

Tariffs and Goods-Market Frictions ^{*}

Pawel M. Krolikowski [†] Andrew H. McCallum [‡]

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Abstract

We study tariffs in a general equilibrium dynamic model with and without search frictions between heterogeneous exporting producers and importing retailers. The model's competitive equilibrium is inefficient because it has matching, participation, and aggregate productivity externalities. We find the global tariffs that internalize all externalities. We characterize optimal import tariffs, chosen by each country's social planner, with and without strategic considerations. We show that optimal tariffs in the model with search frictions may be negative because of market thickness effects. Finally, we characterize how unilateral optimal tariffs with search vary with matching efficiency and relative country size.

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[†]pawel.krolikowski@clev.frb.org, Federal Reserve Bank of Cleveland, 1455 E 6th St, Cleveland, OH 44114

[‡]Corresponding author, andrew.h.mccallum@frb.gov, +1(202)452-5249, Board of Governors of the Federal Reserve System, Washington, DC 20551

1 Introduction

Trade policy and the search for international trading partners are important for the welfare of countries and firms. While a number of studies have examined the impact of tariffs on welfare, including influential work by [Broda, Limao, and Weinstein \(2008\)](#), [Ossa \(2011\)](#), and [Costinot and Rodríguez-Clare \(2014\)](#), little is known about the interactions between tariff policy and the process of building connections with overseas buyers, a prevalent search friction faced by exporters ([Brancaccio, Kalouptsidi, and Papageorgiou, 2020](#); [Krolikowski and McCallum, 2021](#); [Eaton, Jinkins, Tybout, and Xu, 2022](#)).

We show that optimal tariffs with search frictions are lower than in models without search because of two new externalities. These “thick market and congestion” externalities arise because importing retailers and exporting producers do not internalize how searching affects equilibrium matching rates. Our model continues to have typical participation, aggregate productivity, and market power externalities as in [Albrecht, Navarro, and Vroman \(2010\)](#), [Mangin and Julien \(2021\)](#), and [Dhingra and Morrow \(2019\)](#). In a calibrated version of our model, optimal tariffs may be negative, which implies a subsidy for imports paid for by an income tax.

We study optimal tariffs in a Melitz-style general equilibrium dynamic model with goods-market frictions between importing retailers and exporting producers. In the steady-state of this model, an endogenous fraction of exporters are actively looking for importing partners but are unmatched. These unmatched exporters alter the *levels* of aggregate variables and the *changes* in aggregate variables in response to shocks (like tariff shocks) because when producers are unmatched their associated varieties cannot be traded.

We use a calibrated version of our model to solve for the tariffs that maximize the sum of global welfare (globally optimal tariffs) that internalize all externalities. We also characterize optimal unilateral import tariffs with and without strategic considerations. We show that optimal tariffs may be negative, which implies a subsidy for imports paid for by an income tax. Negative tariffs can be optimal because the social planner can increase welfare by making the search market thicker than in the competitive equilibrium, which raises consumption and trade. Finally, we characterize how unilateral optimal tariffs with search vary with matching efficiency and relative country size ([Krugman, 1980](#)).

We also compare our results to past studies of tariff policy and efficiency in settings without and with search frictions. Particularly relevant work is by [Demidova and Rodríguez-Clare \(2009\)](#), which extends the optimal-tariff results in [Gros \(1987\)](#) to a small country Melitz model. Both models are a special case of the model studied in [Felbermayr, Jung, and Larch \(2013\)](#), which characterizes optimal tariffs in cooperative and noncooperative games for two large countries with heterogeneous producers. These models account for small trade flows between countries with large iceberg trade costs. Optimal

tariffs in these models without search frictions are about 30 percent, and typically no smaller than about 15 percent. Our calibrated model without search frictions implies optimal tariffs that are similar to the optimal tariffs in these papers. However, in our calibrated model with search frictions, optimal tariffs are negative (implying an import subsidy) because search frictions can account for trade flows without resorting to large iceberg trade costs. Finally, [Brancaccio, Kalouptsi, Papageorgiou, and Rosaia \(2022\)](#) study efficiency in markets with search but focus on the international transportation sector.

Section 2 outlines the model, which builds on [Krolikowski and McCallum \(2021\)](#). Section 3 defines equilibria and optimal tariffs for the competitive, global social planner, unilateral country social planner, and Nash outcome for country planners. Section 4 presents our calibration. Section 5 provides numerical results for the equilibrium concepts defined in Section 3 and using the calibration in Section 4. Section 6 concludes.

2 The model, aggregation, and steady-state equilibrium

2.1 Model

We use an extension of the continuous-time model model of [Krolikowski and McCallum \(2021\)](#) and outline it here with additional details and equations included in Appendix A. The model features D countries. We index importing countries with d (destination) in the first index position and exporting countries with o (origin) in the second so that, for example, imports from o to d are denoted IM_{do} . We allow for search frictions between producers and retailers in domestic and international goods markets and we focus on steady-state implications.

2.1.1 Consumers

A representative consumer in destination market d has Cobb-Douglas utility, U_d , over a homogeneous non-traded good, $q_d(1)$, and a second good that is a constant elasticity of substitution (CES) aggregate of differentiated varieties, $q_{do}(\omega)$, from all origins. The two goods are combined with exponents $1 - \alpha$ and α , respectively. The differentiated goods are substitutable with constant elasticity, $\sigma > 1$, across varieties and destinations and we denote the value of total consumption as C_d in destination country d . At prices paid by final consumers, we denote the value of consumption of the differentiated good to destination d from origin o as C_{do} . The homogeneous good has price $p_d(1)$. Define P_d as the price index for the bundle of differentiated varieties and P_{do} as the price index for the bundle of varieties produced in country o and consumed in country d , which have price $p_{do}(\omega)$. The ideal price index, defined as Ξ_d , combines $p_d(1)$ and P_d . Details for the consumers' problem are in Appendix A.1 and we discuss the price index more in Section 2.4.

2.2 Producers and retailers

2.2.1 The matching function

A costly process of search governs how producers and retailers find one another, similar to that in [Diamond \(1982a\)](#), [Pissarides \(1985\)](#), and [Mortensen \(1986\)](#). We assume that the flow number of relationships formed at any moment in time between searching retailers and producers is determined by a Cobb-Douglas matching function, with matching efficiency ξ and elasticity with respect to the number of searching producers η . As such, market tightness—the ratio of the mass of searching retailers to the mass of searching producers, which we denote, $\kappa_{do} = v_{do}N_d^m/u_{do}N_o^x$ —is sufficient to determine contact rates on both sides of each do search market. The Poisson rate at which retailers in country d contact producers in country o is given on the left, and the contact rate for producers is given on the right:

$$\chi(\kappa_{do}) = \xi\kappa_{do}^{-\eta}, \quad \kappa_{do}\chi(\kappa_{do}) = \xi\kappa_{do}^{1-\eta}. \quad (1)$$

Only the number of vacancies matters in our model, not the number of retailers. Vacancies can originate from one retailing firm posting all vacancies, all retailers posting one vacancy each, or anything in between. As such, we interpret matches as one retailer to one producer, as in [Pissarides \(2000\)](#), and we refer to vacancies and retailers interchangeably. Details about the matching function are in [Appendix A.2.1](#) with details about continuous time Poisson processes in [Appendix A.2](#) of [Krolikowski and McCallum \(2021\)](#).

2.2.2 Producers

We index producers of goods by their permanent productivity, φ . We assume this productivity is exogenous and has the same distribution in all countries: Pareto with cumulative density function $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$.

There are two production costs for differentiated goods. First, producers face a variable cost indexed by productivity

$$v(q_{do}, w_o, \tau_{do}, \varphi) = q_{do}w_o\tau_{do}\varphi^{-1}. \quad (2)$$

This variable cost function implies a constant-returns-to-scale production function in which labor is the only input. The equilibrium wage in the exporting (origin) country, w_o , is determined by a market clearing condition discussed in [section 2.3](#); $\tau_{do} \geq 1$ is an iceberg cost such that one unit of the differentiated good arrives in destination d when τ_{do} units are sent from origin o and $\tau_{do} - 1$ units are lost to physical destruction; and q_{do} is the amount traded. Second, producers face a fixed cost of production, f_{do} , so that the total production cost is $v(q_{do}, w_o, \tau_{do}, \varphi) + f_{do}$. We could include non-tradeable differentiated goods in our

framework by increasing the number of sectors and setting the iceberg trade costs in some of these sectors to infinity.

At any instant in time, each producer is in one of three mutually exclusive states. The value for each state is described by Bellman Equations (3), (4), and (5). First, the producer could be matched with a retailer with value $X_{do}(\varphi)$ defined by,

$$rX_{do}(\varphi) = n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + \lambda(U_{do}(\varphi) - X_{do}(\varphi)). \quad (3)$$

In this state, the flow payoff is the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable, $v(q_{do}, w_o, \tau_{do}, \varphi)$, and fixed cost of production, f_{do} . The negotiated price, n_{do} , and the quantity traded, q_{do} , are determined through a bargaining process that we describe in Section 2.3. Matches end exogenously at rate λ , which leads to a capital loss as the producer becomes unmatched and the future is discounted at rate r .

Second, the producer could be unmatched but searching with value $U_{do}(\varphi)$ defined by,

$$rU_{do}(\varphi) = -l_{do} + \kappa_{do}\chi(\kappa_{do})(X_{do}(\varphi) - U_{do}(\varphi) - s_{do}). \quad (4)$$

The producer pays a flow cost, l_{do} , to generate contacts with retailers. At endogenous Poisson rate $\kappa_{do}\chi(\kappa_{do})$ the producer contacts a retailer and becomes matched, after paying the sunk cost, s_{do} , of starting up the relationship.

Third, producers have the option of remaining idle and not expending resources to look for a retailer with value $I_{do}(\varphi)$ defined by,

$$rI_{do}(\varphi) = h_{do}. \quad (5)$$

Idle producers receive a constant flow payoff, h_{do} . We include an idle state because without it, all producers would search in all markets, even if they expect to reject all contacts. Allowing producers to optimally choose not to search in each market is both more general and more intuitive. Appendix A.2.2 has more details about the producers' value functions.

