\right) e\left(p, u\right) \\
u &= W_d\left(p, 1\right) e\left(p, u\right) \\
1 &= W_d\left(p, 1\right) e\left(p, 1\right) \\
\frac{1}{e\left(p, 1\right)} &= W_d\left(p, 1\right),
\end{aligned}$$

in which the first line comes from homothetic preferences; the second line follows by plugging in for consumption expenditure $C_d = e\left(p, u\right)$; the third line comes from equation (3.E.1) in MWG that says $W_d\left(p, e\left(p, u\right)\right) = u$ (also known as duality); and in the fourth line we plug in for utility level $u = 1$. The function $e\left(p, u\right)$ is the consumption expenditure function that solves the expenditure minimization problem. Using this result and the fact that the price index is defined as $e\left(p, 1\right) \equiv \Xi_d$ we can show that

$$W_d\left(p, C_d\right) = W_d\left(p, 1\right) C_d = \frac{1}{e\left(p, 1\right)} C_d = \frac{C_d}{\Xi_d}.$$

Hence, as long as preferences are homothetic, we will always get welfare equal to consumption expenditure divided by the price index, $W_d\left(p, Y\right) = C_d/\Xi_d$. The expenditure approach to accounting can be particularly useful for computing aggregate welfare in this setting because, $W_d\left(p, C_d\right) = \frac{C_d}{\Xi_d} = \frac{Y - I_d}{\Xi_d}$.

A.16 Efficiency properties of our model

Our model has the standard matching externality because retailers do not internalize how their vacancies affect matching probabilities for other firms in the economy. As we describe below, however, our model also includes participation and output externalities because producers' productivity is heterogenous and their decision to search is endogenous.

All these externalities can be internalized with adjustments to variable trade costs that attain the efficient level of market tightness in each search market. In contrast, these externalities cannot be internalized by adjusting one bargaining parameter, β, that does not vary by market and so our model differs from standard labor search models and the Hosios (1990) condition. A separate project should carefully characterize the efficiency properties of our model when producers' bargaining power varies by market, and quantify the role of various externalities.

A policymaker could adjust variable trade costs, τ_{do}, in each market to attain the efficient level of market tightness, κ_{do}^{SP} $\forall do$, similar to Brancaccio et al. (2020b). The global social planner's problem in steady state is:

$$\begin{aligned} &\max_{\kappa_{do}\forall do} \sum_d \frac{C_d}{\Xi_d} \\ s.t.\ \frac{u_{do}}{1-i_{do}} &= \frac{\lambda}{\lambda+\kappa_{do}\chi\left(\kappa_{do}\right)}\ \forall do, \\ \pi_{do}\left(\bar{\varphi}_{do}\right) &= F\left(\kappa_{do}\right)\forall do, \\ w_d L_d\left(1+\pi\right) &= C_d + I_d\ \forall d, \\ N_d^x &= \frac{C_d}{1+\pi}\ \forall d, \\ \pi &= \frac{\alpha C}{\sigma\sum_k w_k L_k}, \end{aligned} \tag{A73}$$

in which the social planner maximizes global welfare choosing tightness in each market subject to the steady-state fraction of varieties that are matched and the threshold productivity in each market, the aggregate resource constraint and the number of producers in each country, and the per-capita dividend. The ideal price index, Ξ_d, in the objective function can be obtained from section 2.1. The solution to equation (A73) yields efficient levels of market tightnesses, κ_{do}^{SP} $\forall do$.

The decentralized equilibrium is defined in section 4.3 of the paper. In particular, market tightness in each market satisfies the free-entry condition (equation 19):

$$\frac{c_{do}}{\chi\left(\kappa_{do}\right)} = \int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right)dG\left(\varphi\right),$$

conditional on the other endogenous variables. As depicted in figure 1a by the retailers' entry curve, market tightness, κ_{do}, increases as the right hand side of this equation increases. Additionally, in section 4.5 of the paper we mention that the right hand side of this free-entry condition can be written as an increasing function of the period profits of matched importers, Π_{do}^m, as,

$$\int_{\bar{\varphi}_{do}} M_{do}\left(\varphi\right)dG\left(\varphi\right) = \left(\frac{1}{r+\lambda}\right)\left(1-\frac{u_{do}}{1-i_{do}}\right)^{-1}\left(\frac{1}{N_o^x}\right)\Pi_{do}^m,$$

in which Π_{do}^m is defined in appendix A.14.4 in equation (A72) as:

$$\Pi_{do}^m = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}.$$

These profits are decreasing in the policy-relevant variable trade costs, τ_{do}. As such, a policymaker can change variable trade costs in each market to adjust period profits so that the decentralized market tightnesses, κ_{do}, coincide with the socially optimal market tightnesses, κ_{do}^{SP}, in each market.

There is no producers' bargaining power, β, that attains the social planner's solution, κ_{do}^{SP}, simultaneously in all search markets of our calibrated model (section 6). Figure A1 depicts the equilibrium decentralized market tightness for each market for various producers' bargaining powers, while holding all other parameters at their baseline values in our calibration (section 6.2). For example, the red solid line in the panel titled "US, US market tightness" depicts how market tightness varies with the bargaining power in the search market for U.S. retailers and U.S. producers. As producers' bargaining power rises, entry into retailing falls, lowering equilibrium market tightness. We solve the social planner's problem in equation (A73) to yield the efficient tightness in each market, represented by the blue horizontal dot-dash line. The social planner's solution does not vary with producers' bargaining power because how retailers and producers split match surplus is immaterial to the social planner. The figure suggests that in the US-CH and CH-US search markets, there exist two different producers' bargaining powers that equate the decentralized market tightnesses with the social planner's. Both of these producers' bargaining values are below the matching elasticity, η, depicted by the vertical black dashed line, which implies that the standard Hosios (1990) condition fails in our baseline calibration. In the two domestic markets, the social planner's market tightness and the decentralized market tightness only coincide for producers' bargaining power very close to one. These results are consistent with Brancaccio et al. (2020b) who show that bargaining power should vary by market in order to attain efficiency.

With producers' bargaining powers that vary by market, it is possible that there exist bargaining powers that internalize all the new externalities in our model, but a careful study of these efficiency issues should be the focus of a separate paper. The standard Hosios (1990) condition will not attain efficiency in our setting with an endogenous participation margin and heterogeneous producer productivity because marginal producers choosing between remaining idle and searching for a partner do not internalize their effect on average match productivity. These additional features lead to a participation externality and an output externality in addition to the standard matching externality. In standard labor search contexts, these new externalities typically imply that there is excessive producer entry and vacancy creation compared with the social planners solution (Albrecht et al., 2010; Julien and Mangin, 2017). Results in our context (figure A1) suggest that this intuition may be true in domestic markets, as the decentralized market tightness lies above the social planner's for any bargaining parameter not very close to one, but might not be the case in international markets, as market tightness is below its efficient level when we set the producers' bargaining power equal to the matching elasticity. The intensive margin in our model may also influence our efficiency results, similar to Kudoh and Sasaki (2011), which suggests another reason why the standard Hosios condition might fail. It is possible that there exists a generalized Hosios condition (as in Mangin and Julien, 2020) that internalizes all the new externalities in our model, but understanding and quantifying the various margins of inefficiency and characterizing the appropriate condition(s) should be the focus of a separate paper. This bifurcation of research projects resembles the approach taken by Brancaccio et al. (2020a) and Brancaccio et al. (2020b) in international trade, and Petrosky-Nadeau and Wasmer (2015) and Petrosky-Nadeau et al. (2018) in labor and goods markets.

If we can derive a generalized Hosios condition that internalizes the new externalities in our model, we suspect that a slightly modified version of our baseline economy would attain the efficient equilibrium. Dhingra and Morrow (2019) show that the market implements the first-best allocation in a model with constant elasticity of substitution utility, along with monopolistically competitive firms, heterogeneous firm productivity and free entry. Our model has these features, except free entry into production because in our baseline model we assume that the number of producers is proportional to aggregate consumption for analytic tractability, as in Chaney (2008). However, in appendix A.9.2 we consider the alternative assumption of free entry into search for producers. We show that this assumption yields additional restrictions on equilibrium market tightness, but does not fundamentally alter the implications of search frictions in our setting. So, with a generalized Hosios condition, this change in assumptions about producer entry would guarantee efficiency based on the results in Dhingra and Morrow (2019).

B Changes to welfare, trade flows, and the margins of trade

B.1 Proof of proposition 2: Changes in welfare

We prove proposition 2 assuming monopolistic competition and following steps similar to those used to prove proposition 1 in ACR. With the exception of the search frictions, our functional form assumptions allow us to relate the differentiated goods price index in our model from section 4 to the price index equation (A22) in ACR (p. 123)

$$P_{do}^{1-\sigma} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)\left(P_{do}^{ACR}\right)^{1-\sigma}, \tag{B74}$$

in which $\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x\left(\mu w_o \tau_{do}\right)^{1-\sigma}\Psi_{do}$, and it will be useful to define the important one-sided moment $\Psi_{do} \equiv \int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG(z)$. Also define the elasticity of this integral with respect to the cutoff as $\psi_{do} \equiv \dfrac{\partial \ln(\Psi_{do})}{\partial \ln(\bar{\varphi}_{do})}$. A sufficient condition for $\psi_{do} \leq 0$ is $\sigma > 1$. The overall price index with both homogeneous and differentiated goods is

$$\Xi_d = \left(\frac{p_d(1)}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha},$$

in which $p_d(1)$ is the price of the freely traded homogeneous good and α is the share of consumption devoted to the differentiated goods bundle. In section A.11.2, we show that the price of the freely traded good in equilibrium is the same in all countries $p_d(1) = p(1)$. Lastly, it will be useful to denote the total derivative of the log of a variable x, as $d\ln x = \ln(x'/x) = \ln(\hat{x})$ and so $\exp(d\ln x) = \hat{x}$.

B.1.1 Step 1: Small changes in welfare satisfy

$$d\ln(W_d) = d\ln(C_d) - \alpha d\ln(P_d). \tag{B75}$$

Because the utility function is homogeneous of degree one, welfare is defined by real consumption expenditure $W_d = \dfrac{C_d}{\Xi_d}$. To derive equation (B75) we use the definition of the price index to write

$$W_d = C_d\left(\frac{p(1)}{1-\alpha}\right)^{\alpha-1}\left(\frac{P_d}{\alpha}\right)^{-\alpha}$$

Taking logs gives

$$\ln(W_d) = \ln(C_d) + (\alpha-1)\left[\ln(p(1)) - \ln(1-\alpha)\right] - \alpha\left[\ln(P_d) - \ln(\alpha)\right]$$

We define the price of the freely traded good as the numeraire, $p(1) = 1$, and then totally differentiate

$$d\ln(W_d) = d\ln(C_d) - \alpha d\ln(P_d)$$

ACR rely on two additional simplifications that remove consumption from this expression, which we cannot employ. First, they have that $C_d \propto Y_d$ with a proportionality constant that is only a function of exogenous parameters. We lack this simplification because investment in our setting is not exogenously proportional to output. Second, while ACR do not explicitly invoke restriction R2 here, they do rely on it to get that $\Pi_d \propto Y_d$ with a proportionality constant that is only a function of exogenous parameters. Because L_d is an exogenous endowment and w_d can be normalized, using R2 in ACR ensures that $w_d L_d + \Pi_d = Y_d \propto w_d L_d$ which ensures that $d\ln(w_d L_d) = 0$ implies $d\ln(C_d) = 0$ and welfare is determined soley by the price index.

B.1.2 Step 2: Small changes in the consumer price index satisfy

$$\begin{aligned} d\ln P_d &= \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha\left(1-\sigma+\alpha^{-1}\psi_d\right)} \left[d\ln\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right) \right. \\ &+ \left(1-\sigma+\psi_{dk}\right)\left(d\ln w_k + d\ln \tau_{dk}\right) + d\ln N_k^x + \psi_{dk}\left(\frac{1}{\sigma-1}\right) d\ln\left(F\left(\kappa_{dk}\right)\right) \\ &+ \left. \psi_{dk}\left(\frac{1}{1-\sigma}\right) d\ln\left(C_d\right) \right], \end{aligned} \tag{B76}$$

in which ψ_{do} is defined above and $\psi_d \equiv \sum_{k=1}^{O} \lambda_{dk}\psi_{dk}$.

Equation (B76) is analogous to equation (A33) in ACR (p. 125) (except there is a typo in their first multiplicative term because γ_{ij} should be γ_j). When the utility function has only differentiated goods ($\alpha = 1$) and there are no search frictions ($u_{do} = 0$), equations (A33) and (B76) are the same. The signs on ψ_d and ψ_{do} differ between the models because our model is defined in terms of productivity, while theirs is defined in terms of marginal cost.

We derive equation (B76) by starting with total consumption in destination country d for the differentiated goods bundle from origin country o by integrating over all varieties at final prices. Because CES preferences define the differentiated goods aggregate given in equation (1), this integral is the value of CES demand for the bundle of country o products

$$C_{do} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}}.$$

This can be easily derived from equation (2). Define the consumption share λ_{do} as

$$\lambda_{do} \equiv \frac{C_{do}}{C_d} = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}} \left(\frac{1}{C_d}\right) = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}.$$

Our definition of the consumption share differs from ACR in two important ways. First, we use consumer expenditure instead of output because our model does not guarantee that income and consumption are proportional. Second, consumption, which is what matters for welfare, is measured at final sales prices, while the import share is measured at negotiated import prices. We will work with $P_d^{1-\sigma}$ using the definition of the price index for the differentiated good in the destination market d, given by

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Take the log of this expression to get $(1-\sigma)\ln P_d = \ln\sum_{k=1}^{O} P_{dk}^{1-\sigma}$ and then totally differentiate to get

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{P_{dk}^{1-\sigma}}{P_d^{1-\sigma}} \frac{dP_{dk}^{1-\sigma}}{P_{dk}^{1-\sigma}}.$$

Rearrange λ_{do} to get $\dfrac{\lambda_{do}}{\alpha P_{do}^{1-\sigma}} = \dfrac{1}{P_d^{1-\sigma}}$ and then use this to simplify

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha} d\ln P_{dk}^{1-\sigma}.$$

Taking logs of equation (B74) and totally differentiating gives

$$d\ln P_{do}^{1-\sigma} = d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) + d\ln\left(P_{do}^{ACR}\right)^{1-\sigma}.$$

Employing our functional form assumptions, which gives $\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x\left(\mu w_o \tau_{do}\right)^{1-\sigma}\Psi_{do}$, we can derive

$$d\ln\left(P_{do}^{ACR}\right)^{1-\sigma} = d\ln N_o^x + (1-\sigma)\left(d\ln w_o + d\ln\tau_{do}\right) + \psi_{do} d\ln\left(\bar{\varphi}_{do}\right),$$

in which we use the chain rule to get

$$d\ln\Psi_{do} = \psi_{do} d\ln\left(\bar{\varphi}_{do}\right).$$

Putting these parts together gives

$$(1-\sigma)\, d\ln P_d = \sum_{k=1}^{O} \frac{\lambda_{dk}}{\alpha}\left[d\ln\left(\frac{1-u_{dk}-i_{dk}}{1-i_{dk}}\right) + (1-\sigma)\left(d\ln w_k + d\ln\tau_{dk}\right) + d\ln N_k^x + \psi_{dk} d\ln\left(\bar{\varphi}_{dk}\right)\right]. \tag{B77}$$

If we set $\alpha = 1$ and $c_{do} = 0$ so that $u_{do} = 0$ we match equation (A34) in ACR (p. 125).