2.2.3 Retailers

Each retailer is in one of two states, described by Equations (6) and (7). First, the retailer could be matched with a producer and receive value $M_{do}(\varphi)$ defined by,

$$rM_{do}(\varphi) = p_{do}q_{do} - t_{do}n_{do}q_{do} + \lambda(V_{do} - M_{do}(\varphi)). \quad (6)$$

In this state, the flow payoff is the revenue, $p_{do}q_{do}$, generated by selling q_{do} units of the differentiated good at a final sales price, p_{do} , paid by the consumer less the tariff-inclusive

cost of acquiring these goods, $t_{do}n_{do}q_{do}$. The retailer pays the *ad valorem* tariff, t_{do} , on the imported value, $n_{do}q_{do}$, to the government. The tariff creates a potential wedge between producer revenue, $n_{do}q_{do}$, in Equation (3) and retailer cost, $t_{do}n_{do}q_{do}$ in Equation (6). Tariff revenues are rebated lump-sum from the government to consumers in the destination country as discussed in Section 2.4. When the relationship is destroyed exogenously, at rate λ , the retailing firm loses the capital value of being matched. All retailers are identical before matching but have differential matched values because producers are heterogeneous in their productivity.

Second, a retailer could be unmatched with value V_{do} defined by,

$$rV_{do} = -c_{do} + \chi(\kappa_{do}) \int [\max\{V_{do}, M_{do}(\varphi)\} - V_{do}] dG(\varphi). \quad (7)$$

The flow search cost, c_{do} , generates the search friction between producers and retailers. At endogenous Poisson rate $\chi(\kappa_{do})$, retailing firms meet a producer and, before consummating a match, learn the productivity of the producer. Retailers then choose between matching with that producer or continuing to search. Because they are uncertain about the productivity of the producer they might meet, retailers take the expectation over all productivities they might encounter when computing their continuation value of searching. There is an unbounded mass of potential retailers that could decide to search. We discuss different entry conditions for retailers in Section 3 and Appendix A.2.3 has more details about the retailers' value functions.

2.3 Solving the partial-equilibrium search problem

Retailing and producing firms use backward induction to maximize their value. The second-stage solution results from jointly Nash bargaining over negotiated price, n_{do} , and quantity, q_{do} , after a retailer and producer meet. In the first stage, retailers and producers—taking the solution to the second-stage bargaining problem as given—choose whether to search for a business partner, or to remain idle. Appendix A.3 solves the search problem in detail.

2.3.1 Match surplus

Define the total private surplus as the value of the relationship to the retailer and the producer less their outside options,

$$S_{do}(\varphi) = X_{do}(\varphi) - U_{do}(\varphi) + M_{do}(\varphi) - V_{do}. \quad (8)$$

Importantly, $S_{do}(\varphi)$ excludes the government's value of collecting tariffs from each match and the government is passive during bargaining. Bargaining over quantity, q_{do} , will maximize total private surplus and bargaining over price, n_{do} , will divide the surplus

between the producer and retailer. Appendix A.3.1 derives the surplus in terms of appropriately discounted profits. This appendix also derives the value of a relationship and discusses the expected duration of matches.

2.3.2 Bargaining over the negotiated price

Bargaining over the negotiated price, n_{do} , will divide the private surplus, $S_{do}(\varphi)$, between producers and retailers according to the ‘‘surplus sharing rule’’, which is:

$$X_{do}(\varphi) - U_{do}(\varphi) = \frac{\beta S_{do}(\varphi)}{\beta + t_{do}(1 - \beta)}, \quad M_{do}(\varphi) - V_{do} = \frac{(1 - \beta) t_{do} S_{do}(\varphi)}{\beta + t_{do}(1 - \beta)}. \quad (9)$$

in which β is producers’ bargaining power. Equation (9) nests the sharing rule in Krolikowski and McCallum (2021, Equation 13) when $t_{do} = 1$. In addition, as the tariff rises, retailers receive a larger fraction of the surplus to account for their increased import costs: As $t_{do} \rightarrow \infty$, the fraction of the surplus received by retailers approaches 1.

The negotiated price that splits the surplus according to Equation (9) when we assume free entry into retailer vacancies, $V_{do} = 0$, is

$$n_{do} = (1 - \gamma_{do}) \left(\frac{p_{do}}{t_{do}} \right) + \gamma_{do} \left(\frac{v(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do} \chi(\kappa_{do}) s_{do}}{q_{do}} \right), \quad (10)$$

in which $\gamma_{do} \equiv \frac{(r + \lambda)(1 - \beta)}{r + \lambda + \beta \kappa_{do} \chi(\kappa_{do})} \in [0, 1]$. The equilibrium negotiated price, n_{do} , is a convex combination of the tariff-adjusted final sales price, p_{do}/t_{do} , and the average total production cost less producers’ search costs. Appendix A.3.2 discusses bargaining over price in detail.

2.3.3 Bargaining over quantity

Bargaining over quantity implies that the quantity exchanged within matches equates marginal revenue obtained by retailers from consumers with marginal production cost inclusive of tariffs. Our functional form assumptions result in an equivalent definition for negotiated quantity in terms of the final consumer being a markup over marginal production and tariff costs,

$$p_{do}(\varphi) = t_{do} \mu w_o \tau_{do} \varphi^{-1}, \quad (11)$$

in which $\mu = \sigma / (\sigma - 1)$. Negotiated quantity is obtained by substituting Equation (11) into the demand curve, Equation (A2). Appendix A.3.3 discusses bargaining over quantity in detail.

2.3.4 Producers’ search productivity threshold

In the first stage, producers, taking the solution to this second-stage bargaining problem from Equations (10) and (11) as given, choose whether to search for a business partner or to

remain idle. As such, a zero-value condition, $U_{do}(\bar{\varphi}_{do}) - I_{do}(\bar{\varphi}_{do}) = 0$, which can be written as,

$$\left(\frac{p_{do}(\bar{\varphi}_{do})}{t_{do}} \right) q_{do}(\bar{\varphi}_{do}) - v(q_{do}(\bar{\varphi}_{do}), w_o, \tau_{do}, \bar{\varphi}_{do}) = F(\kappa_{do}), \quad (12)$$

determines producers' minimum productivity threshold, $\bar{\varphi}_{do}$, that makes searching worthwhile. Equation (12) equates tariff-adjusted variable profits from the match with the “effective entry cost.” The latter is defined as

$$F(\kappa_{do}) \equiv f_{do} + \left(\frac{r + \lambda}{\beta \kappa_{do} \chi(\kappa_{do})} \right) l_{do} + \left(1 + \frac{r + \lambda}{\beta \kappa_{do} \chi(\kappa_{do})} \right) h_{do} + \left(\frac{r + \lambda}{\beta} \right) s_{do}, \quad (13)$$

which is the sum of the fixed cost of production, f_{do} , and the (appropriately discounted) flow cost of searching for a retailer, l_{do} , the opportunity cost of remaining idle, h_{do} , and the sunk cost of starting up a relationship, s_{do} .

Solve Equation (12) using our functional forms to get the threshold explicitly as,

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d} \right) \left(\frac{F(\kappa_{do})}{C_d} \right)^{\frac{1}{\sigma-1}} t_{do}^\mu. \quad (14)$$

Detailed discussion of the threshold productivity is in Appendix A.3.4. Appendix A.6.4 of [Krolikowski and McCallum \(2021\)](#) compares the threshold productivity to previous models.

The fraction of idle producers, i_{do} , that choose not to search is defined by the steady-state productivity threshold, $\bar{\varphi}_{do}$, and the exogenous distribution of productivity, $i_{do} = \int_1^{\bar{\varphi}_{do}} dG(\varphi) = G(\bar{\varphi}_{do})$. Appendix A.6.5 of [Krolikowski and McCallum \(2021\)](#) details the importance of including an idle state and its relationship to the threshold productivity.

First-stage entry conditions for retailers that are analogous to Equation (12) vary depending on assumptions about the search market structure. We present those alternatives in Section 3.

2.4 Aggregation

Because of search frictions, in steady state there exists a set of unmatched producers (mass of unmatched product varieties) that are actively looking for a retail partner. This fraction of unmatched producers is given by

$$\frac{u_{do}}{1 - i_{do}} = \frac{\lambda}{\lambda + \kappa_{do} \chi(\kappa_{do})}, \quad (15)$$

in which u_{do} is the fraction of producers that are unmatched and searching and $u_{do}/(1 - i_{do})$ is the fraction of active producers that are unmatched adjusted by the fraction of producers that will ever search, $1 - i_{do}$. Equation (15) is analogous to the labor unemployment rate, which is characterized as the fraction of the labor force that is actively searching for a job.

The unmatched producers result in associated unmatched varieties that cannot be consumed and are therefore absent from imports, the indirect utility (welfare) function, and all other aggregates.

We can move from indexing over an unordered set of varieties that enter utility to indexing using a distribution of productivities using the steps in Appendix A.11.1 of [Krolikowski and McCallum \(2021\)](#). Those steps show that if an unordered set of varieties, Ω_o , has measure $N_o^x = |\Omega_o|$, then the set of varieties above the threshold has measure $(1 - G(\bar{\varphi}_{do})) N_o^x = (1 - i_{do}) N_o^x$ and the set of matched varieties that are above the threshold has measure $(1 - u_{do}/(1 - i_{do})) N_o^x$. The correct measure of goods consumed will feature prominently in any aggregate quantity in the model.

The aggregate resource constraint using the expenditure approach and the measure of matched varieties can then be written as,

$$\begin{aligned}
Y_d = & \underbrace{p_d(1) q_d(1) + \sum_{k=1}^D \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi) q_{dk}(\varphi) dG(\varphi)}_{\text{Consumption } (C_d)} \\
& + \underbrace{N_d^x e_d^x + \sum_{k=1}^D \kappa_{dk} u_{dk} N_k^x c_{dk} + u_{kd} N_d^x (l_{kd} + s_{kd} \kappa_{kd} \chi(\kappa_{kd})) + (1 - u_{kd} - i_{kd}) N_d^x f_{kd}}_{\text{Investment } (I_d)} \\
& + \underbrace{\sum_{k=1}^D \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} (1 - t_{dk}) n_{dk}(\varphi) q_{dk}(\varphi) dG(\varphi)}_{\text{Government } (G_d)}. \tag{16}
\end{aligned}$$

Consumption expenditure, C_d , is the total resources devoted to consumption of both the homogeneous good and the differentiated varieties, evaluated at final consumer prices. Investment expenditure, I_d , is the resources devoted to creating producers, to creating retailer-producer relationships, and to paying for the per-period fixed costs of production. We define investment costs as those that must be paid before producing the first unit of output and that do not scale with output. Government expenditure, G_d , is total tariffs that are levied on retailers at the negotiated price. The government budget is balanced by rebating tariff revenue to (taxing subsidy cost from) consumers. For example, if $t_{do} > 1 \forall do$, then $G_d < 0$ and moving G_d to the resources (left) side of Equation (16), increase resources available for consumption or investment. In contrast, if $t_{do} < 1 \forall do$, then $G_d > 0$ and the import subsidy reduces resources that are available. ([Appendix A.4.1](#) contains details.) Government payments to idle producers are financed by a lump-sum tax on consumption so that they cancel out on the expenditure (right) side of the aggregate resource constraint. Finally, we impose balanced trade, so that net exports do not appear in Equation (16).

The equilibrium wage, w_d , is defined by labor market clearing. We assume that labor is not mobile across countries so that labor supply in an economy is equal to the country's labor endowment, L_d . Labor demand is equal to the labor used to produce investment and the non-traded and differentiated goods. Labor market clearing sets labor supply equal to labor demand resulting in the equilibrium wage

$$w_d = \frac{I_d + (1 - \alpha) C_d + \frac{1}{\mu} \sum_o \frac{C_{od}}{t_{od}}}{L_d}, \quad (17)$$

with details in Appendix A.4.2.

Firms make profits in our framework because of restricted entry, as discussed in Appendix A.4.3. Total resources paid to labor are defined by $Y_d = w_d L_d + \Pi_d$, in which L_d is the exogenous labor endowment, w_d is the equilibrium wage, and Π_d are profits. We discuss three ownership structures of firms in Appendix A.4.4: Consumers in country d own retailers and producers in country d (profits attributed by location); they own retailers in country d and all producers in country o that serve them (profits from vertically firms); or they own shares of a global mutual that collects all retailer and producer profits and then redistributes them in π proportion to the value of the labor endowment in each country, $w_d L_d$. In our analysis we use the global mutual fund approach, as in Chaney (2008), to facilitate comparison to that work. As such, $\Pi_d = w_d L_d \pi$ in which

$$\pi = \frac{\Pi}{\sum_{k=1}^O w_k L_k}, \quad \Pi = \sum_d \Pi_d = \alpha C - \frac{1}{\mu} \sum_d \sum_o \frac{C_{od}}{t_{od}} + G, \quad (18)$$

$C = \sum_d C_d$ is global consumption, and $G = \sum_d G_d$ is global government expenditure.

Using the optimal final sales price from Equation (11) and the other assumptions in Sections 2.1.1 through 2.3, we derive the price index for differentiated goods in country d :

$$P_d = \lambda_2 C_d^{\frac{1}{\theta} - \frac{1}{\sigma-1}} \rho_d, \quad \rho_d \equiv \left(\sum_{k=1}^D \left(1 - \frac{u_{dk}}{1 - i_{dk}} \right) \frac{C_k}{C} (w_k \tau_{dk})^{-\theta} F_{dk}^{-[\frac{\theta}{\sigma-1} - 1]} t_{dk}^{1-\mu\theta} \right)^{-\frac{1}{\theta}}, \quad (19)$$

in which $\lambda_2 \equiv (\theta / (\theta - (\sigma - 1)))^{-\frac{1}{\theta}} (\sigma / \alpha)^{\frac{1}{\sigma-1} - \frac{1}{\theta}} \mu (C / (1 + \pi))^{-\frac{1}{\theta}}$. More details appear in Appendix A.5 and to conserve on notation, we sometimes refer to $F(\kappa_{do})$ as F_{do} . The ideal price index that minimizes expenditure to obtain utility level $U_d = 1$ combines the differentiated and homogeneous goods prices as, $\Xi_d = [p_d(1) / (1 - \alpha)]^{1-\alpha} [P_d / \alpha]^\alpha$.

The gravity equation gives total imports by destination d from origin o in the differentiated goods sector, which is the total value of all imported varieties evaluated at negotiated prices, $n_{do} q_{do}$. As we show in Appendix A.6.1, imports are:

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} t_{do}^{-\mu\theta}, \quad (20)$$

in which the fraction of matched exporters, $1 - u_{do}/(1 - i_{do})$, and the import markup, $1 - b(\cdot)$ reduce imports relative to a model without search (Krolikowski and McCallum, 2021).

The total amount paid by consumers in d for imports from o , C_{do} , must equal the value in the do market of all imported varieties, tariffs, and retailer profits. As such, $C_{do} = IM_{do} + \Pi_{do}^r - G_{do}$, in which Π_{do}^r are retailer profits in the do market and G_{do} is government expenditure in the do market, as defined in Appendix A.6.1. We also show that $C_{do} = t_{do} IM_{do}/(1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}))$ in Appendix A.6.2.

2.5 Steady-state general equilibrium

A steady-state general equilibrium consists of market tightnesses, κ_{do} , and threshold productivities, $\bar{\varphi}_{do}, \forall do$, aggregate consumptions, C_d , and wages, $w_d, \forall d$, and the per-capita dividend, π . In all equilibria discussed in section 3, $\bar{\varphi}_{do}, C_d, w_d$, and π , will jointly satisfy the zero-profit conditions (equation 14), the aggregate resource constraints (equation 16), labor market clearing (equation 17), and redistribution of profits via dividend (equation 18). When not directly chosen by a social planner, market tightnesses, κ_{do} , will be determined by the retailers' free-entry conditions, as described in section 3.1. We elaborate on the definition of the steady-state general equilibrium in Appendix A.7.1.

The exogenous parameters are $\beta, \lambda, r, \eta, \xi, \theta, \sigma, \alpha, e_d^x, L_d, t_{do}, c_{do}, f_{do}, h_{do}, l_{do}$, and s_{do} , in which d and o vary by countries. Tariffs, t_{do} , are exogenous parameters to economic agents, except when they are chosen by a social planner or when they are the outcomes of a Nash equilibrium, as discussed in Section 3.

The main difference between our model's equilibrium definition and the definitions in trade models without search is that we introduce market tightnesses, k_{do} . Our model nests trade models without search frictions if market tightnesses are infinite. Specifically, our model exactly reproduces Chaney (2008) if retailers' search costs are zero, $w_d = 1$, and $G_d = 0, \forall d$, and we make the same parameter value restrictions that he does ($s_{do} = h_{do} = e_d^x = 0, \forall d$, and, $\forall o$). We provide more details for this result in Appendix A.7.2.

3 Different search market structures

In this section we study the decentralized equilibrium and compare it to the solution of a global social planner, and a country social planner with and without strategic considerations. In the decentralized equilibrium, tariffs are taken as given, but social planners choose tariffs to maximize welfare. Welfare (indirect utility) in country d is defined by real consumption expenditure, $W_d = C_d/\Xi_d$, in which C_d is consumption expenditure in country d (defined in

equation 16) and Ξ_d is the ideal price index (defined in section 2.3) because preferences are homothetic in our model (see appendix A.8.1).

3.1 Decentralized search market

This section considers a decentralized search market in which we assume free entry into the market of unmatched retailers, as in Pissarides (1985) and Shimer (2005). This assumption, along with the other equations defining the equilibrium, implies that market tightness in each do search market will be determined by

$$\frac{c_{do}}{\chi(\kappa_{do})} = \left(\frac{1}{r + \lambda} \right) \frac{\Pi_{do}^r}{(1 - u_{do}/(1 - i_{do})) N_o^x}, \quad (21)$$

in which total period profits accruing to importers in matched relationships are given by the difference between final consumption expenditure in d on differentiated products from o and the after-tariff value of those products at negotiated prices (see Appendix A.6.1 for details). Equation (21) defines the equilibrium market tightness, κ_{do} , that equates the expected cost of being an unmatched retailer, on the left, with the expected benefit from matching, on the right. The expected cost of being an unmatched retailer is the expected duration of being unmatched, $1/\chi(\kappa_{do})$, times the flow cost of searching for a producing affiliate, c_{do} . The expected benefit from matching is the discounted average profits accruing to matched retailers each period. Average profits are total profits divided by the mass of products from origin market o that are matched in market d , which is the fraction of products matched times the number of exporting producers in the origin market, N_o^x .

To get intuition from equation (21), notice that as the expected benefit from retailing rises, free entry implies that retailers enter the search market. This entry raises market tightness, $\kappa_{do} = v_{do} N_d^m / u_{do} N_o^x$, and, through congestion effects, reduces the rate at which searching retailers contact searching producers, $\chi(\kappa_{do})$. This increases retailers expected cost of search (the left-hand side) so that, with free entry into retailing, κ_{do} always satisfies equation (21) in equilibrium.

The decentralized equilibrium solves a system of nonlinear equations in the equilibrium variables in which the equilibrium conditions are constraints, our calibration will determine the model parameters (section 4), and the objective function is any constant, including zero.

This problem solves:

$$\left(\boldsymbol{\kappa}^c, \bar{\boldsymbol{\varphi}}^c, \vec{C}^c, \vec{w}^c, \pi^c \right) = \arg \max_{\boldsymbol{\kappa}, \bar{\boldsymbol{\varphi}}, \vec{C}, \vec{w}, \pi} 0 \quad (22a)$$

subject to:

$$\frac{c_{do}}{\chi(\kappa_{do})} = \left(\frac{1}{r + \lambda} \right) \frac{\Pi_{do}^r(\vec{\kappa}_{d*}, \vec{\varphi}_{d*}, \vec{C}, \vec{w}, \vec{t}_{d*}) (1 + \pi)}{(1 - u_{do}(\kappa_{do}) / (1 - i_{do}(\bar{\varphi}_{do}))) C_o} \quad \forall do, \quad (22b)$$

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d(\vec{\kappa}_{d*}, \vec{\varphi}_{d*}, \vec{C}, \pi, \vec{w}, \vec{t}_{d*})} \right) \left(\frac{F(\kappa_{do})}{C_d} \right)^{\frac{1}{\sigma-1}} t_{do}^\mu \quad \forall do, \quad (22c)$$

$$w_d L_d (1 + \pi) = C_d + I_d(\vec{\kappa}_{d*}, \vec{\kappa}_{*d}, \vec{\varphi}_{*d}, \vec{C}, \pi) + G_d(\vec{\kappa}_{d*}, \vec{\varphi}_{*d}, \vec{C}, \pi, \vec{t}_{d*}) \quad \forall d, \quad (22d)$$

$$w_d = \frac{I_d(\vec{\kappa}_{d*}, \vec{\kappa}_{*d}, \vec{\varphi}_{*d}, \vec{C}, \pi) + (1 - \alpha) C_d + \frac{1}{\mu} \left(\vec{C}_{*d}(\vec{\kappa}_{*d}, \vec{\varphi}_{*d}, \vec{C}, w_d, \vec{\tau}_{*d}) / \vec{t}_{*d} \right)' \vec{t}}{L_d} \quad \forall d, \quad (22e)$$

$$\pi = \frac{\vec{t}' \boldsymbol{\Pi}(\boldsymbol{\kappa}, \bar{\boldsymbol{\varphi}}, \vec{C}, \vec{w}, \boldsymbol{t})' \vec{t}}{\vec{w}' \vec{L}} \quad (22f)$$

$$\vec{t}_{d*} = \vec{t}_{d*}^c \quad \forall d, \quad (22g)$$

which uses our assumption that the number of producers exporting from the origin market o is proportional to total consumption in o , as discussed in section 2.3. Equation (22) is expressed as a function of only the endogenous variables and parameters. Equations (22d) and (22e) implicitly define a trade balance condition because we assume that net exports are zero. In section 2.4 we discuss three ownership structures of firms, and in Appendix A.4.4 we show that one of these structures implies an explicit trade balance condition, as in Demidova and Rodríguez-Clare (2009), Felbermayr et al. (2013), and Costinot, Rodríguez-Clare, and Werning (2020).