Next, take the log and total derivative of the cutoff expression from equation (18)

$$d\ln\left(\bar{\varphi}_{do}\right) = -d\ln\left(P_d\right) + d\ln\left(\tau_{do}\right) + \left(\frac{1}{\sigma-1}\right) d\ln\left(F\left(\kappa_{do}\right)\right) - \left(\frac{1}{\sigma-1}\right) d\ln\left(C_d\right) + d\ln\left(w_o\right), \tag{B78}$$

which is the analog to equation (A36) in ACR (p. 126). There are a few differences between equation (B78) here and their equation (A36). First, the signs are reversed because we define everything in terms of productivity, while they use costs. Second, their term ξ_{ij} captures the fixed cost of entry like our term $F\left(\kappa_{do}\right)$ (see equation (A27) on page 124). And while their term ρ_{ij} allows for some foreign labor to be used to enter a foreign country, we do not. Making the same restriction in their model would require setting $h_{ij} = 1$ and hence $\rho_{ij} = 1$. Finally, our threshold expression includes total consumption.

Combining equations (B77) and (B78) gives equation (B76).

B.1.3 Step 3: Small changes in the consumer price index satisfy

$$
\begin{aligned}
d\ln P_d &= \sum_{k=1}^{O} \lambda_{dk} \left(\frac{d\ln(\lambda_{dk}) - d\ln(\lambda_{dd})}{\alpha(1-\sigma+\alpha^{-1}\psi_d)} \right) + \frac{(\alpha\psi_{dd} - \psi_d)\, d\ln(\bar{\varphi}_{dd})}{\alpha(1-\sigma+\alpha^{-1}\psi_d)} \\
&+ \frac{d\ln N_d^x}{(1-\sigma+\alpha^{-1}\psi_d)} \\
&+ \frac{\psi_d d\ln(F(\kappa_{dd}))}{(\sigma-1)\alpha(1-\sigma+\alpha^{-1}\psi_d)} \\
&+ \frac{d\ln\left(\dfrac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{(1-\sigma+\alpha^{-1}\psi_d)} \\
&+ \frac{\psi_d d\ln(C_d)}{(1-\sigma)\alpha(1-\sigma+\alpha^{-1}\psi_d)} \qquad \text{(B79)}
\end{aligned}
$$

If we set $\alpha = 1$ and $c_{do} = 0$ so that $u_{do} = 0$ and $l_{dd} = -h_{dd}$ so that $F(\kappa_{dd})$ is a constant, then (B79) becomes equation (A37) of ACR (p. 126).

Start again with the consumption share $\lambda_{do} = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}$ and form $\frac{\lambda_{do}}{\lambda_{dd}} = \frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}}$. Substitute into the ratio $\frac{\lambda_{do}}{\lambda_{dd}}$ our functional form assumptions for the price index, take logs, and then totally differentiate to get

$$
\begin{aligned}
d\ln(\lambda_{do}) - d\ln(\lambda_{dd}) &= (1-\sigma)(d\ln w_o + d\ln\tau_{do}) + \psi_{do} d\ln(\bar{\varphi}_{do}) - \psi_{dd} d\ln(\bar{\varphi}_{dd}) \\
&+ d\ln N_o^x - d\ln N_d^x \\
&+ d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \qquad \text{(B80)}
\end{aligned}
$$

In obtaining this expression we have simplified terms by recalling that we are considering a foreign shock so that $d\ln\tau_{dd} = 0$ and that our normalization of the price of the freely traded good ensures that $d\ln w_o = d\ln w_d = 0$. We keep $d\ln w_o$ in the expression and ordered the terms as presented in ACR (p. 126) to make comparing the expressions easier.

We can derive two cutoff expressions

$$
d\ln(\bar{\varphi}_{do}) = -d\ln(P_d) + d\ln(\tau_{do}) + \frac{d\ln(F(\kappa_{do}))}{\sigma-1} + d\ln(w_o) - \left(\frac{1}{\sigma-1}\right) d\ln(C_d),
$$

and also

$$
d\ln(\bar{\varphi}_{dd}) = -d\ln(P_d) + \frac{d\ln(F(\kappa_{dd}))}{\sigma-1} - \left(\frac{1}{\sigma-1}\right) d\ln(C_d),
$$

in which we again impose that $\tau_{dd} = 1$ and $d\ln w_d = 0$. Combining these two cutoff expressions gives

$$
d\ln(\bar{\varphi}_{do}) = d\ln(\bar{\varphi}_{dd}) + d\ln(w_o) + d\ln(\tau_{do}) + \frac{d\ln(F(\kappa_{do}))}{\sigma-1} - \frac{d\ln(F(\kappa_{dd}))}{\sigma-1}, \quad \text{(B81)}
$$

which is akin to the last equation of ACR (p. 126) with the exception that they have a typo because the equal sign should be a minus sign. In our model, it is not necessarily the case that $d\ln(F(\kappa_{dd})) = 0$ in response to a foreign shock because the effective entry cost, $F(\kappa_{dd})$, is an endogenous variable and not a parameter.

Combine expression (B81) with (B80) to get

$$\begin{aligned} d\ln(\lambda_{do}) - d\ln(\lambda_{dd}) &= (1-\sigma+\psi_{do})(d\ln w_o + d\ln\tau_{do}) \\ &+ \psi_{do}\left(\frac{d\ln(F(\kappa_{do}))}{\sigma-1} - \frac{d\ln(F(\kappa_{dd}))}{\sigma-1}\right) \\ &+ (\psi_{do}-\psi_{dd})\, d\ln(\bar{\varphi}_{dd}) + d\ln N_o^x - d\ln N_d^x \\ &+ d\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \end{aligned} \tag{B82}$$

Equation (B82) is analogous to equation (A38) in ACR (p. 127) which has a typo because α^*_{ij} should be α^*_{jj}. Substituting equation (B82) into equation (B76) and performing algebra gives equation (B79).

B.1.4 Step 4: Small changes in the consumer price index satisfy

$$\begin{aligned} d\ln P_d &= \frac{d\ln(\lambda_{dd})}{\theta} \\ &- \frac{d\ln N_d^x}{\theta} \\ &- \frac{(\sigma-1-\theta)\, d\ln(F(\kappa_{dd}))}{(\sigma-1)\theta} \\ &- \frac{d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)}{\theta} \\ &- \frac{(\sigma-1-\theta)\, d\ln(C_d)}{(1-\sigma)\theta} \end{aligned} \tag{B83}$$

We depart somewhat from the approach taken in step 4 of ACR (p. 127) in simplifying equation (B79) to derive equation (B83). They invoke macro-level restriction number 3, "R3: The import demand system is such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, $\varepsilon_j^{ii'} = \varepsilon < 0$ if $i = i'$, and zero otherwise." As they describe on page 103, this restriction imposes symmetry on the elasticity of the consumption ratio to changes in variable trade costs. That elasticity in our model in general is given by equation (B89) and need not be symmetric across countries. A sufficient condition to derive equation (B83), however, is that productivity distributions and consumer preferences are symmetric. For now, we impose those restrictions in the following steps but could likely relax them in future work.

The term we need to consider from equation (B79) is $\psi_d \equiv \sum_{k=1}^{O} \lambda_{dk}\psi_{dk}$, which is the consumption share weighted average of the elasticity of the moment of the productivity distribution, in which $\psi_{do} = \dfrac{d\ln(\Psi_{do})}{d\ln(\bar{\varphi}_{do})}$. We assume that productivity $\varphi \in [1, +\infty)$ is Pareto distributed with CDF $G\left[\tilde{\varphi}<\varphi\right]=1-\varphi^{-\theta}$ and PDF $g\left(\varphi\right)=\theta\varphi^{-\theta-1}$ in which, as usual, $\theta>\sigma-1$ in order to close the model. With this distribution, the moment $\Psi_{do}=\dfrac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta-\sigma+1}$ and the elasticity $\psi_{do}=-\dfrac{\bar{\varphi}_{do}^{\sigma}\theta\bar{\varphi}_{do}^{-\theta-1}}{\Psi_{do}}=-\left(\theta-\sigma+1\right)$. Notice that the restriction that $\theta>\sigma-1$ ensures $\psi_{do}<0$ and $\psi_{d}<0$. Also notice that $\psi_{do}=\psi_{dd}$ and the term we are actually interested in becomes

$$\psi_{d}\equiv\sum_{k=1}^{O}\lambda_{dk}\psi_{do}=\alpha\left(\sigma-1-\theta\right), \tag{B84}$$

because by definition consumption shares $\alpha=\sum_{k=1}^{O}\lambda_{dk}$. Substituting equation (B84) into (B79) and also using the fact that Euler's homogeneous function theorem gives $\sum_{k=1}^{O}\lambda_{dk}d\ln\left(\lambda_{do}\right)=0$ provides (B83).

B.1.5 Step 5: Small changes in the number of producers

We cannot make the simplification $d\ln N_{d}^{x}=0$ as done in step 5 of ACR (p. 127) because we have assumed that $N_{d}^{x}=\dfrac{C}{\left(1+\pi\right)}\left(\dfrac{C_{d}}{C}\right)$ and both C_{d} and $\pi=\Pi/\sum_{k}w_{k}L_{k}$ are endogenous objects. Allowing free entry into the market for producers would be an alternative assumption but would then require an additional equation for determining equilibrium market tightness. We discuss that extension more in appendix A.9.

B.1.6 Combining step 1 to step 4 into the general welfare expression

Combining equation (B75) with equation (B83) provides the change in welfare in response to a foreign shock in our model

$$\begin{aligned}
d\ln\left(W_{d}\right) &= -\left(\frac{\alpha}{\theta}\right)d\ln\left(\lambda_{dd}\right)\\
&+\left(1+\left(\frac{\alpha}{\theta}\right)\left(1-\frac{\theta}{\sigma-1}\right)\right)d\ln\left(C_{d}\right)\\
&+\left(\frac{\alpha}{\theta}\right)d\ln N_{d}^{x}\\
&+\left(\frac{\alpha}{\theta}\right)\left(1-\frac{\theta}{\sigma-1}\right)d\ln\left(F\left(\kappa_{dd}\right)\right)\\
&+\left(\frac{\alpha}{\theta}\right)d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)
\end{aligned} \tag{B85}$$

We derive this by substituting the change in the price index from (B83) into the welfare expression from (B75) and simplifying the algebra using, in particular, $\dfrac{\alpha\left(\sigma-1-\theta\right)}{\left(\sigma-1\right)\theta}=\left(\dfrac{\alpha}{\theta}\right)\left(1-\dfrac{\theta}{\sigma-1}\right)$.

B.1.7 The change in welfare in proposition 2

We made an assumption that productivity, $\varphi\in[1,+\infty)$, follows a Pareto distribution with CDF $G\left[\tilde{\varphi}<\varphi\right]=1-\varphi^{-\theta}$ in appendix B.1.4 in order to derive the general equation

(B85). Two additional assumptions are needed to derive proposition 2 in the main text from the general welfare change in equation (B85). The first of these assumptions is that the cost of remaining idle, $-h_{dd}$, is the same as the flow search costs that producers pay to find retailers, l_{dd}, so that $l_{dd} = -h_{dd}$. With this assumption, the effective entry costs become a function of exogenous parameters and $d\ln\left(F\left(\kappa_{dd}\right)\right) = 0$. The second assumption is that the number of domestic producers does not respond to a foreign shock, $d\ln N_d^x = 0$. One could rationalize this assumption by assuming free entry into production or that N_d^x is exogenous.

Applying these two additional assumptions to the general welfare changes in equation (B85) gives

$$d\ln\left(W_d\right) = -\left(\frac{\alpha}{\theta}\right) d\ln\left(\lambda_{dd}\right) + \left(1 + \left(\frac{\alpha}{\theta}\right)\left(1 - \frac{\theta}{\sigma-1}\right)\right) d\ln\left(C_d\right) + \left(\frac{\alpha}{\theta}\right) d\ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right), \tag{B86}$$

which we can integrate to get the welfare response to any foreign shock in proposition 2 of the main text

$$\hat{W}_d = \hat{\lambda}_{dd}^{-\frac{\alpha}{\theta}} \left(1 - \widehat{\frac{u_{dd}}{1-i_{dd}}}\right)^{\frac{\alpha}{\theta}} \hat{C}_d^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)} \tag{B87}$$

B.2 Derivation of consumption elasticity

B.2.1 Relating price indexes

To derive an analogous elasticity in our model, start with the functional form assumptions detailed in section 2. Because, with the exception of the search frictions, these functional form assumptions are the same as in ACR, we can relate the price index in our model given in section 4 to the price index equation (A22) in ACR (p. 123)

$$\left(P_{do}^{ACR}\right)^{1-\sigma} = N_o^x \left(\mu w_o \tau_{do}\right)^{1-\sigma} \Psi_{do},$$

in which it will be useful to define $\Psi_{do} = \int_{\bar{\varphi}_{do}}^{\infty} \varphi^{\sigma-1} dG\left(\varphi\right)$ and the elasticity of this integral with respect to the cutoff $\psi_{do} = \dfrac{\partial\ln\left(\Psi_{do}\right)}{\partial\ln\left(\bar{\varphi}_{do}\right)} \leq 0$ a sufficient condition for which is $\sigma > 1$.

B.2.2 Demand for a country's bundle of goods

We can derive total consumption in destination country d for the goods bundle from origin country o by integrating over all varieties at final prices. Because we have CES preferences, this integral is the value of CES demand for the bundle of country o products

$$C_{do} = \left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) dG\left(\varphi\right) = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma}}.$$

Define the consumption share (which we note in our model is different from the observed trade share) as λ_{do}

$$\lambda_{do} = \frac{C_{do}}{C_d} = \alpha \frac{P_{do}^{1-\sigma} C_d}{P_d^{1-\sigma} C_d} = \alpha \frac{P_{do}^{1-\sigma}}{P_d^{1-\sigma}}.$$

We can also form relative consumption ratios, which is equivalent to ACR (equation 21), page 110, and is just the ratio of the price indexes raised to a power

$$\frac{\lambda_{do}}{\lambda_{dd}} = \frac{C_{do}}{C_{dd}} = \frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}}.$$

Using the country-specific price indexes given above we have

$$\frac{P_{do}^{1-\sigma}}{P_{dd}^{1-\sigma}} = \frac{\left(\dfrac{1-u_{do}-i_{do}}{1-i_{do}}\right)\left(P_{do}^{ACR}\right)^{1-\sigma}}{\left(\dfrac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right)\left(P_{dd}^{ACR}\right)^{1-\sigma}},$$

in which we used the definition of P_{do}^{ACR} . Taking the log of relative consumption ratios therefore gives

$$\ln\left(\frac{C_{do}}{C_{dd}}\right) = \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) + \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) - \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right). \tag{B88}$$

B.2.3 Derivative of consumption ratio with respect to tariffs

The goal is to derive two derivatives. The first is the direct effect of a change in the tariffs τ_{do} on the consumption ratio

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} = \varepsilon_o^{dd}.$$

The second is the indirect effect, which documents how changing tariffs between a third country d' and the origin o changes relative consumption in country d

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \varepsilon_o^{dd'}.$$

B.2.4 Direct effect of tariff changes ($d' = d$ case)

We begin by deriving $\dfrac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} = \varepsilon_o^{dd}$ in the most general form and then apply a few restrictions to compare it to the elasticity in ACR. Normalizing the price of the homogeneous good ensures that $\dfrac{\partial \ln\left(w_d\right)}{\partial \ln\left(\tau_{do}\right)} = 0$ and $\dfrac{\partial \ln\left(w_o\right)}{\partial \ln\left(\tau_{do}\right)} = 0$.

B.2.5 First and second terms of equation (B88) ($d' = d$ case)

Differentiating and simplifying the first term of equation (B88) gives

$$\frac{\partial}{\partial \ln\left(\tau_{do}\right)} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln\left(N_o^x\right)}{\partial \ln\left(\tau_{do}\right)} + (1-\sigma) + \psi_{do}\frac{\partial \ln\left(\bar{\varphi}_{do}\right)}{\partial \ln\left(\tau_{do}\right)},$$

and similarly

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})} + \psi_{dd} \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})}.$$

Combining these gives

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (1-\sigma) + \psi_{do} \frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{do})} - \psi_{dd} \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} \\ &+ \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})}.\end{aligned}$$

The elasticities of the cutoffs $\bar{\varphi}_{do}$ and $\bar{\varphi}_{dd}$ are related because changing tariff τ_{do} changes the price index P_d which changes the cutoff $\bar{\varphi}_{dd}$. We can derive this relationship by differentiating the explicit expression for the cutoff given in equation (18)

$$\begin{aligned}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{do})} &= 1 + \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} \\ &+ \left(\frac{1}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))} \frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))} \frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right].\end{aligned}$$

Substituting this into the elasticity of the general expression for the ratio of relative price indexes and simplifying gives

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} \\ + \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))} \frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})} \right. &- \left. \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))} \frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right] \\ &+ \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})}.\end{aligned}$$

The first line on the right is the same as equation (21) in ACR except that $\psi_{do} \leq 0$ in our case, while $\gamma_{ij} \geq 0$ in the Arkolakis et al.'s expressions because we define our model in terms of productivity, φ, while they define theirs in terms of marginal cost.

B.2.6 Elasticity of destination–origin market unmatched rate

Next, we calculate the elasticity of the destination–origin market unmatched producers' rate. Because we are studying a steady state, we use the definition $\dfrac{1-u_{do}-i_{do}}{1-i_{do}} = \dfrac{\kappa_{do}\chi(\kappa_{do})}{\lambda + \kappa_{do}\chi(\kappa_{do})}$ to derive

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right),$$

in which we used the chain rule to write

$$\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} = \frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \kappa_{do}\chi(\kappa_{do})} \frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} = \frac{1}{\kappa_{do}\chi(\kappa_{do})}\left(\frac{\partial \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right).$$

B.2.7 Elasticity of destination–destination market unmatched rate

Calculating the elasticity of the destination–destination market unmatched producers' rate with respect to τ_{do} also relies on the definition of $\frac{1-u_{dd}-i_{dd}}{1-i_{dd}} = \frac{\kappa_{dd}\chi(\kappa_{dd})}{\lambda+\kappa_{dd}\chi(\kappa_{dd})}$. The steps to derive this will be identical to the ones we took in calculating the destination–origin market unmatched rate with only the sub-indexes changing. The final derivative is

$$\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) = \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right),$$

in which we used the chain rule again to calculate

$$\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} = \frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \kappa_{dd}\chi(\kappa_{dd})}\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})} = \frac{1}{\kappa_{dd}\chi(\kappa_{dd})}\left(\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right).$$

B.2.8 General expression for $d' = d$ case

Here we try to write the most general possible expression only assuming that $\frac{\partial \ln(w_d)}{\partial \ln(\tau_{do})} = 0$ and $\frac{\partial \ln(w_o)}{\partial \ln(\tau_{do})} = 0$. Combining the general term expression in Arkolakis et al. with the elasticity of the finding rate with respect to tariffs gives

$$\begin{aligned}
\frac{\partial}{\partial \ln(\tau_{do})} \ln\left(\frac{C_{do}}{C_{dd}}\right) &= (1-\sigma)+\psi_{do}+(\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})}+\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})}-\frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{do})} \\
&+ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})}\right)-\left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{do})}\right) \\
&+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{do})}-\frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{do})}\right].
\end{aligned}$$

B.2.9 Indirect effect of tariff changes ($d' \neq d$ case)

The second derivative is the indirect effect, which documents how changing tariffs between a third country d' and the origin o changes relative consumption in country d

$$\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{d'o})} = \varepsilon_o^{dd'}.$$

B.2.10 First and second terms of equation (B88) ($d' \neq d$ case)

Following the general pattern used previously, we first derive the change in the price indexes in Arkolakis et al. as

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})}+\psi_{do}\frac{\partial \ln(\bar{\varphi}_{do})}{\partial \ln(\tau_{d'o})},$$

and similarly

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) = \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})}+\psi_{dd}\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})}.$$

Combining these gives

$$\begin{aligned}\frac{\partial}{\partial\ln\left(\tau_{d'o}\right)}\ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right)-\frac{\partial}{\partial\ln\left(\tau_{d'o}\right)}\ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= \psi_{do}\frac{\partial\ln\left(\bar{\varphi}_{do}\right)}{\partial\ln\left(\tau_{d'o}\right)}-\psi_{dd}\frac{\partial\ln\left(\bar{\varphi}_{dd}\right)}{\partial\ln\left(\tau_{d'o}\right)}\\ &+ \frac{\partial\ln\left(N_o^x\right)}{\partial\ln\left(\tau_{d'o}\right)}-\frac{\partial\ln\left(N_d^x\right)}{\partial\ln\left(\tau_{d'o}\right)}.\end{aligned}$$

The elasticities of the cutoffs $\bar{\varphi}_{do}$ and $\bar{\varphi}_{dd}$ with respect to $\tau_{d'o}$ are also related because changing tariff $\tau_{d'o}$ changes the price index P, which changes the cutoff $\bar{\varphi}_{dd}$. We can derive this relationship by differentiating the explicit expression for the cutoff given in equation (18)

$$\frac{\partial\ln\left(\bar{\varphi}_{do}\right)}{\partial\ln\left(\tau_{d'o}\right)} = -\frac{\partial\ln\left(P\right)}{\partial\ln\left(\tau_{d'o}\right)}+\left(\frac{1}{\sigma-1}\right)\frac{\partial\ln\left(F_{do}\right)}{\partial\ln\left(\tau_{d'o}\right)}$$

and symmetrically

$$\frac{\partial\ln\left(\bar{\varphi}_{dd}\right)}{\partial\ln\left(\tau_{d'o}\right)}=-\frac{\partial\ln\left(P_d\right)}{\partial\ln\left(\tau_{d'o}\right)}+\left(\frac{1}{\sigma-1}\right)\frac{\partial\ln\left(F_{dd}\right)}{\partial\ln\left(\tau_{d'o}\right)}.$$

So the relationship between the two cutoff elasticities is

$$\begin{aligned}\frac{\partial\ln\left(\bar{\varphi}_{do}\right)}{\partial\ln\left(\tau_{d'o}\right)} &= \frac{\partial\ln\left(\bar{\varphi}_{dd}\right)}{\partial\ln\left(\tau_{d'o}\right)}\\ &+ \left(\frac{1}{\sigma-1}\right)\left[\frac{\partial\ln\left(F_{do}\right)}{\partial\ln\left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial\ln\left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial\ln\left(\tau_{d'o}\right)}-\frac{\partial\ln\left(F_{dd}\right)}{\partial\ln\left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial\ln\left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial\ln\left(\tau_{d'o}\right)}\right],\end{aligned}$$

and we use the chain rule to expand the derivatives with respect to the finding rate. Substituting the relationship between the cutoffs into the general expression for the ratio of relative prices and simplifying gives

$$\begin{aligned}\frac{\partial}{\partial\ln\left(\tau_{d'o}\right)}\ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right)-\frac{\partial}{\partial\ln\left(\tau_{d'o}\right)}\ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= \left(\psi_{do}-\psi_{dd}\right)\frac{\partial\ln\left(\bar{\varphi}_{dd}\right)}{\partial\ln\left(\tau_{d'o}\right)}+\frac{\partial\ln\left(N_o^x\right)}{\partial\ln\left(\tau_{d'o}\right)}-\frac{\partial\ln\left(N_d^x\right)}{\partial\ln\left(\tau_{d'o}\right)}\\ &+\left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial\ln\left(F_{do}\right)}{\partial\ln\left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial\ln\left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial\ln\left(\tau_{d'o}\right)}-\frac{\partial\ln\left(F_{dd}\right)}{\partial\ln\left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial\ln\left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial\ln\left(\tau_{d'o}\right)}\right].\end{aligned}$$

B.2.11 Elasticity of destination–origin market matched rate

We continue to follow the pattern used previously and calculate the elasticity of the destination–origin market matched producers' rate. Because we are studying a steady state, we use the definition $\dfrac{1-u_{do}-i_{do}}{1-i_{do}}=\dfrac{\kappa_{do}\chi\left(\kappa_{do}\right)}{\lambda+\kappa_{do}\chi\left(\kappa_{do}\right)}$ to derive

$$\frac{\partial}{\partial\ln\left(\tau_{d'o}\right)}\ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial\ln\kappa_{do}\chi\left(\kappa_{do}\right)}{\partial\ln\left(\tau_{d'o}\right)}\right),$$

in which we used the chain rule to write

$$\frac{\partial\ln\kappa_{do}\chi\left(\kappa_{do}\right)}{\partial\ln\left(\tau_{d'o}\right)}=\frac{\partial\ln\kappa_{do}\chi\left(\kappa_{do}\right)}{\partial\kappa_{do}\chi\left(\kappa_{do}\right)}\frac{\partial\kappa_{do}\chi\left(\kappa_{do}\right)}{\partial\ln\left(\tau_{d'o}\right)}=\frac{1}{\kappa_{do}\chi\left(\kappa_{do}\right)}\frac{\partial\kappa_{do}\chi\left(\kappa_{do}\right)}{\partial\ln\left(\tau_{d'o}\right)}.$$

For the record, the elasticity of the third term boils down to

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right).$$

The things that matter are the unmatched rate and the elasticity of the finding rate with respect to tariffs.

B.2.12 Elasticity of destination–destination market matched rate

The fourth term requires that we calculate

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right).$$

The steps are identical to the ones we took in calculating the destination–origin market matched rate derivative with only the sub-indexes changing. The end result is

$$\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\frac{1-u_{dd}-i_{dd}}{1-i_{dd}}\right) = \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right),$$

in which we again used the chain rule to write

$$\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \kappa_{dd}\chi(\kappa_{dd})}\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})} = \frac{1}{\kappa_{dd}\chi(\kappa_{dd})}\left(\frac{\partial \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right).$$

B.2.13 General expression for $d' \neq d$ case

Here we try to write the most general possible expression only assuming that $\frac{\partial \ln(w_d)}{\partial \ln(\tau_{d'o})} = 0$ and $\frac{\partial \ln(w_o)}{\partial \ln(\tau_{d'o})} = 0$. The general term expression in Arkolakis et al. was

$$\begin{aligned}\frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{do}^{ACR}\right)^{1-\sigma}\right) - \frac{\partial}{\partial \ln(\tau_{d'o})} \ln\left(\left(P_{dd}^{ACR}\right)^{1-\sigma}\right) &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right].\end{aligned}$$

Combining these with the elasticity of unmatched rates gives

$$\begin{aligned}\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{d'o})} &= (\psi_{do}-\psi_{dd})\frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} + \frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(N_d^x)}{\partial \ln(\tau_{d'o})} \\ &+ \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{d'o})}\right) - \left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi(\kappa_{dd})}{\partial \ln(\tau_{d'o})}\right) \\ &+ \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln(F_{do})}{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}\frac{\partial \ln(\kappa_{do}\chi(\kappa_{do}))}{\partial \ln(\tau_{d'o})} - \frac{\partial \ln(F_{dd})}{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}\frac{\partial \ln(\kappa_{dd}\chi(\kappa_{dd}))}{\partial \ln(\tau_{d'o})}\right].\end{aligned}$$

B.2.14 Final general elasticity

The final expression is

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_o^{dd'} = \begin{cases} \quad (1-\sigma)+\psi_{do}+\left(\psi_{do}-\psi_{dd}\right)\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}+\frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{do}\right)}-\frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{do}\right)} & \\ +\quad \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{do}\right)}\right)-\left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}\right) & \\ +\quad \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{do}\right)}-\frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{do}\right)}\right] & if\ d'=d \\ \quad \left(\psi_{do}-\psi_{dd}\right)\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}+\frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{d'o}\right)}-\frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{d'o}\right)} & \\ +\quad \left(\frac{u_{do}}{1-i_{do}}\right)\left(\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right)-\left(\frac{u_{dd}}{1-i_{dd}}\right)\left(\frac{\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}\right) & \\ +\quad \left(\frac{\psi_{do}}{\sigma-1}\right)\left[\frac{\partial \ln \left(F_{do}\right)}{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}\frac{\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)}-\frac{\partial \ln \left(F_{dd}\right)}{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}\frac{\partial \ln \left(\kappa_{dd}\chi\left(\kappa_{dd}\right)\right)}{\partial \ln \left(\tau_{d'o}\right)}\right] & if\ d'\neq d \end{cases}. \tag{B89}$$

B.2.15 Consumption elasticity as retailer search costs approach zero

As the search costs that retailers pay to find producers approaches zero in all destination-origin markets, $c_{do} \to 0\ \forall do$, the following three things happen: 1) the fraction of unmatched searching producers goes to zero, $u_{do} \to 0\ \forall do$, 2) the effective entry costs become a function of exogenous parameters, $\partial \ln \left(F_{do}\right)/\partial \ln \left(\kappa_{do}\chi\left(\kappa_{do}\right)\right) \to 0\ \forall do$, and 3) the value of imports converges to the value of consumption, $IM_{do} \to C_{do}\ \forall do$. These three facts together imply that the consumption elasticity converges to

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_o^{ACRdd'} = \begin{cases} (1-\sigma)+\psi_{do}+\left(\psi_{do}-\psi_{dd}\right)\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)}+\frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{do}\right)}-\frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{do}\right)}\ if\ d'=d \\ \\ \left(\psi_{do}-\psi_{dd}\right)\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)}+\frac{\partial \ln \left(N_o^x\right)}{\partial \ln \left(\tau_{d'o}\right)}-\frac{\partial \ln \left(N_d^x\right)}{\partial \ln \left(\tau_{d'o}\right)}\ if\ d'\neq d \end{cases}. \tag{B90}$$

First, we highlight that this is the elasticity of imports with respect to variable trade costs that would result in a model that has the same structure but no search frictions. Second, if we are willing to assume that $\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} = \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{do})}$ and $\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{d'o})}$, then equation (B90) becomes

$$\frac{\partial \ln \left(C_{do}/C_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \frac{\partial \ln \left(IM_{do}/IM_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} = \varepsilon_o^{ACRdd'} = \begin{cases} (1-\sigma)+\psi_{do}+\left(\psi_{do}-\psi_{dd}\right)\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{do}\right)} & if\ d'=d \\ \\ \left(\psi_{do}-\psi_{dd}\right)\frac{\partial \ln \left(\bar{\varphi}_{dd}\right)}{\partial \ln \left(\tau_{d'o}\right)} & if\ d'\neq d \end{cases}, \tag{B91}$$

in which $\psi_{do} = \dfrac{\partial \ln \left(\Psi_{do}\right)}{\partial \ln \left(\bar{\varphi}_{do}\right)} \geq 0$ and $\Psi_{do} = \int_{\bar{\varphi}_{do}}^{\infty} \varphi^{\sigma-1} dG\left(\varphi\right)$. Equation (B91) is exactly the trade elasticity in the Melitz (2003) model as derived in ACR, equation (21) except that $\psi_{do} \leq 0$ while $\gamma_{ij} \geq 0$. This sign difference occurs because we define our model in terms of productivity, while they define theirs in terms of marginal cost.