We denote these solutions to the decentralized competitive equilibrium defined by Eq. (22) with ‘‘c’’ superscripts. We also define vectors as collections of the variables across subindexes and matrices are denoted as bold. For example, search market tightnesses are collected into the following

$$\vec{\kappa}_{*o} = \begin{pmatrix} \kappa_{1o} \\ \kappa_{2o} \\ \vdots \\ \kappa_{Do} \end{pmatrix}, \quad \vec{\kappa}_{d*} = \begin{pmatrix} \kappa_{d1} & \kappa_{d2} & \dots & \kappa_{dD} \end{pmatrix}, \quad \boldsymbol{\kappa} = \begin{bmatrix} \kappa_{11} & \dots & \kappa_{1D} \\ \vdots & \ddots & \vdots \\ \kappa_{D1} & \dots & \kappa_{DD} \end{bmatrix}, \quad (23)$$

so that rows index destinations and columns index origins. $\vec{\kappa}_{d*}$ is the d th row of $\boldsymbol{\kappa}$ and $\vec{\kappa}_{*o}$ is the o th column of $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}$ is a square matrix. The column vector of D aggregate

consumption expenditures in each d economy is collected in \vec{C} , \vec{v} is a $D \times 1$ column vector of ones, and \vec{L} is a column vector of D labor endowments. Division of matrices is element by element. Variable trade costs, τ^c , are exogenous in the decentralized competitive economy. Higher variable export costs to country d , $\vec{\tau}_{d*}^c$, directly lower importers' period profits, Π_{do}^m , and raise the price index, P_d (equation 19), but only affect investment, I_d , through other equilibrium variables.

3.2 Efficient equilibrium with search

The decentralized equilibrium in our model is not efficient in general, so that global welfare in the decentralized equilibrium does not necessarily attain the global welfare in the social planner's solution. The inefficiency arises because retailers and producers do not internalize how searching affects equilibrium matching probabilities. Specifically, when market tightness rises, producers find retailers more easily ("thick market" externality), but retailers find producers more slowly ("congestion" externality). These matching externalities are common to search models (Diamond, 1982b; Pissarides, 2000). Our model also has participation and output externalities because the threshold producer does not internalize their effect on average match productivity, as in Albrecht et al. (2010) and Julien and Mangin (2017).

3.2.1 Global social planner picks market tightness

The global social planner internalizes all externalities by choosing the market tightness in each search market to maximize global welfare. Global welfare is the sum of welfare (sum of real consumption) in all countries: $\sum_d W_d = \sum_d C_d / \Xi_d$. As such, the global social planner solves the following problem:

$$\left(\kappa^e, \vec{\varphi}^e, \vec{C}^e, \vec{w}^e, \pi^e \right) = \arg \max_{\kappa} \sum_d \frac{C_d}{\Xi_d \left(\vec{\kappa}_{d*}, \vec{\varphi}_{d*}, \vec{C}, \pi, \vec{w}, \vec{\tau}_{d*} \right)} \quad (24a)$$

subject to: Eqs. (22c) through (22g).

This solution is constrained efficient in that the global social planner chooses tightness in each market and is subject to the optimal entry decision of producers, the aggregate resource constraint, and the same goods-market frictions, labor market clearing, and profit redistribution as in the decentralized economy. We assume the global social planner faces the same level of tariffs as agents in the decentralized equilibrium, τ^c . We denote these constrained efficient solutions to Eq. (24) with "e" superscripts. These efficient market tightnesses, κ^e , contrast with market tightnesses, κ^c , determined by the retailer free entry condition in the competitive equilibrium from Eq. (21).

The global social planner trades off increasing market tightness to speed up the creation

of matches against the cost of greater resources being expended on forming those matches. The resources expended to create matches enter investment, $I_d(\vec{\kappa}_{d*}, \vec{\kappa}_{*d}, \vec{\varphi}_{*d}, \vec{C}, \pi)$, in the aggregate accounting identity (Eq. 16). Pushing market tightness to infinity would match all producers in steady state, minimizing the price index, but would also exhaust all resources in the economy and result in zero real consumption. In other words, while the social planner is not directly constrained by Eq. (21) as in the decentralized equilibrium, the expected cost of forming a match, $c_{do}/\chi(\kappa_{do})$, will be finite through the aggregate resource constraint and its implications for real consumption.

3.2.2 Comparing decentralized and efficient equilibria

In this section we derive conditions that ensure that the decentralized and social planner equilibria coincide, similar to the exercise in Hosios (1990). In addition to the standard matching externality, our decentralized equilibrium features a participation externality and an output externality because of an endogenous participation margin and heterogeneous producer productivity. These features imply that marginal producers choosing between remaining idle and searching for a partner do not internalize their effect on average match productivity, as in Albrecht et al. (2010) and Julien and Mangin (2017). As such, the standard Hosios (1990) condition, which sets producers' bargaining power, β , equal to the matching elasticity, η , does not ensure efficiency in our setting. In fact, adjusting one bargaining parameter cannot simultaneously internalize the externalities in all search markets. If bargaining parameters vary by do market there exists a generalized Hosios condition that internalizes the matching, participation, and output externalities, as in Mangin and Julien (2021) and Brancaccio et al. (2022). Finally, because the trade part of our model is similar to the model in Dhingra and Morrow (2019), their results suggest that, together with our generalized Hosios condition, our decentralized equilibrium attains the social planner's solution. The global social planner's problem in Eq. (24) is similar to the one in Hosios (1990) because it maximizes steady-state welfare, but it differs in that our consumers are not risk neutral and we study segmented search markets that are connected by general-equilibrium variables, such as the price index.

Proposition 1. *Assume $l_{do} = -h_{do}$ and $s_{do} = 0$ so that $F(\kappa_{do}) = f_{do} + h_{do}$ is not a function of β_{do} . Then there exist bargaining parameters β_{do}^c for all do markets such that the decentralized equilibrium coincides with the social planner's solution*

$$\left(\kappa^c, \vec{\varphi}^c, \vec{C}^c, \vec{w}^c, \pi^c\right) = \left(\kappa^e, \vec{\varphi}^e, \vec{C}^e, \vec{w}^e, \pi^e\right).$$

Proof. See Appendix A.8.2. □

The simplifications $l_{do} = -h_{do}$ and $s_{do} = 0$ make our framework comparable to previous work. In Pissarides (2000, Chapter 8), for example, the socially optimal solution is not

affected by β_{do} . In contrast, β_{do} would affect the optimal solution in our model without our simplification via the effective entry cost, Eq. (13). For example, when producers receive none of the match surplus, $\beta_{do} = 0$, they are unable to recover any sunk costs, $s_{do} > 0$, of forming matches so that they remain idle and aggregate exports are zero. Julien and Mangin (2017, section 2) also make a restriction on the relationship between the idle payoff and the bargaining parameter to characterize efficiency in a model with labor force participation.

3.3 Global social planner picks all tariffs

This section considers a global social planner that chooses tariffs, domestic and foreign, to maximize welfare subject to the equilibrium constraints in Eq. (22). Specifically, instead of the global planner choosing market tightnesses directly, we allow them to manipulate tariffs, $\boldsymbol{\tau}$, in order to influence market tightnesses and other equilibrium variables subject to the decentralized retailer entry condition defined in Eq. (21). Formally, this problem is given by

$$\left(\boldsymbol{\kappa}^\tau, \vec{\varphi}^\tau, \vec{C}^\tau, \vec{w}^\tau, \pi^\tau, \boldsymbol{\tau}^\tau \right) = \arg \max_{\boldsymbol{\tau}} \sum_d \frac{C_d}{\Xi_d \left(\vec{\kappa}_{d*}, \vec{\varphi}_{d*}, \vec{C}, \pi, \vec{w}, \vec{\tau}_{d*} \right)} \quad (25a)$$

subject to: Eqs. (22b) through (22f).

We denote the solutions to the global social planner's choice of $\boldsymbol{\tau}$ defined by Eq. (25) with “ τ ” superscripts. These market tightnesses, $\boldsymbol{\kappa}^\tau$, differ from market tightnesses, $\boldsymbol{\kappa}^c$, which are determined by the retailer free entry condition in the competitive equilibrium from Eq. (21) when tariffs are exogenous at level $\boldsymbol{\tau}^c$.

When the global social planner chooses tariffs, the market tightnesses, and all equilibrium variables, are the same as when they choose market tightnesses directly. The benefits from entry into retailing in each do search market on the right hand side of Eq. (21) are decreasing in trade costs in that market, τ_{do} . These benefits vanish to zero when $\tau_{do} \rightarrow \infty$ so that retailer entry and market tightness fall to zero. Conversely, if the global social planner subsidizes the marginal cost of production in the do market so that production is free, $\tau_{do} \rightarrow 0$, then the benefits from retailer entry in that market can become as large as the value of the whole economy. With such large benefits, retailer entry rises and market tightnesses tend to infinity. As such, the global social planner can change variable trade costs in each market to adjust period profits so that in each market the decentralized tightnesses, $\boldsymbol{\kappa}^\tau$, coincide with the socially optimal tightnesses, $\boldsymbol{\kappa}^e$.

3.4 Country social planner picks own tariffs holding foreign tariffs fixed

This section considers a country social planner that unilaterally chooses import tariffs to maximize its own country's welfare without considering the welfare of other countries. While this country can set import tariffs, τ_{d*} , we assume it cannot choose domestic taxes and the

import tariffs of other countries. Rather, these are set to their efficient levels defined in section 3.3 so that $\tau_{dd} = \tau_{dd}^\tau$ and $\vec{\tau}_{o*} = \vec{\tau}_{o*}^\tau$, $\forall o \neq d$. This country's social planner remains constrained by the decentralized retailer entry condition and the other equilibrium constraints in all countries defined in Eq. (22). Formally, this problem is given by

$$\left(\kappa^u, \bar{\varphi}^u, \vec{C}^u, \vec{w}^u, \pi^u, \vec{\tau}_{d*}^u \right) = \arg \max_{\vec{\tau}_{d*}} \left(\frac{C_d}{\Xi_d \left(\vec{\kappa}_{d*}, \vec{\varphi}_{d*}, \vec{C}, \pi, \vec{w}, \vec{\tau}_{d*} \right)} \right) \quad (26a)$$

subject to: Eqs. (22b) through (22f),

$$\vec{\tau}_{o*} = \vec{\tau}_{o*}^\tau \quad \forall o \neq d, \quad (26b)$$

$$\tau_{dd} = \tau_{dd}^\tau \quad \forall d. \quad (26c)$$

We denote the solutions to the unilateral problem defined by Eq. (26) with ‘‘u’’ superscripts.