Our baseline calibration assumes that productivity, φ, follows a Pareto distribution with CDF $G\left[\tilde{\varphi} < \varphi\right] = 1 - \varphi^{-\theta}$. This assumption simplifies terms in equation (B91) that depend on moments of the productivity distribution as shown in B.2.16 and leads to

$$\frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \frac{\partial \ln\left(IM_{do}/IM_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \varepsilon_o^{ACRdd'} = \begin{cases} -\theta & if\ d' = d \\ \\ 0 & if\ d' \neq d \end{cases}. \tag{B92}$$

Consumption and trade elasticities are equivalent in these models because trade and consumption are both evaluated at final sales prices. Equation (B92) is the consumption and trade elasticity if $c_{do} \to 0\ \forall do$ and productivity is Pareto distributed. This elasticity is identical to the Melitz (2003) model with the same productivity distribution. We compare the effects of search frictions on the consumption and trade elasticities from equation (28) and 3 to standard trade models without search frictions in section 5.2 using our baseline calibration and equation (B92).

B.2.16 Consumption elasticity with Pareto distributed productivity

The elasticity of the moment of the productivity distribution defined by $\psi_{do} = \dfrac{d\ln\left(\Psi_{do}\right)}{d\ln\left(\bar{\varphi}_{do}\right)}$ takes a particularly simple form if productivity $\varphi \in [1, +\infty)$ is Pareto distributed with CDF $G\left[\tilde{\varphi} < \varphi\right] = 1 - \varphi^{-\theta}$ and PDF $g\left(\varphi\right) = \theta\varphi^{-\theta-1}$. As usual, assume that $\theta > \sigma - 1$ in order to close the model, which also ensures that $\psi_{do} < 0$. With this distribution, the moment $\Psi_{do} \equiv \int_{\bar{\varphi}_{do}}^{\infty} z^{\sigma-1} dG\left(z\right) = \dfrac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta-\sigma+1}$ and the elasticity $\psi_{do} = -\dfrac{\bar{\varphi}_{do}^{\sigma}\theta\bar{\varphi}_{do}^{-\theta-1}}{\Psi_{do}} = \sigma - 1 - \theta$. Importantly, this implies that $\psi_{do} = \psi_{dd}$. The $d' = d$ case of the consumption elasticity therefore simplifies to

$$\left(1-\sigma\right) + \psi_{do} + \left(\psi_{do} - \psi_{dd}\right)\frac{\partial \ln\left(\bar{\varphi}_{dd}\right)}{\partial \ln\left(\tau_{do}\right)} = \left(1-\sigma\right) + \left(\sigma - 1 - \theta\right) = -\theta, \tag{B93}$$

and the $d' \neq d$ case of the consumption elasticity simplifies to

$$\left(\psi_{do} - \psi_{dd}\right)\frac{\partial \ln\left(\bar{\varphi}_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = 0. \tag{B94}$$

B.2.17 The consumption elasticity in proposition 3

Four additional assumptions are needed to derive equation (28) in the main text from the general elasticity equation (B89). The first of these is that the cost of remaining idle, $-h_{do}$, is the same as the flow search costs that producers pay to find retailers, l_{do}, so that $l_{do} = -h_{do}\ \forall do$. With this assumption, the effective entry costs become a function of exogenous parameters and $\partial \ln\left(F_{do}\right)/\partial \ln\left(\kappa_{do}\chi\left(\kappa_{do}\right)\right) = 0\ \forall do$. The second assumption is that $\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{do})} = \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{do})}$ and $\frac{\partial \ln(N_o^x)}{\partial \ln(\tau_{d'o})} = \frac{\partial \ln\left(N_d^x\right)}{\partial \ln(\tau_{d'o})}$. One could rationalize equality between these elasticities by studying symmetric equilibria or ensure that the elasticities are zero by either assuming free entry into production or that N_d^x and N_o^x are exogenous. The third assumption is that the functional form of the matching function is $\kappa_{do}\chi\left(\kappa_{do}\right) = \xi\kappa_{do}^{1-\eta}$ so that $\dfrac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \kappa_{do}} = 1 - \eta$ and

$$\frac{\partial}{\partial \ln\left(\tau_{do}\right)} \ln\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right) = \left(\frac{u_{do}}{1-i_{do}}\right)\frac{\partial \ln \kappa_{do}\chi\left(\kappa_{do}\right)}{\partial \ln \kappa_{do}}\frac{\partial \ln \kappa_{do}}{\partial \ln\left(\tau_{do}\right)} = \left(\frac{u_{do}}{1-i_{do}}\right)\left(1-\eta\right)\frac{\partial \ln \kappa_{do}}{\partial \ln \tau_{do}}. \tag{B95}$$

The fourth assumption is that productivity, $\varphi \in [1, +\infty)$, follows a Pareto distribution with CDF $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$. Appendix B.2.16 shows that this assumption implies that the terms in the elasticity that depend on moments of the productivity distribution simplify to

$$(1-\sigma) + \psi_{do} + (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{do})} = -\theta \qquad \text{and} \qquad (\psi_{do} - \psi_{dd}) \frac{\partial \ln(\bar{\varphi}_{dd})}{\partial \ln(\tau_{d'o})} = 0$$

for the $d' = d$ and $d' \neq d$ cases of the consumption elasticity, respectively. Applying these four assumptions to the general elasticity equation (B89) gives

$$\frac{\partial \ln(C_{do}/C_{dd})}{\partial \ln(\tau_{d'o})} = \begin{cases} -\theta + \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial \ln \kappa_{do}}{\partial \ln \tau_{d'o}} - \left(\frac{u_{dd}}{1-i_{dd}}\right)(1-\eta)\frac{\partial \ln \kappa_{dd}}{\partial \ln \tau_{d'o}} & if\ d' = d \\ \left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{\partial \ln \kappa_{do}}{\partial \ln \tau_{d'o}} - \left(\frac{u_{dd}}{1-i_{dd}}\right)(1-\eta)\frac{\partial \ln \kappa_{dd}}{\partial \ln \tau_{d'o}} & if\ d' \neq d \end{cases}. \tag{B96}$$

B.2.18 Markup response to tariff changes in our baseline

We know that raising tariffs, τ_{do}, reduces producers' finding rate, $\kappa_{do}\chi(\kappa_{do})$, and that higher marginal costs reduce the markup in the *do* market in our calibration, $\partial \ln(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})) / \partial \ln(\tau_{do}) \leq 0$. Higher tariffs, τ_{do}, also increase the markup in the *dd* market so that $\partial \ln(1 - b(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd})) / \partial \ln(\tau_{do}) \geq 0$. Intuitively, this is because raising tariffs in the *do* market protects the *dd* market and makes being a retailer in the *dd* market more valuable, inducing entry and increasing the finding rate for producers in the *dd* market, which allows producers to negotiate higher prices. Because the effects of both *do* and *dd* price markups on the import elasticity are weakly negative, equation (28) shows that the import elasticity in our model is more negative than our consumption elasticity. The remainder of this section discusses this result in more detail.

The sign of the elasticity of the markup between consumption and imports with respect to iceberg costs,

$$\frac{\partial \ln(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))}{\partial \ln(\tau_{do})}$$

and

$$\frac{\partial \ln(1 - b(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}))}{\partial \ln(\tau_{do})},$$

respectively, only depend on market tightness κ_{do} because tariffs do not directly affect the $b(\cdot)$ term. The relevant partial derivative in the first case is

$$\frac{\partial \ln(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))}{\partial \ln(\tau_{do})} = -\frac{\partial b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})}{\partial \kappa_{do}} \frac{\kappa_{do}}{(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))} \frac{\partial \ln(\kappa_{do})}{\partial \ln(\tau_{do})}$$

We argue below that $\dfrac{\partial \ln \kappa_{do}\chi(\kappa_{do})}{\partial \ln(\tau_{do})} \leq 0$, which implies that $\dfrac{\partial \ln \kappa_{do}}{\partial \ln(\tau_{do})} \leq 0$ as well. The term $\dfrac{\kappa_{do}}{(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))} \geq 0$ and so it remains to consider the sign of $\dfrac{\partial b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})}{\partial \kappa_{do}}$. Our baseline calibration has $l_{do} = -h_{do}$ so that $F_{do}(\kappa_{do})$ is not a function of κ_{do},

$$F_{do} = f_{do} + h_{do} + \frac{(r+\lambda)}{\beta} s_{do}.$$

This assumption simplifies the desired derivative to

$$\frac{\partial b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\partial\kappa_{do}} = \frac{\gamma_{do}}{\sigma\theta}\left(\frac{\theta-(\sigma-1)}{F_{do}}\right)\left[\frac{\partial}{\partial\kappa_{do}}\kappa_{do}\chi(\kappa_{do})s_{do}\right] + \frac{b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\gamma_{do}}\frac{\partial}{\partial\kappa_{do}}\gamma_{do}$$

in which $\frac{\partial}{\partial\kappa_{do}}\kappa_{do}\chi(\kappa_{do}) \geq 0$ as mentioned above and $\frac{\partial\gamma_{do}}{\partial\kappa_{do}} \leq 0$ because $\gamma_{do} = \frac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})}$ so that $\frac{\partial\gamma_{do}}{\partial\kappa_{do}} = -\frac{\gamma_{do}}{r+\lambda+\beta\kappa_{do}\chi(\kappa_{do})} \leq 0$. Even with our restriction $l_{do} = -h_{do}$, the sign is ambiguous. In particular, $\frac{\partial b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\partial\kappa_{do}}$ will be negative if $s_{do} = 0$ or if the first term is smaller than the second. Our baseline parameterization has that $\frac{\partial b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\partial\kappa_{do}} \leq 0$. This fact implies that

$$\frac{\partial\ln(1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}))}{\partial\ln(\tau_{do})} \leq 0.$$

As tariffs increase, the aggregate markup term, $1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})$, the difference between final sales prices and negotiated prices, declines. We get the same result with the assumption that $s_{do} = 0$. We do not need the additional assumption that $l_{do} = -h_{do}$. With $s_{do} = 0$, the derivative simplifies to

$$\frac{\partial b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do})}{\partial\kappa_{do}} = \frac{\frac{\partial\gamma_{do}}{\partial\kappa_{do}}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}(\theta-(\sigma-1))\right) + \frac{\gamma_{do}}{\sigma\theta}\left(\frac{\delta_{do}}{F_{do}^2}(\theta-(\sigma-1))\frac{\partial F_{do}}{\partial\kappa_{do}}\right)$$

We know that $\frac{\partial\gamma_{do}}{\partial\kappa_{do}} < 0$ and that $\frac{\partial F_{do}}{\partial\kappa_{do}} < 0$ so that this derivative is negative. As before, this implies that

$$\frac{\partial\ln(1-b(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}))}{\partial\ln(\tau_{do})} \leq 0.$$

Similar logic applies for $\frac{\partial\ln(1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}{\partial\ln(\tau_{do})}$ because each term will have the same sign as before except that $\frac{\partial\ln(\kappa_{dd})}{\partial\ln(\tau_{do})} \geq 0$ so that

$$\begin{aligned}\frac{\partial\ln(1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}{\partial\ln(\tau_{do})} &= -\frac{\partial b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd})}{\partial\kappa_{do}}\frac{\kappa_{dd}}{(1-b(\sigma,\theta,\gamma_{dd},\delta_{dd},F_{dd}))}\frac{\partial\ln(\kappa_{dd})}{\partial\ln(\tau_{do})}\\ &\geq 0\end{aligned}$$

B.3 Proof of proposition 3: Trade elasticity

B.3.1 Relating the consumption and trade elasticities

We derive the trade elasticity in our model by relating it to the consumption elasticity. Imports evaluated at negotiated prices and total sales evaluated at final prices are related through the gravity equation (25) and equation (26) as

$$IM_{do} = \left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) C_{do},$$

in which

$$\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) = \left(1 - \frac{\gamma_{do}}{\sigma\theta}\left(\theta - \frac{\delta_{do}}{F_{do}}\left(\theta - (\sigma - 1)\right)\right)\right).$$

This equation is not a general relationship and depends on the assumptions we have made about preferences, bargaining, and the productivity distribution. Forming the import ratio in markets do and dd in our model therefore gives

$$\frac{IM_{do}}{IM_{dd}} = \frac{\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) C_{do}}{\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right) C_{dd}}.$$

It is straightforward to see that the trade elasticity in our model is related to the consumption elasticity according to

$$\frac{\partial \ln\left(IM_{do}/IM_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \frac{\partial \ln\left(C_{do}/C_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} + \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} - \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)},$$

which is equation (28) in the main text.

The model's baseline trade elasticity presented in proposition 3 simply combines equation (28) with equation (B96) (which makes the four restrictions) to get

$$\frac{\partial \ln\left(IM_{do}/IM_{dd}\right)}{\partial \ln\left(\tau_{d'o}\right)} = \begin{cases} -\theta + \left(\frac{u_{do}}{1 - i_{do}}\right)(1-\eta)\frac{\partial \ln \kappa_{do}}{\partial \ln \tau_{d'o}} - \left(\frac{u_{dd}}{1 - i_{dd}}\right)(1-\eta)\frac{\partial \ln \kappa_{dd}}{\partial \ln \tau_{d'o}} & \\ + \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} - \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} & if\ d' = d \\ \left(\frac{u_{do}}{1 - i_{do}}\right)(1-\eta)\frac{\partial \ln \kappa_{do}}{\partial \ln \tau_{d'o}} - \left(\frac{u_{dd}}{1 - i_{dd}}\right)(1-\eta)\frac{\partial \ln \kappa_{dd}}{\partial \ln \tau_{d'o}} & \\ + \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} - \frac{\partial \ln\left(1 - b\left(\sigma, \theta, \gamma_{dd}, \delta_{dd}, F_{dd}\right)\right)}{\partial \ln\left(\tau_{d'o}\right)} & if\ d' \neq d \end{cases}. \tag{B97}$$

The trade elasticity in our model differs from the standard trade elasticity because it is affected by the endogenous markup change between the negotiated and final sales prices in addition to the change in the mass of unmatched varieties.

B.3.2 Import elasticity is more negative than in a model without search

Using equation (19), we know that raising tariffs, τ_{do}, reduces the value of importing, $M_{do}(\varphi)$, and therefore reduces market tightness, κ_{do}, and producers' finding rate, $\kappa_{do}\chi(\kappa_{do})$. This comparative static implies that $\partial \ln \kappa_{do}\chi(\kappa_{do}) / \partial \ln(\tau_{do}) \leq 0$. Conversely, raising tariffs in the do market raises the price index, P_d, making the domestic market more attractive for retailers, thereby encouraging domestic retailer entry, and thus raising domestic market tightness, κ_{dd}, and the domestic producers' finding rate, which implies

$\partial \ln \kappa_{dd}\chi\left(\kappa_{dd}\right)/\partial \ln\left(\tau_{do}\right) \geq 0$. Because both do and dd unmatched rates of producers are weakly positive, the consumption elasticity in our model is at least as negative as the analogous elasticity in the class of models from ACR that satisfy equation (B92). Appendix B.2.18 shows that the effect of the markup terms are weakly negative as well. Together these results imply that the import elasticity in our model is at least as negative as in a model without search frictions.

B.4 Intensive and extensive margins of trade

Combine the value of imports from proposition 1 and imports evaluated at final consumption prices from equation (26) to get that

$$IM_{do} = \left(1 - b_{do}\left(\cdot\right)\right)\left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) dG. \tag{B98}$$

Taking logs and differentiating equation (B98) with respect to a parameter will give the intensive and extensive margins responses of imports.

B.4.1 Margins with respect to search costs

Decomposing the elasticity of imports in response to search costs, $c_{d'o}$, for $d' = d$, gives

$$\begin{aligned} \frac{d\ln\left(IM_{do}\right)}{d\ln c_{do}} &= \underbrace{\underbrace{\left(\sigma-1\right)\frac{d\ln P_d}{d\ln c_{do}} + \frac{d\ln C_d}{d\ln c_{do}}}_{\text{Final sales elasticity}} + \underbrace{\frac{d\ln\left(1-b_{do}\left(\cdot\right)\right)}{d\ln c_{do}}}_{\text{Markup elasticity}}}_{\text{Intensive margin elasticity}} \\ &+ \underbrace{\underbrace{\left(\sigma-\theta-1\right)\left(\left(\frac{1}{\sigma-1}\right)\frac{d\ln F\left(\kappa_{do}\right)}{d\ln c_{do}} - \left(\frac{1}{\sigma-1}\right)\frac{d\ln C_d}{d\ln c_{do}} - \frac{d\ln P_d}{d\ln c_{do}}\right)}_{\text{Threshold elasticity}} + \underbrace{\left(\frac{u_{do}}{1-i_{do}}\right)\left(1-\eta\right)\frac{d\ln \kappa_{do}}{d\ln c_{do}}}_{\text{Matched elasticity}}}_{\text{Extensive margin elasticity}}, \end{aligned}$$

which is the expression in proposition 4.

To get intuition about how search, variable, and fixed costs affect the intensive and extensive margins in our model assume the indirect effect of market tightness and all general equilibrium effects are small (as in Chaney, 2008, footnote 20) so that

$$\begin{gathered} \frac{d\ln P_d}{d\ln x_{do}} = \frac{d\ln C_d}{d\ln x_{do}} = \frac{d\ln\left(1-b_{do}\left(\cdot\right)\right)}{d\ln x_{do}} = 0, \text{ for } x_{do} = \left\{c_{do}, \tau_{do}, F_{do}\right\}, \\ \frac{d\ln F\left(\kappa_{do}\right)}{d\ln x_{do}} = 0 \text{ for } x_{do} = \left\{c_{do}, \tau_{do}\right\}, \text{and } \frac{d\ln F\left(\kappa_{do}\right)}{d\ln F_{do}} = 1. \end{gathered} \tag{B99}$$

These restrictions eliminate all but the matched elasticity from proposition 4 leaving

$$\frac{d\ln\left(IM_{do}\right)}{d\ln c_{do}} = \underbrace{\left(\frac{u_{do}}{1-i_{do}}\right)\left(1-\eta\right)\frac{d\ln \kappa_{do}}{d\ln c_{do}}}_{\text{Matched elasticity=Extensive margin}}. \tag{B100}$$

Search costs affect trade flows through the extensive margin and via the matched elasticity margin, specifically.

To derive our approximation in equation (31) from this equation, start with the retailer entry condition (19) rewritten as equation (A61). Next, substitute the retailer contact rate implied by the matching function (4) into equation (A61), take logs of both sides, totally differentiate, and rearrange to get

$$\frac{d\ln\kappa_{do}}{d\ln c_{do}} = -1/\left[\left(\frac{u_{do}}{1-i_{do}}\right)+\left(1-\frac{u_{do}}{1-i_{do}}\right)\eta\right].$$

When the unmatched rate is close to zero, this becomes $d\ln\kappa_{do}/d\ln c_{do} = -1/\eta$. Conceptually, this approximation assumes that retailers' benefits from being matched, the right hand side of equation (19), is constant with respect to changes in search costs. We discuss this issue in greater detail in appendix D.4.

B.4.2 Margins with respect to variable costs

Variable trade costs in our model affect trade flows through the usual intensive margin, final sales elasticity, and also the extensive margin, threshold elasticity for $d' = d$ according to

$$\begin{aligned}\frac{d\ln(IM_{do})}{d\ln\tau_{do}} &= \underbrace{\underbrace{(\sigma-1)\frac{d\ln P_d}{d\ln\tau_{do}}+\frac{d\ln C_d}{d\ln\tau_{do}}+(1-\sigma)}_{\text{Final sales elasticity}}+\underbrace{\frac{d\ln(1-b_{do}(\cdot))}{d\ln\tau_{do}}}_{\text{Markup elasticity}}}_{\text{Intensive margin elasticity}} \qquad \text{(B101)}\\ &\quad+\underbrace{\underbrace{(\sigma-\theta-1)\left(1+\left(\frac{1}{\sigma-1}\right)\frac{d\ln F(\kappa_{do})}{d\ln\tau_{do}}-\left(\frac{1}{\sigma-1}\right)\frac{d\ln C_{do}}{d\ln\tau_{do}}-\frac{d\ln P_{do}}{d\ln\tau_{do}}\right)}_{\text{Threshold elasticity}}+\underbrace{\left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{d\ln\kappa_{do}}{d\ln\tau_{do}}}_{\text{Matched elasticity}}}_{\text{Extensive margin elasticity}}.\end{aligned}$$

Making the restrictions detailed in equations (B99) implies that

$$\frac{d\ln(IM_{do})}{d\ln\tau_{do}} = \underbrace{\underbrace{(1-\sigma)}_{\text{Final sales elasticity}}}_{\text{Intensive margin elasticity}} + \underbrace{\underbrace{(\sigma-\theta-1)}_{\text{Threshold elasticity}} + \underbrace{\left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\frac{d\ln\kappa_{do}}{d\ln\tau_{do}}}_{\text{Matched elasticity}}}_{\text{Extensive margin elasticity}}. \qquad \text{(B102)}$$

This equation again predicts that the main difference between our model and Chaney (2008) is the search margin's effect on the extensive margin. This result is confirmed in table 7 for our baseline calibration.

The introduction of search frictions adds a matched margin which is the main reason why our normalized trade elasticity in (29) differs from the standard elasticity $-\theta$.

B.4.3 Margins with respect to fixed costs

To compare the import response to changes in fixed costs that are comparable to Chaney (2008) and most other trade models, we need to change not only, f_{do}, but also producer search costs, l_{do}, the opportunity cost of remaining idle, h_{do}, and the sunk cost of, s_{do}, by the same amount, $d\ln F_{do}$. We will refer to these simultaneous changes as $d\ln F_{do}$ despite the slight abuse of notation. All together the response to these parameters is given by

$$\begin{aligned}\frac{d\ln\left(IM_{do}\right)}{d\ln F_{do}} &= \underbrace{\underbrace{(\sigma-1)\,\frac{d\ln P_d}{d\ln F_{do}}+\frac{d\ln C_d}{d\ln F_{do}}}_{\text{Final sales elasticity}}+\underbrace{\frac{d\ln\left(1-b_{do}\left(\cdot\right)\right)}{d\ln F_{do}}}_{\text{Markup elasticity}}}_{\text{Intensive margin elasticity}} \\ &+ \underbrace{\underbrace{(\sigma-\theta-1)\left(\left(\frac{1}{\sigma-1}\right)\frac{d\ln F\left(\kappa_{do}\right)}{d\ln F_{do}}-\left(\frac{1}{\sigma-1}\right)\frac{d\ln C_{do}}{d\ln F_{do}}-\frac{d\ln P_{do}}{d\ln F_{do}}\right)}_{\text{Threshold elasticity}}+\underbrace{\left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\,\frac{d\ln\kappa_{do}}{d\ln F_{do}}}_{\text{Matched elasticity}}}_{\text{Extensive margin elasticity}}.\end{aligned}$$

Again making the restrictions detailed in equation (B99) gives intuition for the effect of fixed barriers in our model as

$$\frac{d\ln\left(IM_{do}\right)}{d\ln F_{do}} = \underbrace{\underbrace{\left(1-\frac{\theta}{\sigma-1}\right)}_{\text{Threshold elasticity}}+\underbrace{\left(\frac{u_{do}}{1-i_{do}}\right)(1-\eta)\,\frac{d\ln\kappa_{do}}{d\ln F_{do}}}_{\text{Matched elasticity}}}_{\text{Extensive margin elasticity}}.$$

The threshold elasticity of imports with respect to the effective entry cost is the same as the prediction in Chaney (2008) and the introduction of search frictions operates through the extensive-matched margin.

C Calibration appendix

C.1 Numerical details

C.1.1 Solution algorithm and weighting matrix

We use MATLAB's *fmincon* function and randomly selected starting parameter vectors to solve for a global minimum, $\left(\hat{\Omega}, \hat{\Phi}\right)$, to the constrained optimization problem in equation (32). Among the 15,000 initial parameter vectors, about 1,000 are feasible. We start at these 1,000 feasible points and solve equation (32) for 100 iterations each, which yields 1,000 preliminary solutions. We then sort these 1,000 preliminary solutions from smallest objective function value to largest and resolve equation (32) starting at the preliminary solutions until about 400 local minima are found. Our baseline internally calibrated parameter values reported in table 1 panel B are the parameters associated with the minimum objective function value among all the 400 local minima. Of these 400 local minima, about 230 minima have objective function values, parameter estimates, and endogenous variables that are the same as the minima we selected implying we attain a global minimum.

In our weighting matrix in equation (32), W, we choose relatively high weights for the manufacturing capacity utilization, fraction of exporting firms, and log-linear trade elasticity moments because they are influential in for determining the retailers' search cost, c_{do}, and the matching elasticity, η. In particular, we choose a weight of five for these five moments and a weight of one for all other moments.

C.1.2 Nonlinear constraints

Equation (32) includes additional linear and nonlinear equilibrium and parameter inequality constraints, $\Psi(\Phi, \Omega)$, which we list here:

1. The fraction of matched producers cannot be negative and cannot exceed one:

$$0 \leq 1 - u_{do} - i_{do} \leq 1 \; \forall do$$

2. The effective entry cost must be non-negative:

$$F_{do} \geq 0 \; \forall do$$

3. The effective entry cost must be weakly less than total imports:

$$F_{do} \leq IM_{do} \; \forall do$$

4. Retailers' profit margin must be weakly smaller than the overall profit margin:

$$1 + b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}) \leq \mu \; \forall do$$

5. Retailers' profit margin must be weakly larger than one:

$$1 + b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}) \geq 1 \; \forall do$$

6. Iceberg costs must be weakly greater than one:

$$\tau_{do} \geq 1 \; \forall do$$

7. Investment cannot exceed output:

$$I_d \leq Y_d \; \forall d$$

8. Output must be non-negative:

$$Y_d \geq 0 \; \forall d$$

9. The labor endowment must be weakly smaller than output:

$$L_d \leq Y_d \; \forall d$$

10. The idle rate must be non-negative:

$$i_{do} \geq 0 \; \forall do$$

11. The threshold productivity in the domestic market must be weakly less than the threshold productivity in the foreign market:

$$\bar{\varphi}_{oo} \leq \bar{\varphi}_{do} \forall o$$

12. Persistence in export status cannot exceed one and must be weakly greater than zero:

$$0 \leq \beta_{du} \leq 1 \; \forall d \in \{u, c\}$$

See appendix C.1.3 for a definition of export persistence, β_{do}, in the context of the model.

C.1.3 Export persistence

Suppose we have a linear regression that relates export status of a firm this period, y_{it}, with export status last period, $y_{i,t-1}$:

$$y_{it} = \alpha + \beta y_{i,t-1} + \epsilon_{it}.$$

in which we drop *do* notation. Notice that

$$\mathbb{E}\left[y_{it} | y_{i,t-1}\right] = \alpha + \beta y_{i,t-1}$$

and recall that

$$\begin{aligned}
\mathbb{E}\left[y_{it}|y_{i,t-1}\right] &= \mathbb{E}\left[y_{it}|y_{it}=1, y_{i,t-1}\right] \mathbb{P}\left[y_{it}=1|y_{i,t-1}\right] + \mathbb{E}\left[y_{it}|y_{it}=0, y_{i,t-1}\right] \mathbb{P}\left[y_{it}=0|y_{i,t-1}\right] \\
&= 1 \times \mathbb{P}\left[y_{it}=1|y_{i,t-1}\right] + 0 \times \mathbb{P}\left[y_{it}=0|y_{i,t-1}\right] \\
&= \mathbb{P}\left[y_{it}=1|y_{i,t-1}\right].
\end{aligned}$$

This implies that

$$\mathbb{P}\left[y_{it}=1|y_{i,t-1}\right]=\alpha+\beta y_{i,t-1}.$$

For reference, note that

$$\mathbb{P}\left[y_{it}=1|y_{i,t-1}=1\right]=\alpha+\beta$$

$$\mathbb{P}\left[y_{it}=0|y_{i,t-1}=1\right]=1-\mathbb{P}\left[y_{it}=1|y_{i,t-1}=1\right]=1-(\alpha+\beta)$$

$$\mathbb{P}\left[y_{it}=1|y_{i,t-1}=0\right]=\alpha$$

$$\mathbb{P}\left[y_{it}=0|y_{i,t-1}=0\right]=1-\mathbb{P}\left[y_{it}=1|y_{i,t-1}=0\right]=1-\alpha.$$

In our model, we know that separation shocks occur at Poisson rate λ, which means that the probability that separation occurs during one unit of time (one unit is one year in our calibration) is $1-e^{-\lambda}$:

$$\mathbb{P}\left[y_{it}=0|y_{i,t-1}=1\right]=\left(1-e^{-\lambda}\right).$$

Therefore

$$\begin{aligned}
\mathbb{P}\left[y_{it}=1|y_{i,t-1}=1\right] &= 1-\mathbb{P}\left[y_{it}=0|y_{i,t-1}=1\right]\\
&= 1-\left(1-e^{-\lambda}\right)\\
&= e^{-\lambda}.
\end{aligned}$$

The probability of becoming an exporter means that you have to make contact with a retailer, which occurs at rate $\kappa\chi\left(\kappa\right)$, that the producer has productivity above the threshold, and the producer was searching:

$$\begin{aligned}
\mathbb{P}\left[y_{it}=1|y_{i,t-1}=0\right] &= \left(1-e^{-\kappa\chi(\kappa)}\right)\mathbb{P}\left[\varphi>\bar{\varphi}\right]\frac{u}{1-i}\\
&= \left(1-e^{-\kappa\chi(\kappa)}\right)\left(1-i\right)\frac{u}{1-i}\\
&= \left(1-e^{-\kappa\chi(\kappa)}\right)u.
\end{aligned}$$

Finally, notice that

$$\begin{aligned}
\beta &= (\alpha+\beta)-\alpha\\
&= \mathbb{P}\left[y_{it}=1|y_{i,t-1}=1\right]-\mathbb{P}\left[y_{it}=1|y_{i,t-1}=0\right]\\
&= e^{-\lambda}-\left(1-e^{-\kappa\chi(\kappa)}\right)u\\
&= e^{-\lambda}-\left(1-e^{-\kappa\chi(\kappa)}\right)u.
\end{aligned}$$

C.2 Identifying retailers' search costs

C.2.1 Identifying importing retailers' search costs

In this section we describe how data on the fraction of exporters in uc and cu markets can be used to help inform the importing retailers' search cost parameters, c_{uc} and c_{cu}. As an example, consider the probability that a U.S. firm exports to China:

$$\mathbb{P}\left[export_{cu}\right]=\mathbb{P}\left[export_{cu}|\varphi>\bar{\varphi}_{cu}\right]\mathbb{P}\left[\varphi>\bar{\varphi}_{cu}\right]+\mathbb{P}\left[export_{cu}|\varphi\leq\bar{\varphi}_{cu}\right]\mathbb{P}\left[\varphi\leq\bar{\varphi}_{cu}\right]$$
$$\mathbb{P}\left[export_{cu}\right]=1-u_{cu}-i_{cu}$$

We divide this term by $1 - i_{cu}$ to obtain the fraction of producers that are matched in the cu market. We know that this matched rate is monotonically increasing in κ_{cu} because the producers' finding rate, $\kappa_{cu}\chi(\kappa_{cu})$, is increasing in market tightness. From the free-entry condition (equation 19) we know that c_{cu} is important in determining equilibrium κ_{cu}. In particular, as the retailers' search cost rises, there is less entry into retailing and κ_{cu} falls. Therefore, we can use observed data on the fraction of U.S. firms that export to China to pin down the c_{cu} parameter. Similarly, we can use the fraction of Chinese firms that export to the United States to pin down the c_{uc} parameter.