This exercise allows us to compare welfare in the globally-efficient solution with welfare when one country optimally defects. This exercise does not allow for strategic considerations, something we analyze in section 3.5. We show in [Krolikowski and McCallum \(2021\)](#) that imports are more elastic to tariff changes in a model with search frictions than without them. This implies that unilateral deviations from the globally-efficient tariffs will be smaller in a model with search than in a model without search and can even be negative. In other words, export supply elasticities in a model with endogenous search are always more elastic than in a model without search. The optimal tariff literature since [Broda et al. \(2008\)](#) shows that the optimal tariff choice depends on the export supply elasticity and lower export supply elasticities result in lower tariffs. Additional externalities in our model could result in unilateral tariffs being set below the efficient level. For example, if foreign producer search costs, l_{do} are large, these will be internalized by the global social planner but not the country social planner. These results will depend on specific parameter values, which we address in section 5.

3.5 Nash equilibrium import tariffs for all countries

This section considers optimal unilateral tariffs in a strategic environment. We define and solve for a D -country pure-strategy Nash equilibrium in which countries choose import tariffs. We assume countries cannot choose domestic taxes, which are set to their efficient levels defined in section 3.3 so that $\tau_{dd} = \tau_{dd}^\tau \quad \forall d$. The Nash equilibrium import tariffs are defined by tariffs that maximize each country's welfare, subject to the equilibrium conditions

and the Nash tariffs set by other countries. Formally, this problem is given by

$$\text{Find } \left\{ \boldsymbol{\kappa}^n, \bar{\boldsymbol{\varphi}}^n, \vec{C}^n, \vec{w}^n, \pi^n, \boldsymbol{\tau}^n \right\} \text{ subject to} \quad (27a)$$

$$\left\{ \boldsymbol{\kappa}^n, \bar{\boldsymbol{\varphi}}^n, \vec{C}^n, \vec{w}^n, \pi^n, \vec{\tau}_{d*}^n \right\} = \arg \max_{\vec{\tau}_{d*}} \left(\frac{C_d}{\Xi_d \left(\vec{K}_{d*}, \bar{\boldsymbol{\varphi}}_{d*}, \vec{C}, \pi, \vec{w}, \vec{\tau}_{d*} \right)} \right) \quad \forall d, \quad (27b)$$

subject to: Eqs. (22b) through (22f),

$$\vec{\tau}_{o*} = \vec{\tau}_{o*}^n \quad \forall o \neq d, \quad (27c)$$

$$\tau_{dd} = \tau_{dd}^n \quad \forall d. \quad (27d)$$

We solve for the Nash equilibrium using the Nikaidô-Isoda (NI) function (Nikaidô and Isoda, 1955) given by

$$\Psi(\boldsymbol{\tau}, \boldsymbol{\zeta}) = \sum_{d=1}^D \left[\mathcal{L}_d(\boldsymbol{\tau}, \boldsymbol{\zeta}) - \sup_{\hat{\boldsymbol{\tau}}_{d*}, \hat{\boldsymbol{\zeta}}_{d*}} \mathcal{L}_d(\hat{\boldsymbol{\tau}}, \hat{\boldsymbol{\zeta}}) \right], \quad (28)$$

in which the Lagrangian, $\mathcal{L}_d(\boldsymbol{\tau}, \boldsymbol{\zeta})$, is written as a function of the exogenous tariffs, $\boldsymbol{\tau}$, and exogenous matrix of Lagrange multipliers, $\boldsymbol{\zeta}$, corresponding to all of the constraints defined in equation (27). The Lagrangian is also a function of the endogenous variables— $\boldsymbol{\kappa}, \bar{\boldsymbol{\varphi}}, \vec{C}, \vec{w}, \pi$ —that define the economy's equilibrium, but those are determined by satisfying the constraints in equation (27) for given values of $\boldsymbol{\tau}$ and $\boldsymbol{\zeta}$. As such, we do not write out the endogenous variables explicitly in equation (28). The Appendix includes more details and provides the Lagrangian explicitly.

Intuitively, each summand of the NI function (28) can be thought of as the difference in equilibrium welfare for a country d and that country's best response. When the summand for country d is zero, that country has no unilateral incentive to deviate. When the sum for all countries is zero, no country has a unilateral incentive to deviate. Hence, the Nash equilibrium is defined as $\Psi(\boldsymbol{\tau}^n, \boldsymbol{\zeta}^n) = 0$ because this is when no country can benefit by unilaterally changing their tariffs. It is easy to show that $\Psi(\boldsymbol{\tau}^n, \boldsymbol{\zeta}^n) = 0$ is a global maximum because $\Psi(\boldsymbol{\tau}, \boldsymbol{\zeta}) \leq 0$.

4 Calibration

We use data for China and the United States in 2016 to calibrate our model, as in Krolikowski and McCallum (2021), but we can generalize our approach to include more trading partners or a different time period. The calibration proceeds in two steps. First, we externally calibrate parameters that can be normalized or that are standard in the literature. Second, we internally calibrate the remaining parameters by minimizing the distance between moments in the data and the decentralized model (Eq. 22) with search frictions subject to that model's equilibrium constraints. Formally, this minimization is accomplished

by solving a mathematical program with equilibrium constraints (MPEC) following [Dubé, Fox, and Su \(2012\)](#) and [Su and Judd \(2012\)](#).

5 Quantitative experiments

5.1 Decentralized optimal tariffs with and without search

5.2 Optimal tariffs with and without search

5.3 Strategic optimal tariffs with and without search

5.4 Welfare gains from changing pre-war tariffs to optimal tariffs

5.5 Welfare losses from the China-U.S. trade war relative to pre-war tariffs

6 Conclusion

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Appendix to ‘‘Tariffs and Goods-Market Frictions’’

Pawel M. Krolkowski*

Andrew H. McCallum†

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A Model appendix

A.1 Consumers

We assume the representative consumer in destination market d has Cobb-Douglas utility, U_d , over a homogeneous good and a second good that is a constant elasticity of substitution (CES) aggregate of differentiated varieties, indexed by ω , from all origins, indexed by $k \in \{1, \dots, O\}$. The two goods are combined with exponents $1 - \alpha$ and α , respectively. The differentiated goods are substitutable with constant elasticity, $\sigma > 1$, across varieties and destinations and we denote the value of total consumption as C_d in destination country d . Formally the consumer’s problem is

$$\begin{aligned} \max_{q_d(1), q_{dk}(\omega)} \quad & q_d(1)^{1-\alpha} \left[\sum_{k=1}^O \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha \left(\frac{\sigma}{\sigma-1}\right)} \\ \text{s.t. } C_d = \quad & p_d(1) q_d(1) + \sum_{k=1}^O \int_{\omega \in \Omega_{dk}} p_{dk}(\omega) q_{dk}(\omega) d\omega, \end{aligned} \quad (\text{A1})$$

which results in the following demand for the homogeneous good and each differentiated variety, respectively

$$q_d(1) = \frac{(1 - \alpha) C_d}{p_d(1)}, \quad q_{do}(\omega) = \alpha C_d \frac{p_{do}(\omega)^{-\sigma}}{P_d^{1-\sigma}}. \quad (\text{A2})$$

Cobb-Douglas preferences across sectors imply that the consumer allocates share $1 - \alpha$ of total consumption expenditure to the homogeneous good and share α to the differentiated goods. We could easily extend our framework to any number of Cobb-Douglas sectors, as in [Chaney \(2008\)](#).

The value of consumption of the differentiated good in the do market is defined as the integral over all varieties, ω , of the value of $q_{do}(\omega)$ units evaluated at final sales prices, $p_{do}(\omega)$:

$$C_{do} = \int_{\omega \in \Omega_{dk}} p_{do}(\omega) q_{do}(\omega) d\omega. \quad (\text{A3})$$

The homogeneous good has price $p_d(1)$. Define P_d as the price index for the bundle of differentiated varieties and P_{do} as the price index for the bundle of varieties produced in

*pawel.krolkowski@clev.frb.org, Federal Reserve Bank of Cleveland, 1455 E 6th St, Cleveland, OH 44114

†Corresponding author, andrew.h.mccallum@frb.gov, Board of Governors of the Federal Reserve System, Washington, DC 20551

country o and consumed in country d :

$$P_d = \left[\sum_{k=1}^O \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = \left[\sum_{k=1}^O P_{dk}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (\text{A4})$$

The ideal price index including the homogeneous good that minimizes expenditure to obtain utility level $U_d = 1$ is

$$\Xi_d = [p_d(1) / (1 - \alpha)]^{1-\alpha} [P_d / \alpha]^\alpha. \quad (\text{A5})$$

We solve the consumer's utility maximization and expenditure minimization problems explicitly in [Krolikowski and McCallum \(2021\)](#) Appendix A.1.

A.2 Matching and value functions for producers and retailers

A.2.1 The matching function

The matching function, denoted by $m(u_{do}N_o^x, v_{do}N_d^m)$, gives the flow number of relationships formed at any moment in time as a function of the stock number of unmatched producers, $u_{do}N_o^x$, and unmatched retailers, $v_{do}N_d^m$, in the do market. N_o^x and N_d^m represent the total mass of producing firms in country o and retailing firms in country d regardless of their match status. The fraction of producers in country o looking for retailers in country d is u_{do} . The fraction of retailers that are searching for producing firms in this market is v_{do} .

As in many studies of the labor market ([Pissarides, 1985](#); [Shimer, 2005](#)), we assume that the matching function takes a Cobb-Douglas form:

$$m(u_{do}N_o^x, v_{do}N_d^m) = \xi (u_{do}N_o^x)^\eta (v_{do}N_d^m)^{1-\eta}, \quad (\text{A6})$$

in which ξ is the matching efficiency and η is the elasticity of matches with respect to the number of searching producers.

The matching function in Equation (A6) is homogeneous of degree one. Therefore, market tightness, $\kappa_{do} = v_{do}N_d^m / u_{do}N_o^x$, which is the ratio of the mass of searching retailers to the mass of producers in a given market, is sufficient to determine contact rates on both sides of that market. In particular, the rate at which retailers in country d contact producers in country o , $\chi(\kappa_{do})$, is the number of matches formed each instant over the number of searching retailers:

$$\chi(\kappa_{do}) = \frac{m(u_{do}N_o^x, v_{do}N_d^m)}{v_{do}N_d^m} = \frac{\xi (u_{do}N_o^x)^\eta (v_{do}N_d^m)^{1-\eta}}{v_{do}N_d^m} = \xi \kappa_{do}^{-\eta}. \quad (\text{A7})$$

Notice that retailers' contact rate falls with market tightness ($d\chi(\kappa_{do})/d\kappa_{do} < 0$) because with more retailers relative to producers, the search market becomes congested with retailers.