C.2.2 Identifying domestic retailers' search costs

In this section, we describe how data on manufacturing capacity utilization can help identify domestic retailers' search cost parameters, c_{uu} and c_{cc}. The capacity utilization rate is mainly determined from two measures collected from manufacturing plants. The first measure is the market value of actual production during a time period. The second measure is the full production capability for that time period assuming normal downtime, fully available inputs, and with currently available machinery and equipment. The manufacturing capacity utilization rate is the sum of all plants' market value of actual production divided by the sum of all plants' full production capability.

The quantity in our model that is analogous to the capacity utilization rate in the data for producers in country o is the value of all sales divided by the value of sales if there were no search frictions:

$$\frac{\sum_k IM_{ko}}{\sum_k IM_{ko} / \left(1 - \frac{u_{ko}}{1-i_{ko}}\right)}.$$

In the main text, we restrict our attention to capacity utilization in the domestic market only to make the exercise transparent. For the United States, domestic capacity utilization is defined as:

$$\frac{IM_{uu}}{IM_{uu} / \left(1 - \frac{u_{uu}}{1-i_{uu}}\right)} = 1 - \frac{u_{uu}}{1 - i_{uu}} = \frac{\kappa_{uu}\chi(\kappa_{uu})}{\lambda + \kappa_{uu}\chi(\kappa_{uu})}.$$

As mentioned before, this quantity is monotonically increasing in κ_{uu}, which is negatively related to c_{uu}, and so monotonically decreasing in c_{uu}. We use observed data on U.S. and Chinese manufacturing capacity utilization to identify the domestic retailers' search costs in each country.

Using capacity utilization to inform the level of domestic search frictions follows Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2017) and Petrosky-Nadeau et al. (2018). Michaillat and Saez (2015, equation 16) shows that the steady-state number of matched producers is closely related to capacity utilization. They also discuss that visits are analogous to vacancies in the labor market because visits capture the process buyers must follow to obtain an item. We use the reasoning from these papers to connect capacity utilization to the fraction of matched producers and discipline search costs in domestic markets.

Capacity underutilization has averaged about 20 percent in post-war U.S. data and is higher than labor underutilization. FRB capacity utilization data (FRB, 2020) are based on the Quarterly Survey of Plant Capacity Utilization (QPC) from the U.S. Census Bureau (CB) (CB, 2018), which covers manufacturing and publishing sectors. The capacity utilization rate is defined as the value of actual output (in dollars) divided by the value of sustainable maximum output (in dollars). The value of sustainable maximum output is the greatest level of output a plant can maintain within the framework of a realistic work schedule, after factoring in normal downtime and assuming sufficient availability of inputs to operate the capital in place. The capacity underutilization rate is one minus the capacity utilization rate and has averaged about 20 percent in the United States since 1948 and was about 25 percent in 2016, as shown in figure A2.

We prefer the FRB measure of capacity utilization to the ISM measure because the ISM measure is based on a smaller sample of plants and overweights smaller producers. First, the ISM measure is based on a limited sample—approximately 350 respondents, whereas the QPC includes about 7,500 establishments from a systematic sampling frame. Second, in order to compute the aggregate measure, ISM reports the simple average of capacity utilization by each respondent whereas the QPC uses the sum of actual output divided by the sum of maximum output. Given that large firms make up most of aggregate production (Axtell, 2001) the simple average computed by the ISM overweights small plants and does not accurately measure aggregate capacity utilization. The ISM manufacturing capacity utilization rate is about 82 percent in 2016 (ISM, 2016) which is similar to the FRB measure of 75 percent.

C.2.3 Search cost symmetry

Intuitively, the cost retailers pay to search for producers should be similar whether they are searching for a domestic or foreign producer. As such, we assume that international search costs are simply the domestic search cost plus a symmetric international premium so that $c_{uc} = c' + c_{uu}$, $c_{cu} = c' + c_{cc}$, and $c' \geq 0$. This symmetry assumption implies, for example, that the cost a Chinese retailer pays to search for a U.S. producer is the same as the cost that Chinese retailer would pay to search for a Chinese producer plus c'. We find this structure for these costs intuitively appealing because, as Kneller and Pisu (2011) report, "identifying the first contact" and "establishing initial dialogue" are examples of search costs and these are likely to be mainly symmetric. We are comfortable imposing that international retailers' search costs at least exceed domestic retailers' search costs. Finally, we note that this restriction provides additional identification for the matching elasticity with respect to the number of searching producers, η.

C.3 Calibrating the matching elasticity

We show in proposition 3 that the trade elasticity in our model differs substantially from the standard elasticity. To calibrate our model to standard trade elasticities, we must first derive a log-linear estimating equation implied by our model that matches the specifications in the literature. We do this by rearranging the gravity equation (25) to collect similar indices of observation:

$$\begin{aligned} \ln\left(IM_{do}\right) &= \ln\left(\frac{\alpha}{C}\right) + \ln\left(\frac{C_d}{\rho_d^{-\theta}}\right) + \ln\left(C_o w_o^{-\theta}\right) + \\ &+ \ln\left[\left(1 - \frac{u_{do}}{1 - i_{do}}\right)\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\right] + \ln\left(\tau_{do}^{-\theta}\right). \end{aligned} \tag{C103}$$

Because the first three terms in equation (C103) are either a constant or only vary by destination or origin, we can simplify notation by writing

$$\phi = \ln\left(\frac{\alpha}{C}\right), \qquad \phi_d = \ln\left(\frac{C_d}{\rho_d^{-\theta}}\right), \qquad \text{and } \phi_o = \ln\left(C_o w_o^{-\theta}\right).$$

Also define the log of the terms introduced into the gravity equation by search frictions as

$$z_{do} = \ln\left[\left(1 - \frac{u_{do}}{1 - i_{do}}\right)\left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\right].$$

Thus, the log-linear gravity equation from our model can be expressed as

$$\ln\left(IM_{do}\right) = \phi + \phi_d + \phi_o - \theta \ln\left(\tau_{do}\right) + \beta_z z_{do}. \tag{C104}$$

Most gravity specifications in the literature do not include search frictions, z_{do}, and instead estimate

$$\ln\left(IM_{do}\right) = \zeta + \zeta_d + \zeta_o - \hat{\theta} \ln\left(\tau_{do}\right). \tag{C105}$$

We estimate equation (C105) to obtain the $\hat{\theta}$ implied by our model and target an analogous standard estimate in the literature.

Targeting standard estimates of the trade elasticity informs the elasticity of matches with respect to the number of searching producers, η. Estimating equation (C105) when the true model is equation (C104) results in omitted variable bias for estimates of θ (the trade elasticity in a Melitz-Pareto model without search frictions) characterized by

$$\mathbb{E}\left[-\hat{\theta} \mid \ln\left(\tau_{do}\right), \zeta, \zeta_d, \zeta_o\right] = -\theta + \beta_z \beta_{z,\tau}. \tag{C106}$$

In this equation, $\beta_{z,\tau}$ is the coefficient from a regression of z_{do} on $\ln\left(\tau_{do}\right)$ and fixed effects given by

$$z_{do} = \psi + \psi_d + \psi_o + \beta_{z,\tau} \ln\left(\tau_{do}\right).$$

The definition of z_{do} implies that $\beta_z = 1$ and we prove that the bias $\beta_{z,\tau}$ in equation (C106) is negative under weak restrictions below. In appendix B.4 we show that $\beta_{z,\tau}$ can be written as

$$\beta_{z,\tau} = \mathbb{E}\left[\left(\frac{u_{do}}{1 - i_{do}}\right)(1-\eta)\frac{d\ln\kappa_{do}}{d\ln\tau_{do}} + \frac{d\ln\left(1 - b_{do}\left(\cdot\right)\right)}{d\ln\tau_{do}} - \left(\frac{\theta}{\sigma-1} - 1\right)\frac{d\ln F\left(\kappa_{do}\right)}{d\ln\tau_{do}} \mid \psi, \psi_d, \psi_o, \ln\left(\tau_{do}\right)\right].$$

This expression, together with equation (C106), suggests that targeting standard estimates of $\hat{\theta}$ informs the elasticity of matches with respect to the number of searching producers, η, which appears in the first term under the expectation. For example, as $\eta > 0$ rises, the response of z_{do} to changes in iceberg trade costs, $\ln\left(\tau_{do}\right)$, falls so that $\beta_{z,\tau}$ is closer to zero and $-\hat{\theta}$ gets closer to zero in our calibration. The markup elasticity and the effective entry cost elasticity are small in our calibration, as shown in table 7. The elasticity of matches is partially informed by a symmetry assumption about retailers' search costs, as discussed in appendix C.2.3. Targeting the standard trade elasticity provides important additional identification of the elasticity of matches. As in the literature, when estimating the trade elasticity in our calibration, we only use international (do with $d \neq o$) observations and not domestic (dd) observations.

We consider the conditions that ensure that $\beta_{z,\tau} \leq 0$. First, the correlation between iceberg costs and the matched rate is negative because higher variable trade costs lower retailer entry and market tightness, which lowers the matched rate. Let $\rho(x, y)$ denote the correlation between x and y, then $\rho(\ln \tau_{do}, \ln[1 - u_{do}/(1 - i_{do})]) \leq 0$. Second, $\rho(\ln \tau_{do}, \ln(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))) < 0$ if $s_{do} = 0$, as shown in appendix B.2.18. The correlation between the effective entry cost term, $F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}$, in z_{do} and τ_{do} cannot be signed in general because that correlation depends on the empirical relationship between the parameters that define it, for example, f_{do}, and τ_{do}. Typically countries with high variable costs also have high effective fixed costs, leading to a weakly negative correlation between $F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}$ and τ_{do}, but it is sufficient to assume that any positive correlation between $F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}$ and τ_{do} is smaller than the other terms.

These inequalities imply that $\beta_{z,\tau} \leq 0$ so that the bias in equation (C106) is negative. Because the bias is negative, omitting the search friction terms from a standard gravity equation as in equation (C105) implies that the resulting estimate of $-\theta$ with respect to variable trade costs is *more negative* than if one included the search friction terms in the estimating equation. Lastly, if z_{do} is observable so that estimating equation (C104) is possible, the coefficient on $\ln(\tau_{do})$ will be $\mathbb{E}\left[-\hat{\theta} \mid x_{do}, \phi, \phi_d, \phi_o\right] = -\theta$, but this is not the trade elasticity in our model according to proposition 3.

C.4 Calibrating producers' fixed, sunk, and flow search costs

We follow di Giovanni and Levchenko (2012) and di Giovanni and Levchenko (2013) and calibrate the fixed costs of production, f_{do}, by using data from the Doing Business Indicators (DBI) database (WB, 2019a). For each country in the database, these measures document the time and costs associated with starting a new business and with exporting and importing a 20-foot dry-cargo container. We use the cost to start a business in the United States and China to discipline f_{uu} and f_{cc}, respectively. This cost is about \$600 in the United States and about \$30 in China. To identify the fixed costs associated with international production, f_{uc} and f_{cu}, we use the sum of the cost of exporting and importing from the Trading Across Borders module of the DBI. For example, to discipline f_{uc}, we use the cost of exporting from China plus the cost of importing into the United States. These trading costs are about \$675 in both the United States and China.

The threshold productivity in equation (18) is defined by the effective entry cost, F_{do}, which is a linear function of the producers' fixed cost, f_{do}, search cost, l_{do}, their idle flow payoff, h_{do}, and their sunk cost, s_{do}. The fact that these costs enter linearly makes it difficult to separately identify them. Because we are ultimately interested in the effective entry cost, F_{do}, as a whole and are less concerned about its individual components, we set l_{do} and s_{do} to zero. Setting $l_{uu} = l_{cc} = 0$ is also consistent with small domestic search costs found in Eaton et al. (2014) and $s_{do} = 0 \ \forall do$ also matches the treatment of sunk costs in most steady-state trade models such as Melitz (2003), Chaney (2008), and Allen (2014).

C.5 Identifying producers' flow idle benefit

The minimum productivity draw identifies the flow payoff from being idle, h_{do}. In particular, we assume that when search costs, tariffs, and the U.S. input cost premium are all zero and each country has a price index equal to the autarky price index, the threshold productivity in each country is equal to one. These assumptions are contradictory—no tariffs but autarky price indexes—but are exactly the restrictions we want to impose when solving for h_{do} because they ensure we choose an h_{do} such that that all equilibria in our counterfactual exercises will have $\bar{\varphi}_{do} \geq 1$.

We implement this procedure by computing another set of equilibrium variables ($\bar{\varphi}_{do}$ $\forall do$, C_d $\forall d$, and π) under the additional restrictions, using equations (18), (22), and (23). Since search costs are zero, we know that market tightness and producers' findings rates in this equilibrium will be infinite using equation (19).

With these restrictions, the cutoff in the *do* market is determined only by global variables, including consumption, and the ratio of h_{dd} and h_{do}. Further imposing that the cutoff is equal to one implies that the domestic producers' flow idle benefit, h_{dd}, does not vary by country, and that the international producers' flow idle benefit are symmetric, $h_{do} = h_{od}$.