The rate at which producers in country o contact retailers in country d is the number of matches formed each instant over the number of searching producers, so that the producer contact rate is,

$$\kappa_{do}\chi(\kappa_{do}) = \xi \kappa_{do}^{1-\eta}. \quad (\text{A8})$$

Producers' contact rate rises with tightness ($d\kappa_{do}\chi(\kappa_{do})/d\kappa_{do} > 0$), also called a market thickness effect. Market tightness is defined from the perspective of producers so that the market is tighter when there are relatively more retailers than producers. Equations (A7)

and (A8) are restated in Equation (1) of the main text.

A.2.2 Producers' value functions

The value of a producer with productivity φ being matched to a retailer, $X_{do}(\varphi)$, can be summarized by a value function in continuous time defined in Equation (3). That asset equation states that the flow return at the risk-free rate, r , from the value of producing must equal the flow payoff plus the expected capital gain from operating as an exporting producer. Each producer is indexed by exogenous productivity, φ . The flow payoff consists of $n_{do}q_{do}$, the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable, Equation (2), and fixed costs of production, f_{do} . The last term in Equation (3) is the value from the dissolution of the match, which occurs at exogenous rate λ and leads to a capital loss of $U_{do}(\varphi) - X_{do}(\varphi)$ as the producer loses value $X_{do}(\varphi)$ but gains the value of being an unmatched producer, $U_{do}(\varphi)$. In writing Equation (3), we explicitly write the value $X_{do}(\varphi)$ as a function of the producer's productivity, φ , but we conserve on notation by omitting this argument from the negotiated price, n_{do} , and traded quantity, q_{do} .

The value that an unmatched producer receives from looking for a retail partner without being in a business relationship is defined by Equation (4). The flow search cost, l_{do} , is what the producer pays when looking for a retailer. Examples of which are the costs of maintaining foreign sales offices, sending sales representatives abroad, researching potential foreign buyers, and paying for a web presence. The second term captures the expected capital gain, in which $\kappa_{do}\chi(\kappa_{do})$ is the endogenous rate at which producing firms contact retailers, and s_{do} is the sunk cost of starting up the relationship. The producer considers the difference between being in a business relationship, $X_{do}(\varphi)$, and searching, $U_{do}(\varphi)$, rather than these quantities separately. As such, any additive term that enters both Equations (3) and (4) will not affect producers' decisions.

The producing firm also has the option of remaining idle and not expending resources to look for a retailer. For producers, the value of not searching, $I_{do}(\varphi)$, is given by Equation (5). The value to a producer of remaining idle can be interpreted, for example, as the value of the stream of payments after liquidation or the flow payoff from home production if these firms are viewed as entrepreneurs.

A.2.3 Retailers' value functions

The value of a retailing firm in a business relationship with a producer of productivity φ , is defined by the asset Equation (6). The flow payoff from being in a relationship is the revenue generated by selling q_{do} units of the product to a representative consumer at a final sales price, p_{do} , — determined in Appendix A.3.3 — less the tariff inclusive cost of acquiring these goods from producers at negotiated price n_{do} . As stated in the main text, tariff revenue is collected from the retailers by the government and rebated lump-sum to consumers. When the relationship is destroyed exogenously, at rate λ , the retailing firm loses the capital value of being matched. All retailers are identical before matching but have differential matched values because producers are heterogeneous in their productivity.

Retailers do not use the product as an input in another stage of production but only facilitate the match between producers and consumers and collect tariffs that are paid to the government. In the event that the relationship undergoes an exogenous separation, at rate λ , the retailing firm loses the capital value of being matched, $V_{do} - M_{do}(\varphi)$.

The value of being an unmatched retailer, V_{do} , satisfies Equation (7). Retailers need to pay a flow cost, c_{do} , to search for a producing affiliate. At endogenous Poisson rate $\chi(\kappa_{do})$,

retailing firms meet a producer of unknown productivity. Producers' productivities are ex-ante unknown to retailers so retailers take the expectation over all productivities they might encounter when computing the expected continuation value of searching. As a result, the value, V_{do} , is not a function of a producer's productivity, φ , but rather a function of the expected payoff. We assume that upon meeting, but before consummating a match, retailers learn the productivity of the producer. Depending on the producer's productivity, φ , retailers choose between matching with that producer, which generates value $M_{do}(\varphi)$, or continuing the search, which generates V_{do} . Hence, the capital gain to retailers from meeting a producer with productivity φ can be expressed as $\max\{V_{do}, M_{do}(\varphi)\} - V_{do}$. In an equilibrium with free entry into retailing, this approach is equivalent to retailers observing producers' productivity after matches are formed.

A.3 Solving the partial-equilibrium search problem

A.3.1 The surplus, value, and expected duration of a relationship

To derive the surplus in terms of model primitives, substitute Equations (3), (4), (6), and free entry for retailers, $V_{do} = 0$, into Equation (8) to write the surplus as,

$$\left(r + \lambda + \frac{\beta\kappa_{do}\chi(\kappa_{do})}{\beta + t_{do}(1 - \beta)}\right) S_{do}(\varphi) = p_{do}q_{do} + n_{do}q_{do}(1 - t_{do}) - v(q_{do}, w_o, \tau_{do}, \varphi) - \delta_{do}, \quad (\text{A9})$$

in which we define,

$$\delta_{do} \equiv f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})s_{do}. \quad (\text{A10})$$

Now substitute the negotiated price from Equation (10) into Equation (A9) and use the definition of

$$\gamma_{do} \equiv \frac{(r + \lambda)(1 - \beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})} \quad (\text{A11})$$

to write surplus as,

$$S_{do}(\varphi) = \left(\frac{\beta + t_{do}(1 - \beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}\right) \left(\frac{p_{do}q_{do}}{t_{do}} - v(q_{do}, w_o, \tau_{do}, \varphi) - \delta_{do}\right). \quad (\text{A12})$$

The surplus created by a match is the appropriately discounted after-tariff flow profit, with the search cost l_{do} and the sunk cost s_{do} also entering the surplus equation because being matched avoids paying these costs.

There are four things to notice about Equation (A12). First, when $t_{do} = 1$, it becomes the surplus in Appendix A.3. Equation (A33) of [Krolikowski and McCallum \(2021\)](#). Second, the surplus from a match is a function of productivity. We show in appendix [A.3.4.3](#) that matches that include a more productive exporting firm lead to greater surplus, that is, $S'_{do}(\varphi) > 0$. Third, the value of a relationship depends on aggregate endogenous quantities such as the price index, consumption, and finding rate $\kappa_{do}\chi(\kappa_{do})$, among others. Finally, surplus is greater than or equal to zero if and only if after-tariff total profits are. That is, when

$$\frac{p_{do}q_{do}}{t_{do}} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + s_{do}\kappa_{do}\chi(\kappa_{do}) \geq 0. \quad (\text{A13})$$

The value of the relationship to the producer is, of course, $X_{do}(\varphi)$ and to the retailer

$M_{do}(\varphi)$. Therefore, the total value of a matched relationship is,

$$R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi). \quad (\text{A14})$$

We can express Equation (A14) in terms of surplus and then Equation (8) with $V_{do} = 0$, by adding and subtracting Equation (4), substituting in Equation (9) for $X_{do}(\varphi) - U_{do}(\varphi)$ and then simplifying to get

$$R_{do}(\varphi) = \left[\frac{r(\beta + t_{do}(1 - \beta)) + \beta\kappa_{do}\chi(\kappa_{do})}{r(\beta + t_{do}(1 - \beta))} \right] S_{do}(\varphi) - \left[\frac{l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r} \right]. \quad (\text{A15})$$

Equation (A15) can be expressed in terms of model primitives using (A12) and the definitions for those functions provided in Equations (11), (2), and (A2). Relationships are destroyed at Poisson rate λ in the model, which implies the average duration of each match is $1/\lambda$. Because the destruction rate is exogenous and does not vary in our model, the average duration of each match is constant. The value of a relationship in product markets has been of recent interest in Monarch and Schmidt-Eisenlohr (2023) and Heise (2016). Finally, Equation (A15) is the same as the $R_{do}(\varphi)$ Equation on page 7 of Appendix A.3 of Krolikowski and McCallum (2021) when $t_{do} = 1$ and $s_{do} = 0$.

A.3.2 Bargaining over the negotiated price

Upon meeting, the retailer and producer bargain over the negotiated price, n_{do} , and quantity, q_{do} , simultaneously. We assume that these objects are determined by the generalized Nash bargaining solution, which, as shown by Nash (1950) and Osborne and Rubinstein (1990), is equivalent to maximizing the following Nash product:

$$\max_{q_{do}, n_{do}} [X_{do}(\varphi) - U_{do}(\varphi)]^\beta [M_{do}(\varphi) - V_{do}]^{1-\beta}, \quad 0 \leq \beta < 1, \quad (\text{A16})$$

in which β is producers' bargaining power. To solve Equation (A16), first solve for $X_{do}(\varphi) - U_{do}(\varphi)$ by combining Equations (3) and (4) to get that:

$$X_{do}(\varphi) - U_{do}(\varphi) = \frac{n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})}. \quad (\text{A17})$$

Next rearrange Equation (6) to get that:

$$M_{do}(\varphi) - V_{do} = \frac{p_{do}(q_{do})q_{do} - t_{do}n_{do}q_{do} - rV_{do}}{r + \lambda}. \quad (\text{A18})$$

Substitute Equations (A17) and (A18) into (A16), then log and differentiate with respect to the n_{do} to get the relevant first order condition:

$$\beta \frac{q_{do}/(r + \lambda)}{X_{do}(\varphi) - U_{do}(\varphi)} + (1 - \beta) \frac{-t_{do}q_{do}/(r + \lambda)}{M_{do}(\varphi) - V_{do}} = 0. \quad (\text{A19})$$

We do not need to calculate the partial derivative with respect κ_{do} , w_o , or other endogenous variables, because we assume individual varieties are too small to influence aggregate values. Hence, when they meet, the firms bargain taking everything but n_{do} and q_{do} as given.

Furthermore, the partial of the value of a vacancy, $\partial V_{do}/\partial n_{do} = 0$, because bargaining takes place over each variety, φ , individually. As long as the distribution of varieties is continuous, $\partial V_{do}/\partial n_{do}$ does not have an effect on the expectation in the continuation value in Equation (7).