C.6 Fitting domestic absorption

The model has difficulty matching domestic absorption for two reasons. First, the observed data are based on an interaction of many countries whereas our calibration is based on a two-country model. Targeting aggregate consumption, C_u, in our model requires, for example, fitting the model (m) aggregate consumption $C_u^m = (1-\alpha)C_u^m + C_{uc}^m + C_{uu}^m$ for the United States to data (d) that is generated by $C_u^d = (1-\alpha)C_u^d + C_{uc}^d + C_{uu}^d + \sum_{o \neq c,u} C_{uo}^d$. The model can match consumption sourced from China well by, for example, adjusting iceberg costs so that $C_{uc}^m \approx C_{uc}^d$. To close the remaining gap between C_u^m and C_u^d caused by omitting $\sum_{o \neq c,u} C_{uo}^d$, the calibration pushes up domestically-produced consumption above what is observed in the data until $C_{uu}^m \approx C_{uu}^d + \sum_{o \neq c,u} C_{uo}^d$. This implies that domestic absorption in the model is $C_{uu}^m > C_{uu}^d$ and one way to rectify this is to reduce aggregate consumption below its observed value because $C_{uu}^m = C_u^m - (1-\alpha)C_u^m - C_{uc}^m$. These are the main reasons why $IM_{uu}^d = \$2.8$ tril. but $IM_{uu}^m = \$4.3$ tril. and $C_u^d = \$12.8$ tril. and $C_u^d = \$10.5$ tril. This discussion abstracts from the effect of the endogenous markup term in converting $IM_{uu} = (1 - b_{uu}(\cdot))\, C_{uu}$ but in our baseline calibration $1 - b_{uu}(\cdot) = 0.95$ and it is not the major source of this discrepancy between model and data.

Second, the real-world trade balances for each country contributes to this discrepancy. The U.S. consumption to GDP ratio is 68 percent in the data but is only 39 percent for China reflecting the substantial U.S. trade deficit and Chinese trade surplus. In other words, the United States consumes more in the data than our model would predict because the United States is borrowing internationally to fund current consumption and China is saving internationally. The model does not allow for unbalanced trade and so pushes the U.S. consumption to GDP ratio in the model down and pushes up the Chinese consumption to GDP ratio as visible in the last two rows of table 2. Because the model pushes down (up) U.S. (Chinese) consumption it pushes up (down) the U.S. (Chinese) domestic absorption consumption ratio above (below) what is observed in the data.

D Robustness results appendix

In this appendix we show that our quantitative results are robust to search costs that are much lower than our baseline level.

D.1 Model fit for different levels of search costs

Lower levels of search frictions increase capacity utilization and the fraction of exporting producers. We solve for the equilibrium endogenous variables using three lower levels of retailers' flow search costs ($c_{do}\ \forall\ do$)—at 50, 10, and 1 percent of their baseline values—while holding all other parameters at their baseline values. For these exercises we lower the retailers' flow cost of search in all *do* markets simultaneously. With lower search frictions, capacity utilization and the fraction of exporting producers rise, as shown in table A2. For example, lowering search frictions to 10 percent of their baseline values (column 4), increases U.S. and Chinese capacity utilization rates substantially—by at least 13 percentage points (pp)—and increases the fraction of exporting firms in the *cu* and *uc* markets by 17 and 27 pp, respectively. (Increases in capacity utilization and the fraction of firms exporting are consistent with our identification intuition in section 6.1).

D.2 Welfare attenuation for different levels of search costs

Search frictions attenuate welfare responses even when they are small. Welfare increases from reducing tariffs are attenuated relative to a model without search for various levels of search frictions, as shown in table A3. As search costs fall, the degree of welfare attenuation also falls. But, changes in welfare are still about 25 percent smaller than in the model without search even when search costs are only one percent of our baseline calibration (column 5).

D.3 The trade elasticity in our model for different levels of search frictions

The trade elasticity in our economy with search remains more negative than in a model without search frictions for various levels of search costs, as shown in table A4. As search costs fall, the difference between the trade elasticity in a model with and without search declines. But, even with search costs at one percent of our baseline calibration (column 5), we find that the trade elasticity is 25 percent higher than in a model without search frictions.

D.4 Replicating tariffs' effects with higher search costs

Doubling retailers' search costs mimics reductions in trade flows and aggregate welfare of a 10 percent increase in bilateral tariffs. Flow search costs affect market tightness and producers' matched rates, which affect the price index in the same way as tariffs. In our numerical example, when flow search costs in foreign markets rise to mimic bilateral tariff increases, average search costs rise in one market but fall in the other. This is in contrast to labor-search models that have benefits from matching that are unresponsive to c_{do} Pissarides (2000, as in) so that average search costs do not change in response to changes in c_{do}.

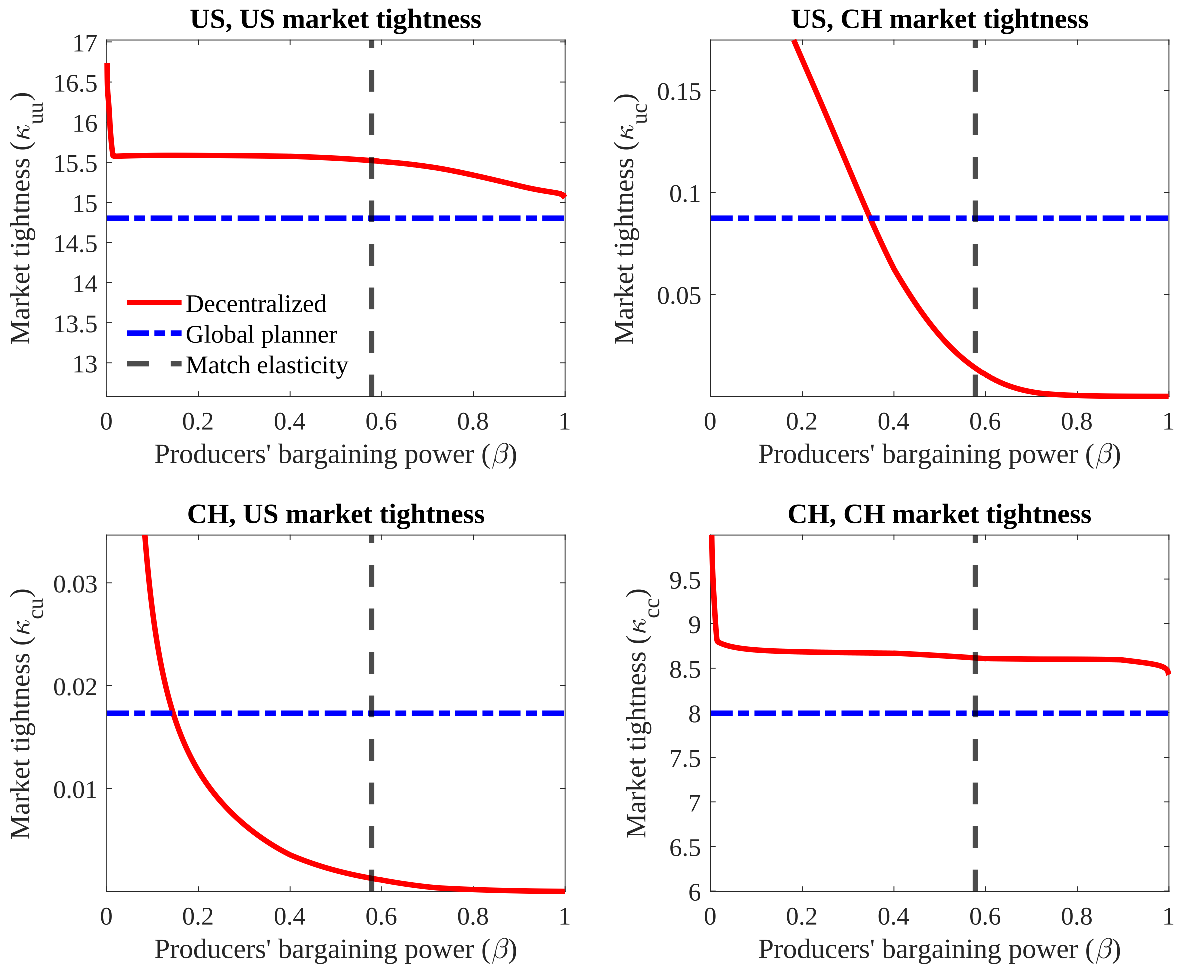
We first quantify the effects of a 10 percent increase in bilateral tariffs, so that $\tau'_{do} = 1.1 \times \tau_{do}$ for $d \neq o$. These results appear in column (2) of table A5. Solving the model with these higher bilateral tariffs, but keeping all other parameters at the baseline values from table 1, implies that welfare in both countries falls. The United States experiences a 0.7 percent reduction in welfare, whereas China's welfare falls by 0.2 percent. U.S. imports from China fall by about 35 percent and Chinese imports from the United States fall by about 40 percent.

To mimic these tariffs' effects with search frictions, we set all parameters to the baseline values in table 1, but increase search costs, c_{cu} and c_{uc}, so that imports decline by the same amount as in column (2). Mimicking the reduction to imports requires more than doubling retailers' search costs in the CH-US and US-CH markets and reducing producers' matched rates by 3 pp. in the CH-US market and 7 pp. in the US-CH market, as shown in column (3). Aside from the changes in producers' matched rates in foreign markets, these search cost increases mimic the tariff increase and, in particular, increase the price index and reduce welfare in the two countries by the same amount. We present other equilibrium quantities in column (3) of table A5.

In our model, changes in average search costs respond to changes in flow search costs—unlike in Pissarides (2000)—and changes in average search costs are not informative about changes in producers' matched rates. To mimic the tariff increases in column (2) of table A5, average search costs rise by 5.6 percent in the US-CH and fall by 0.1 percent in the CH-US market, despite more than doubling flow search costs in both markets. In the standard search model Pissarides (2000, equation 1.9), average retailer search costs (the left hand side of equation 19) equal the benefits of matching, which is a bundle of parameters, on the right hand side. This implies that changes in flow search costs, c_{do}, are offset by retailer entry and changes in market tightness, so that average search costs, $c_{do}/\chi\left(\kappa_{do}\right)$, remain unchanged in equilibrium. In our model, the right hand side of the free entry condition (equation 19) is a decreasing function of the producer matched rate (equation A61) so that the benefits of retailing are also affected by retailers' search decisions. In particular, when flow search costs rise and retailer entry falls, producers' matched rates fall, which raises the benefits of retailing and encourages retailer entry. The total effect on average search costs is ambiguous despite an unambiguous decline in market tightness.

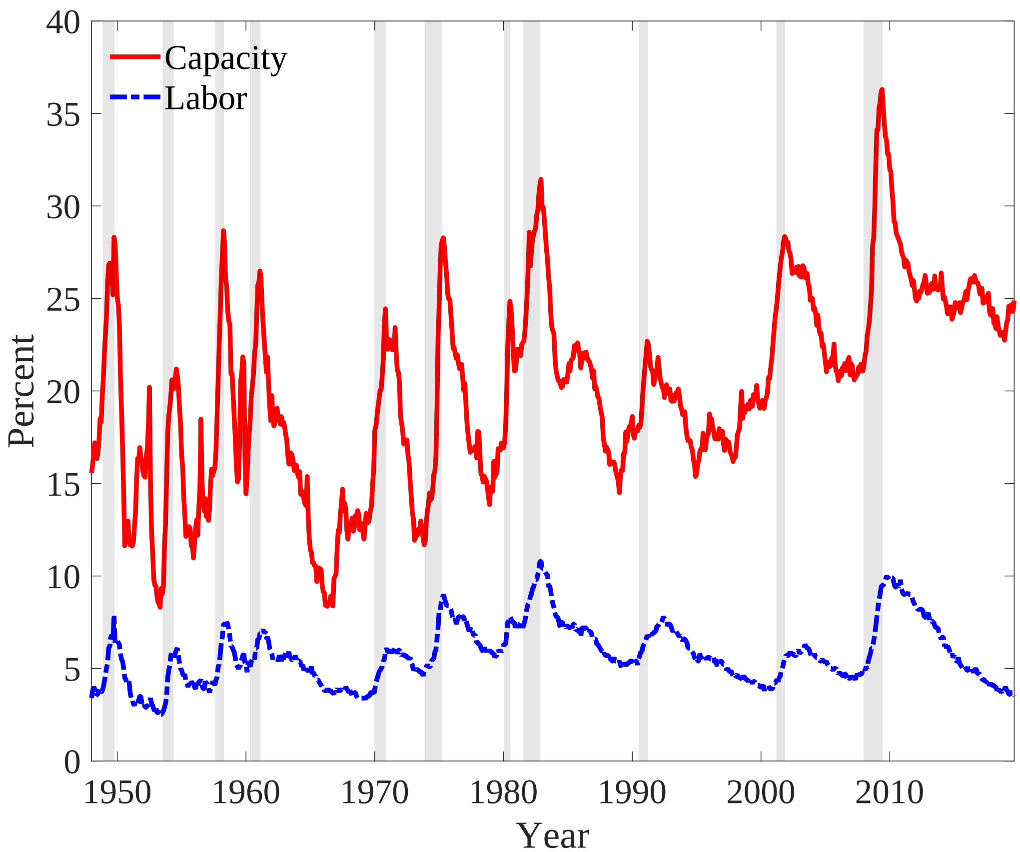
E Appendix figures and tables

Figure A1: Decentralized and global planner market tightness by bargaining power



Note: There is no producers' bargaining power, β, that attains the social planner's solution in each market of our calibrated model. We solve our model for various producers' bargaining powers, while holding all other parameters at their baseline values, and depict how the decentralized market tightnesses vary with β in each market. We solve the social planner's problem in equation (A73) to yield the efficient tightness in each market, represented by the blue horizontal line. In the two international markets there exist two different bargaining powers that equate the decentralized market tightnesses with the social planner's. In the two domestic markets, the social planner's market tightness and the decentralized market tightness coincide for producers' bargaining power very close to one. Our calibration sets producers' bargaining power to 0.5. ‘‘US’’ stands for the United States, and ‘‘CH’’ stands for China. See section 4.3 and appendix A.16 for details.

Figure A2: Capacity and labor underutilization



Note: Capacity underutilization in the manufacturing sector has averaged about 20 percent in the United States since 1948 and was about 25 percent in 2016. Labor underutilization, as measured by the unemployment rate, tends to be substantially lower than capacity underutilization. The FRB capacity utilization rate measures the value of output divided by the value of sustainable maximum output—the greatest level of output a plant can maintain within the framework of a realistic work schedule, after factoring in normal downtime and assuming sufficient availability of inputs to operate the capital in place. The unemployment rate is defined as the number of people who are without a job but available and looking for a job (the unemployed) divided by the number of unemployed and employed (the labor force). Capacity underutilization data come from FRB (2020) and the unemployment rate data from BLS (2020). This figure graphs January 1948 through September 2019. See section 6.1 and appendix C.2.2 for details.