For any variety that is traded, $q_{do} > 0$, Equation (A19) can be written as,

$$\beta (M_{do}(\varphi) - V_{do}) = (1 - \beta) t_{do} (X_{do}(\varphi) - U_{do}(\varphi)). \quad (\text{A20})$$

Using Equations (A20) and (8) delivers the surplus sharing rule, Equation (9). To find the negotiated price show in Equation (10), use the equilibrium free entry condition $V_{do} = 0$ and substitute (A17) and (A18) into (A20), then solve for n_{do} .

A.3.3 Bargaining over the quantity

Substitute Equations (A17) and (A18) into (A16), then log and differentiate with respect to q_{do} to get the relevant first order condition for quantity:

$$\beta \frac{\left(n_{do} - \frac{\partial v(q_{do}, w_o, \tau_{do}, \varphi)}{\partial q_{do}} \right)}{[X_{do}(\varphi) - U_{do}(\varphi)](r + \lambda)} + (1 - \beta) \frac{\left(p_{do}(q_{do}) + \frac{\partial p_{do}(q_{do})}{\partial q_{do}} q_{do} - t_{do} n_{do} \right)}{[M_{do}(\varphi) - V_{do}](r + \lambda)} = 0, \quad (\text{A21})$$

in which we use the same reasoning for $\partial V_{do}/\partial n_{do} = 0$ as in Appendix A.3.2.

Considering only solutions with positive values for Equation (A18) and $q_{do} > 0$, we plug Equation (A20) into (A21) rearrange to get:

$$p_{do}(q_{do}) + \frac{\partial p_{do}(q_{do})}{\partial q_{do}} q_{do} = t_{do} \frac{\partial v(q_{do}, w_o, \tau_{do}, \varphi)}{\partial q_{do}}. \quad (\text{A22})$$

This expression says that the quantity produced and traded equates marginal revenue earned from consumers to tariff-inclusive marginal production cost paid by producers. Equation (A22) is the same in a model with or without search frictions implying that search does not change the quantity traded within each match. In a model of search, parties agree upon a quantity that equates marginal revenue and marginal tariff-inclusive cost because that quantity maximizes surplus.

CES utility implies the consumer's price elasticity of demand from Equation (A2) is

$$\frac{\partial q_{do} p_{do}}{\partial p_{do} q_{do}} = -\sigma. \quad (\text{A23})$$

Indexing an individual variety by ω is equivalent to indexing by φ and we have treated these interchangeably when using Equation (A2) here. The equivalence of indexing variables contrasts with changing from a measure over a set of goods indexed by ω to a distribution of goods indexed by φ , which is subtle and discussed in detail in Krolikowski and McCallum (2021) Appendix A.11.1.

Combining Equation (A23) with the fact that $\partial p_{do}/\partial q_{do} = 1/(\partial q_{do}/\partial p_{do})$, we can write Equation (A22) as

$$p_{do}(q_{do}) + \frac{\partial p_{do}(q_{do})}{\partial q_{do}} q_{do} = p_{do} \left(\frac{\sigma - 1}{\sigma} \right) = t_{do} \frac{\partial v(q_{do}, w_o, \tau_{do}, \varphi)}{\partial q_{do}}. \quad (\text{A24})$$

Rearranging Equation (A24) and computing marginal costs from Equation (2) gives Equation (11).

Finally, setting price equal to average total cost (ATC) gives zero profit for any variety. As such, Equation (11) is always at least as high as ATC for all traded varieties above the threshold defined in Equation (14) and therefore defines the equilibrium price.

A.3.4 Producers' search productivity threshold

There are two productivity thresholds to consider. First, there is a productivity threshold, $\bar{\varphi}_{do}$, that makes the producer indifferent between searching and remaining idle defined by, $U_{do}(\bar{\varphi}_{do}) - I_{do}(\bar{\varphi}_{do}) = 0$. Second, there is a weakly lower productivity threshold, φ_{do} , which makes that producer indifferent between consummating a relationship upon contacting a retailer and continuing to search defined by, $X_{do}(\varphi_{do}) - U_{do}(\varphi_{do}) = 0$. We derive these two thresholds and show in Appendix A.3.4.2 that the binding threshold is defined by $\bar{\varphi}_{do}$, because $\bar{\varphi}_{do} \geq \varphi_{do}$ if and only if $l_{do} + h_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} \geq 0$.

The productivity threshold nests the threshold from Krolikowski and McCallum (2021, Equation 18) and we compare it to productivity thresholds in other models in Appendix A.6.4 of the same paper.

A.3.4.1 Solving for the binding productivity threshold

Combine Equations (4) and (5) to get

$$U_{do}(\varphi_{do}) - I_{do}(\varphi_{do}) = \frac{-l_{do} + \kappa_{do}\chi(\kappa_{do})(X_{do}(\varphi) - U_{do}(\varphi) - s_{do}) - h_{do}}{r} \quad (\text{A25})$$

The threshold productivity, $\bar{\varphi}_{do}$, is given by $U_{do}(\bar{\varphi}) - I_{do}(\bar{\varphi}_{do}) = 0$ so evaluate Equation (A25) at $\bar{\varphi}_{do}$, set the left hand side to zero, and rearrange to get

$$X_{do}(\bar{\varphi}_{do}) - U_{do}(\bar{\varphi}_{do}) = \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do}. \quad (\text{A26})$$

Substitute Equation (A17) into Equation (A26) and suppress $\bar{\varphi}_{do}$ for simplicity to derive

$$\begin{aligned} \frac{n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} &= \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do} \\ \Rightarrow n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} &= (r + \lambda + \kappa_{do}\chi(\kappa_{do})) \frac{s_{do}\kappa_{do}\chi(\kappa_{do}) + l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} \\ \Rightarrow n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} + l_{do} + \kappa_{do}\chi(\kappa_{do})s_{do} &= (r + \lambda)s_{do} + s_{do}\kappa_{do}\chi(\kappa_{do}) + (r + \lambda) \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + l_{do} + h_{do} \\ \Rightarrow n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} &= (r + \lambda)s_{do} + (r + \lambda) \frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + h_{do} \\ \Rightarrow n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} &= \left(\frac{r + \lambda}{\kappa_{do}\chi(\kappa_{do})} \right) l_{do} + \left(1 + \frac{r + \lambda}{\kappa_{do}\chi(\kappa_{do})} \right) h_{do} + (r + \lambda) s_{do}. \end{aligned}$$

Now, use the negotiated price, n_{do} , from Equation (10), to get

$$\begin{aligned} (1 - \gamma_{do}) \left(\frac{p_{do}}{t_{do}} \right) q_{do} + \gamma_{do} (v(q_{do}, w_o, \tau_{do}, \varphi) + f_{do} - l_{do} - \kappa_{do}\chi(\kappa_{do})s_{do}) - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} \\ = \left(\frac{r + \lambda}{\kappa_{do}\chi(\kappa_{do})} \right) l_{do} + \left(1 + \frac{r + \lambda}{\kappa_{do}\chi(\kappa_{do})} \right) h_{do} + (r + \lambda) s_{do}. \end{aligned}$$

which can be rearranged to obtain

$$\begin{aligned} & \left(\frac{p_{do}}{t_{do}} \right) q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - f_{do} \\ & = (1 - \gamma_{do})^{-1} \left[\left(\frac{r + \lambda}{\kappa_{do}\chi(\kappa_{do})} \right) l_{do} + \left(1 + \frac{r + \lambda}{\kappa_{do}\chi(\kappa_{do})} \right) h_{do} + (r + \lambda) s_{do} + \gamma_{do} [l_{do} + \kappa_{do}\chi(\kappa_{do}) s_{do}] \right]. \end{aligned}$$

Further simplification of the terms on the right hand side with γ_{do} delivers Equation (12) in the main text.

Using the price charged to consumers by retailers from Equation (11) we can write retailer revenue as proportional to variable production costs, Equation (2):

$$p_{do}(\varphi) q_{do}(\varphi) = (t_{do}\mu w_o \tau_{do} \varphi^{-1}) q_{do}(\varphi) = t_{do}\mu v(q_{do}, w_o, \tau_{do}, \varphi). \quad (\text{A27})$$

Then Equation (A27) implies that after-tariff variable profits are,

$$\left(\frac{p_{do}(\varphi_{do})}{t_{do}} \right) q_{do}(\varphi_{do}) - v(q_{do}, w_o, \tau_{do}, \varphi_{do}) = \frac{p_{do}(\varphi) q_{do}(\varphi)}{\sigma t_{do}}. \quad (\text{A28})$$

Or alternatively,

$$\left(\frac{p_{do}(\varphi_{do})}{t_{do}} \right) q_{do}(\varphi_{do}) - v(q_{do}, w_o, \tau_{do}, \varphi_{do}) = (\mu - 1) v(q_{do}, w_o, \tau_{do}, \varphi). \quad (\text{A29})$$

Substitute Equations (11) and (A2) into Equation (A28) and then substitute the resulting expression into the left hand side of Equation (12) to get that:

$$\frac{\alpha}{\sigma} C_d P_d^{\sigma-1} (\mu w_o \tau_{do})^{1-\sigma} t_{do}^{-\sigma} \bar{\varphi}_{do}^{\sigma-1} = F(\kappa_{do}). \quad (\text{A30})$$

Solving this expression for $\bar{\varphi}_{do}$ gives Equation (14).

Finally, all matches must have positive surplus so we can check that $S_{do}(\bar{\varphi}_{do}) \geq 0$ by using Equations (9) and (A26) to write,

$$S_{do}(\bar{\varphi}_{do}) = \left(\frac{\beta + t_{do}(1 - \beta)}{\beta} \right) \left(\frac{l_{do} + h_{do}}{\kappa_{do}\chi(\kappa_{do})} + s_{do} \right). \quad (\text{A31})$$

Equation (A31) puts restrictions on the parameters because they must be such that $S_{do}(\bar{\varphi}_{do}) \geq 0$. For example, h_{do} cannot be so negative as to make Equation (A31) negative. Does it matter that this is only private surplus?