Table A1: Changes in producer matched rates, imports, price indexes, and welfare when search frictions are reduced

	(1) Baseline search frictions	(2) No search frictions		(3) Reducing int'l search frictions to domestic search frictions	
	(1.1) Levels	(2.1) Levels	(2.2) Δ from baseline	(3.1) Levels	(3.2) Δ from baseline
Producer matched rate in US-US mkt.	76%	100%	24pp	73%	-3pp
Producer matched rate in CH-US mkt.	7%	100%	93pp	50%	43pp
Producer matched rate in US-CH mkt.	19%	100%	81pp	76%	58pp
Producer matched rate in CH-CH mkt.	71%	100%	29pp	69%	-3pp
US absorption of domestic prod.	$4.3 tril.	$3.1 tril.	-27.8%	$2.9 tril.	-33.4%
Chinese imports from U.S.	$92.4 bil.	$875.1 bil.	847.4%	$635.9 bil.	588.5%
US imports from China	$595.1 bil.	$2110.7 bil.	254.7%	$2069.5 bil.	247.8%
CH absorption of domestic prod.	$2.2 tril.	$1.6 tril.	-28.7%	$1.6 tril.	-25.5%
US price index for US goods	$35.5 mil./util	$32.5 mil./util	-8.5%	$36.4 mil./util	2.4%
CH price index for US goods	$102.2 mil./util	$39.5 mil./util	-61.3%	$50.4 mil./util	-50.7%
US price index for CH goods	$67.4 mil./util	$37.1 mil./util	-44.9%	$40.8 mil./util	-39.5%
CH price index for CH goods	$36.3 mil./util	$32.5 mil./util	-10.5%	$37 mil./util	1.9%
US price index	$368.5 mil./util	$331.1 mil./util	-10.2%	$348.9 mil./util	-5.3%
Chinese price index	$378.3 mil./util	$335 mil./util	-11.5%	$363.9 mil./util	-3.8%
US welfare	28.4 thous. utils	31.7 thous. utils	11.4%	30 thous. utils	5.6%
Chinese welfare	12.9 thous. utils	14.6 thous. utils	13%	13.4 thous. utils	4%
Retailer matched rate in US-US mkt.	17%	0%	-17pp	21%	4pp
Retailer matched rate in CH-US mkt.	97%	0%	-97pp	50%	-48pp
Retailer matched rate in US-CH mkt.	88%	0%	-88pp	17%	-72pp
Retailer matched rate in CH-CH mkt.	22%	0%	-22pp	26%	3pp
US consumption	$10.5 tril.	$10.5 tril.	0.1%	$10.5 tril.	-0.004%
CH consumption	$4.9 tril.	$4.9 tril.	0.1%	$4.9 tril.	0.025%
US dom. sales at final sales prices	$4.6 tril.	$3.1 tril.	-31.4%	$3.1 tril.	-33%
CH imports from U.S. at fin. sales prices	$105.1 bil.	$875.1 bil.	733%	$694.6 bil.	561.2%
US imports from CH. at fin. sales prices	$670.8 bil.	$2110.9 bil.	214.7%	$2177 bil.	224.6%
Chinese dom. sales at final sales prices	$2.3 tril.	$1.6 tril.	-32.8%	$1.8 tril.	-25.2%
US GDP	$19.4 tril.	$19.4 tril.	0.005%	$19.4 tril.	0.0003%
Chinese GDP	$11.6 tril.	$11.6 tril.	0.005%	$11.6 tril.	0.0003%

Note: Lowering search frictions increases welfare by reducing the price index through reallocating production across countries. The table presents deviations from the baseline calibration discussed in section 6 and has more details than table 3. Column (1) presents the baseline calibration. Column (2) eliminates search frictions altogether and shows that the associated welfare gains are large. Column (3) reduces retailers' search costs in international markets to their domestic levels. For example, U.S. retailers' search cost for a partner in China are reduced to search costs for a partner in the U.S. See section 7.1 for further details. "pp" stands for percentage point. "CH" stands for China and "US" stands for the United States.

Table A2: Model fit

Moment in the data	(1) Data	(2) Baseline search frictions	(3) 50% of baseline search frictions	(4) 10% of baseline search frictions	(5) 1% of baseline search frictions
Log-linear import elasticity	-6	-6.9	-6.7	-5.8	-4.5
US mfg. capacity utilization rate	75%	76%	81%	89%	96%
Percent of US firms exporting to CH	6%	7%	10%	24%	54%
Percent of CH firms exporting to US	21%	19%	25%	46%	74%
CH mfg. capacity utilization rate	74%	71%	77%	87%	95%
Cost of business start up in US	$550	$550	$550	$550	$550
Fixed foreign trade costs (CH-US)	$683	$683	$683	$683	$683
Fixed foreign trade costs (US-CH)	$664	$664	$664	$664	$664
Cost of business start up in CH	$28	$28	$28	$28	$28
US absorption of domestic prod. (IM_{uu})	$2.8 tril.	$4.3 tril.	$4.2 tril.	$3.9 tril	$3.4 tril.
CH imports from US (IM_{cu})	$116 bil.	$92 bil.	$128 bil.	$260 bil	$521 bil.
US imports from CH (IM_{uc})	$463 bil.	$595 bil.	$755 bil.	$1175 bil	$1666 bil.
CH absorption of domestic prod. (IM_{cc})	$2.7 tril	$2.2 tril	$2.2 tril	$2.1 tril	$1.9 tril
US dom. absorp. consump. ratio (IM_{uu}/C_u)	22.2%	41.4%	40.2%	36.7%	32.8%
CH-US export consump. ratio (IM_{cu}/C_u)	0.9%	0.9 %	1.2%	2.5%	5%
US-CH export consump. ratio (IM_{uc}/C_c)	10.5%	12.2%	15.4%	24%	34%
CH dom. absorp. consump. ratio (IM_{cc}/C_c)	61.5%	45.1%	44.8%	42.7%	37.9%
Average relationship duration	1	1	1	1	1
GDP in US	$18.7 tril.	$19.4 tril.	$19.4 tril.	$19.4 tril	$19.4 tril.
GDP in CH	$11.2 tril.	$11.6 tril.	$11.6 tril.	$11.6 tril	$11.6 tril.
Consumption in US	$12.8 tril.	$10.5 tril.	$10.5 tril.	$10.5 tril	$10.5 tril.
Consumption in CH	$4.4 tril.	$4.9 tril.	$4.9 tril.	$4.9 tril	$4.9 tril.
US consumption to GDP share	68%	54%	54%	54%	54%
CH consumption to GDP share	39%	42%	42%	42%	42 %

Note: Reduced search frictions increase the model's capacity utilization and fraction of exporting firms. Column (1) of this table presents the value of the moment in the data. Column (2) is the baseline calibration in the main text and matches the "Model" column in table 2 of the main text. Subsequent columns present the value of the equivalent moment in the model at different levels of search frictions, c_{do} $\forall do$. We lower these search costs in all markets by the fraction indicated. "CH" stands for China, "US" stands for the United States, and "GDP" stands for Gross Domestic Product. See section 7.1 and appendix D.1 for further details.

Table A3: Decomposing the ex-ante Chinese welfare response to a unilateral tariff increase

Determinants of welfare change	(1) No search frictions and 10% unilateral tariff	(2) Baseline search frictions and 10% unilateral tariff	(3) 50% of baseline search frictions and 10% unilateral tariff	(4) 10% of baseline search frictions and 10% unilateral tariff	(5) 1% of baseline search frictions and 10% unilateral tariff
Pre-tariff dom. consump. share (λ_{cc})	0.321	0.4785	0.47	0.44	0.384
Post-tariff dom. consump. share (λ'_{cc})	0.354	0.487	0.482	0.46	0.414
Ratio of dom. consump. shares $\left(\hat{\lambda}_{cc} = \lambda'_{cc}/\lambda_{cc}\right)$	1.103	1.0177	1.024	1.046	1.078
Dom. consump. shares' effect on welfare $\left(\hat{\lambda}_{cc}^{-\frac{\alpha}{\theta}}\right)$	0.985	0.9972	0.996	0.993	0.988
Pre-tariff dom. matched rate $\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)$	1	0.713	0.772	0.871	0.946
Post-tariff dom. matched rate $\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)'$	1	0.715	0.773	0.873	0.947
Ratio of dom. matched rates $\widehat{\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)}$	1	1.002	1.002	1.003	1.002
Dom. matched rates' effect on welfare $\widehat{\left(1 - \frac{u_{cc}}{1-i_{cc}}\right)}^{\frac{\alpha}{\theta}}$	1	1.0004	1.0004	1.0004	1.0003
Pre-tariff dom. consump. level (C_c)	\$4.9 tril	\$4.9 tril	\$4.9 tril	\$4.9 tril	\$4.9 tril
Post-tariff dom. consump. level (C'_c)	\$4.9 tril	\$4.9 tril	\$4.9 tril	\$4.9 tril	\$4.9 tril
Ratio of dom. consump. levels $\left(\hat{C}_c = C'_c/C_c\right)$	1	1	1	1	1
Dom. consump. levels' effect $\left(\hat{C}_c^{1+\frac{\alpha}{\theta}\left(1-\frac{\theta}{\sigma-1}\right)}\right)$	1	1	1	1	1
Welfare as fraction of pre-tariff welfare $\left(\hat{W}_c\right)$	0.985	0.998	0.997	0.993	0.989
Welfare percent change $\left(100 \times \left[\hat{W}_c - 1\right]\right)$	-1.53	-0.24	-0.33	-0.66	-1.15
Pre-tariff CH price index (Ξ_c)	\$335 mil./util	\$378.3 mil./util	\$372.2 mil./util	\$360.7 mil./util	\$348 mil./util
Post-tariff CH price index (Ξ'_c)	\$340.2 mil./util	\$379.2 mil./util	\$373.4 mil./util	\$363.1 mil./util	\$352 mil./util
Price index percent change $\left(100 \times \left[\hat{\Xi}_c - 1\right]\right)$	1.55	0.24	0.33	0.66	1.16

Note: Our welfare attenuation result is robust to search costs that are much smaller than our baseline calibration. Using proposition 2 in the main text, we decompose the response of welfare in this table. Column (1) presents the response without search frictions, which is the same as ACR and is completely determined by the ratio of the domestic consumption shares and model parameters α, θ, and σ. Some rows in column (1) are exactly 1 because those factors do not change in a model without search frictions. Column (2) presents the decomposition of the effect in our model with search frictions. Subsequent columns present the decomposition with smaller levels of search frictions, c_{do} $\forall do$. As search costs fall, welfare attenuation also falls. But, changes in welfare are still about 25 percent smaller than in the model without search frictions even when search frictions are one percent of our baseline calibration (column 5). See section 7.2 and appendix D.2 for further details.

Table A4: Decomposing the Chinese consumption and trade elasticities

	(1) No search frictions and 10% unilateral tariff	(2) Baseline search frictions and 10% unilateral tariff	(3) 50% of baseline search frictions and 10% unilateral tariff	(4) 10% of baseline search frictions and 10% unilateral tariff	(5) 1% of baseline search frictions and 10% unilateral tariff
Pareto shape parameter ($-\theta$)	-3.18	-3.18	-3.18	-3.18	-3.18
Elasticity of CH producers	0	0	0	0	0
Elasticity of US producers	0	0	0	0	0
Elasticity of the CH-US matched rate	0	-2.12	-1.98	-1.49	-0.69
Elasticity of the CH-CH matched rate	0	0.02	0.03	0.03	0.02
Effect of CH-CH & CH-US eff. entry costs	0	-0.13	-0.13	-0.1	-0.06
Consumption elasticity	-3.18	-5.45	-5.31	-4.8	-3.94
Elasticity of CH-US markup	0	-0.01	-0.01	-0.03	-0.04
Elasticity of CH-CH markup	0	0	0	0	0
Trade elasticity	-3.18	-5.47	-5.33	-4.83	-3.99

Note: Our trade elasticity result is robust to search costs that are much smaller than our baseline calibration. The table presents equilibrium variables in response to a 10 percent increase in unilateral tariffs on imports into China from the United States. The decomposition is based on proposition 3, along with (B89) and (B92) in appendix B.2. Column (1) presents the response of the consumption and trade shares to a foreign tariff shock with no search frictions, which is $-\theta$ (equation B92). Column (2) presents the decomposition of these elasticities into their components in our model with search frictions. Subsequent columns present the decomposition with smaller levels of search frictions, $c_{do}\ \forall do$. As search costs fall, the trade elasticity also falls. But, even with search frictions at one percent of our baseline calibration (column 5), we find that the trade elasticity is 25 percent higher than in a model without search frictions. See section 7.4 and appendix D.3 for further details. ‘‘eff’’ stands for effective.

Table A5: Changes in welfare, imports, and the unmatched rate in response to tariff and search cost changes

	(1) Baseline search frictions	(2) Baseline search frictions and 10% bilateral tariff		(3) Search costs equiv. to 10% bilateral tariff	
	(1.1)	(2.1)	(2.2)	(3.1)	(3.2)
	Levels	Levels	Δ from baseline	Levels	Δ from baseline
Producer matched rate in US-US mkt.	76%	77%	0pp	77%	0pp
Producer matched rate in CH-US mkt.	7%	6%	-1pp	4%	-3pp
Producer matched rate in US-CH mkt.	19%	16%	-3pp	12%	-7pp
Producer matched rate in CH-CH mkt.	71%	71%	0pp	71%	0pp
US absorption of domestic prod.	\$4.3 tril.	\$4.6 tril.	5.1%	\$4.6 tril.	5.1%
Chinese imports from U.S.	\$92.4 bil.	\$55.8 bil.	-39.5%	\$55.8 bil.	-39.5%
US imports from China	\$595.1 bil.	\$389.1 bil.	-34.6%	\$389.1 bil.	-34.6%
CH absorption of domestic prod.	\$2.2 tril.	\$2.2 tril.	1.8%	\$2.2 tril.	1.8%
US price index for US goods	\$35.5 mil./util	\$35.4 mil./util	-0.3%	\$35.4 mil./util	-0.3%
CH price index for US goods	\$102.2 mil./util	\$121.4 mil./util	18.8%	\$121.3 mil./util	18.7%
US price index for CH goods	\$67.4 mil./util	\$78.6 mil./util	16.7%	\$78.6 mil./util	16.6%
CH price index for CH goods	\$36.3 mil./util	\$36.3 mil./util	-0.1%	\$36.3 mil./util	-0.1%
US price index	\$368.5 mil./util	\$371.1 mil./util	0.7%	\$371.1 mil./util	0.7%
Chinese price index	\$378.3 mil./util	\$379.2 mil./util	0.2%	\$379.2 mil./util	0.2%
US welfare	28.4 thous. utils	28.2 thous. utils	-0.7%	28.2 thous. utils	-0.7%
Chinese welfare	12.9 thous. utils	12.9 thous. utils	-0.2%	12.9 thous. utils	-0.2%
Retailer matched rate in US-US mkt.	17%	17%	0pp	17%	0pp
Retailer matched rate in CH-US mkt.	97%	98%	1pp	99%	1pp
Retailer matched rate in US-CH mkt.	88%	91%	2pp	94%	5pp
Retailer matched rate in CH-CH mkt.	22%	22%	0pp	22%	0pp
US consumption	\$ 10.5 tril.	\$10.5 tril.	-0.003%	\$10.5 tril.	-0.003%
CH consumption	\$ 4.9 tril.	\$4.9 tril.	-0.003%	\$4.9 tril.	-0.003%
US dom. sales at final sales prices	\$4.6 tril.	\$4.8 tril.	5.1%	\$4.8 tril.	5%
CH imports from U.S. at fin. sales prices	\$105.1 bil.	\$63.6 bil.	-39.5%	\$63.6 bil.	-39.4%
US imports from CH. at fin. sales prices	\$670.8 bil.	\$439.5 bil.	-34.5%	\$440.8 bil.	-34.3%
Chinese dom. sales at final sales prices	\$2.3 tril.	\$2.4 tril.	1.8%	\$2.4 tril.	1.8%
US GDP	\$ 19.4 tril.	\$19.4 tril.	-0.0002%	\$19.4 tril.	-0.0002%
Chinese GDP	\$11.6 tril.	\$11.6 tril.	-0.0002%	\$11.6 tril.	-0.0002%

Note: Search frictions play an important role in the response of welfare to tariff changes. The table presents deviations from the baseline calibration in section 6. Column (1) presents the baseline calibration. Columns (2) and (3) present the two exercises in appendix D.4. Column (2) increases bilateral tariffs by 10 percent. Column (3) shows that, by affecting producers' matched rates, increases in the average cost for retailers to contact foreign producers attain the same welfare changes as in column (2). "pp" stands for percentage point, "CH" stands for China, and "US" stands for the United States.