A.3.4.2 Solving for the non-binding productivity threshold

The threshold productivity that is indifferent between matching and not, φ_{do} , is defined by

$$X_{do}(\varphi_{do}) - U_{do}(\varphi_{do}) = 0. \quad (\text{A32})$$

We can be sure that $X_{do}(\bar{\varphi}_{do}) - U_{do}(\bar{\varphi}_{do}) \geq X_{do}(\varphi_{do}) - U_{do}(\varphi_{do})$ as long as $(l_{do} + h_{do})/\kappa_{do}\chi(\kappa_{do}) + s_{do} \geq 0$. This result implies that as long as $X_{do}(\varphi) - U_{do}(\varphi)$ is increasing in φ , then $\bar{\varphi}_{do} \geq \varphi_{do}$. In appendix A.3.4.3, we show the very general conditions under which $X_{do}(\varphi) - U_{do}(\varphi)$ is increasing in φ . The binding productivity threshold

defining the mass of producers that have retail partners is the greater of these two thresholds and hence $\bar{\varphi}_{do}$. In other words, the productivity necessary to induce a producer to search for a retail partner is greater than the productivity necessary to consummate a match after meeting a retailer due to the costs that are incurred while searching. Similarly, the productivity necessary to form a match is greater than the productivity to maintain one already in place. Note that $\bar{\varphi}_{do} > \varphi_{do}$ if $(l_{do} + h_{do}) / \kappa_{do} \chi(\kappa_{do}) + s_{do} > 0$, which is true if and only if the cost of forming a relationship is positive, $l_{do} + h_{do} + \kappa_{do} \chi(\kappa_{do}) s_{do} > 0$.

A.3.4.3 The value of importing is strictly increasing in productivity

Here we show that the value of importing, $M_{do}(\varphi)$, is strictly increasing with the producer's productivity level, φ . This result leads to three implications. First, it allows us to replace the integral of the max over V_{do} and $M_{do}(\varphi)$ in Equation (7) with the integral of $M_{do}(\varphi)$ from the threshold from Equation (14). Second, in equilibrium, because $M'_{do}(\varphi) > 0$, Equation (9) implies that $S'_{do}(\varphi) > 0$ and therefore that $X'_{do}(\varphi) - U'_{do}(\varphi) > 0$. Third, it allows us to show that $\bar{\varphi}_{do} \geq \varphi_{do}$, as we did in Appendix A.3.4.2.

Starting with Equation (6) and $V_{do} = 0$, substituting in negotiated prices from Equation (10), and using the relationship between retailer revenue and variable costs from Equation (A27) we can write,

$$M_{do}(\varphi) = \frac{\sigma^{-1} \gamma_{do} p_{do}(\varphi) q_{do}(\varphi) - t_{do} \gamma_{do} \delta_{do}}{r + \lambda} \quad (\text{A33})$$

Remember that δ_{do} from Equation (A10) and γ_{do} from Equation (A11) are functions of tightness, κ_{do} , but not productivity, φ . It is clear from the integral in the import relationship creation, Equation (A69), that κ_{do} is not a function of φ . Given these facts, we can prove our result by differentiating both sides of Equation (A33) with respect to φ and showing that $M'_{do}(\varphi) = (\partial M_{do}(\varphi) / \partial q_{do}(\varphi)) \cdot (\partial q_{do}(\varphi) / \partial \varphi) > 0$. Using demand from Equation (A2), first write inverse demand $p_{do}(q_{do}(\varphi))$ then

$$M'_{do}(\varphi) = \frac{\sigma^{-1} \gamma_{do}}{r + \lambda} \left(p_{do}(q_{do}(\varphi)) + \frac{\partial p_{do}(\varphi)}{\partial q_{do}(\varphi)} q_{do} \right) \frac{\partial q_{do}(\varphi)}{\partial \varphi}. \quad (\text{A34})$$

The partial derivative in parentheses is marginal revenue, which we know in equilibrium will be equal to marginal cost times the tariff as shown in Equation (A22). Using this fact and applying the chain rule to $\partial q_{do}(\varphi) / \partial \varphi$ leads to our final expression,

$$M'_{do}(\varphi) = \frac{\sigma^{-1} \gamma_{do}}{r + \lambda} \left(t_{do} \frac{\partial v(q_{do}, w_o, \tau_{do}, \varphi)}{\partial q_{do}(\varphi)} \right) \frac{\partial q_{do}(\varphi)}{\partial p_{do}(\varphi)} \frac{\partial p_{do}(\varphi)}{\partial \varphi}. \quad (\text{A35})$$

As long as $\gamma_{do} > 0$ (which holds for finite κ_{do} and $\beta < 1$), marginal cost is positive, demand is downward sloping, and higher productivity varieties cost less, then $M'_{do}(\varphi) > 0$. These general conditions are satisfied for the functional forms of our model.

We can use the fact that $M'_{do}(\varphi) > 0$ to demonstrate the way in which many other important quantities depend on the producer's productivity level, φ . The surplus sharing rule, Equation (9), can be rewritten as

$$\beta M_{do}(\varphi) = (1 - \beta) t_{do} (X_{do}(\varphi) - U_{do}(\varphi)), \quad (\text{A36})$$

We know that in equilibrium, because $M'_{do}(\varphi) > 0$, it must be that $X'_{do}(\varphi) - U'_{do}(\varphi) > 0$.

Differentiating both sides of Equation (4) gives $rU'_{do}(\varphi) = \kappa_{do}\chi(\kappa_{do})(X'_{do}(\varphi) - U'_{do}(\varphi)) > 0$. We can combine these facts to show $X'_{do}(\varphi) > U'_{do}(\varphi) > 0$. Using the definition of the joint surplus of a match, Equation (8), we get $S'_{do}(\varphi) > 0$. Likewise, the value of a relationship, $R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi)$, has $R'_{do}(\varphi) > 0$.

A.4 Aggregation

A.4.1 Government expenditure

We assume that retailers in the do market pay a tariff, $t_{do} - 1$, on the value of imported differentiated goods, $n_{do}(\varphi)q_{do}(\varphi)$. Our assumption implies that for each unit that is traded of the differentiated good in market do , the government receives revenue $(1 - t_{do})n_{do}(\varphi)q_{do}(\varphi)$. This revenue is negative if $t_{do} > 1$ because it reduces consumption. Integrating over all the products and summing over all the origin countries yields

$$G_d = \sum_{k=1}^D G_{dk} = \sum_{k=1}^D \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} (1 - t_{dk}) n_{dk}(\varphi) q_{dk}(\varphi) dG(\varphi), \quad (\text{A37})$$

which matches the G_d term in Equation (16) in the main text. Notice that when $t_{dk} = 1 \forall k$ then $G_d = 0$ because there are no import tariffs or subsidies.

XXXX add relationship between government and imports XXXX add relationship between government and consumption

A.4.2 Labor market clearing

The labor market clearing condition determines the wage in each country d that ensures that labor supply is equal to labor demand:

$$L_d = \frac{I_d}{w_d} + q_d(1) + \sum_o \left(1 - \frac{u_{od}}{1 - i_{od}}\right) N_d^x \int_{\bar{\varphi}_{od}} q_{od}(\varphi) \tau_{od} \varphi^{-1} dG(\varphi).$$

Labor supply in country d is fixed and exogenously given by L_d . Labor demand is the labor used to create firms, pay fixed costs, and form matches captured by the investment term, I_d . (Investment must be divided by the wage to yield units of labor.) Labor demand also includes all labor used to produce the non-traded good, which is made with one unit of labor in each country. Finally, labor demand includes all labor used to produce the differentiated goods for the domestic and all foreign markets. Demand for $q_d(1)$ is given by Equation (A2), in which $p_d(1) = w_d$. Labor used to produce the differentiated good can be written as

$$\left(1 - \frac{u_{od}}{1 - i_{od}}\right) N_d^x \int_{\bar{\varphi}_{od}} q_{od}(\varphi) \varphi^{-1} dG(\varphi) = \frac{C_{od}}{\mu w_d t_{od}}.$$

In short, labor market clearing can be written as

$$L_d = \frac{I_d}{w_d} + \frac{(1 - \alpha) C_d}{w_d} + \frac{1}{\mu} \sum_o \frac{C_{od}}{w_d t_{od}}.$$

Re-arranging slightly yields the equilibrium wage, which is equation (17) in the main text.

A.4.3 Number of producers

Similar to [Chaney \(2008\)](#), we assume that the number of producers in the origin market that take a draw from the productivity distribution is proportional to consumption expenditure in the economy, C_o . The basic intuition behind this is that larger economies have a larger stock of potential entrepreneurs. To make this explicit, we denote the total mass of potential entrants as $N_o^x = \xi_o C_o$, in which the proportionality constant $\xi_o \in [0, \infty)$ captures exogenous structural factors that affect the number of potential entrants in country k . Among others, these could include such factors as literacy levels and attitudes toward entrepreneurship. As discussed in appendix [A.4.4](#), because the number of producers is fixed, the economy has profits. We assume that a global mutual fund collects worldwide profits and redistributes them as π dividends per share to each worker who owns w_o shares. We assume that $\xi_o = \frac{1}{1 + \pi}$ so that

$$N_o^x = \frac{C}{(1 + \pi)} \frac{C_o}{C} \quad (\text{A38})$$

in which we have multiplied and divided by global consumption, C .

A.4.4 Profits and the global mutual fund

In this appendix, we present three ownership structures for the profits earned by retailers and producers. First, we assume ownership of firms by location: Consumers in country d own retailers and producers in country d . Second, we assume that ownership is vertically integrated over firms: Consumers in country d own retailers in country d and the profits generated by producers in the potentially many countries, indexed by o , that produce for country d . This ownership structure results in simple trade balance condition and could be used to study multinational firms in a world with search frictions. Third, we assume that consumers in country d own $w_d L_d$ shares of a global mutual fund that owns all retailers and producers. The mutual fund redistributes profits derived anywhere proportionally to each country in the form of π dividends per share (equation [18](#)). We implement the third ownership structure in this paper to facilitate comparisons with [Chaney \(2008\)](#) but it would be straightforward to implement either of the other two structures and our conclusions about optimal tariffs with and without search frictions would remain unchanged.

A.4.4.1 Profits attributed by location

Assume that consumers in country d own retailers and producers in country d . This assumption implies that total profits in country d are profits from retailers in country d selling products from potentially many origin k markets and profits from producers in country d selling to potentially many other markets k markets, in which $k = 1, \dots, D$.

Retailer profits in country d are defined as

$$\Pi_d^r = \sum_{k=1}^D \Pi_{dk}^r = \sum_{k=1}^D \left(1 - \frac{u_{dk}}{1 - i_{dk}} \right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi) q_{dk}(\varphi) - t_{dk} n_{dk}(\varphi) q_{dk}(\varphi) dG(\varphi). \quad (\text{A39})$$

Producer profits in country d are defined as

$$\Pi_d^p = \sum_{k=1}^D \Pi_{kd}^p = \sum_{k=1}^D \left(1 - \frac{u_{kd}}{1 - i_{kd}} \right) N_d^x \int_{\bar{\varphi}_{kd}} n_{kd}(\varphi) q_{kd}(\varphi) - v(q_{kd}, w_d, \tau_{kd}, \varphi) dG(\varphi). \quad (\text{A40})$$

Notice that Π_d^r and Π_d^p sum over different indices. Multiply equation (10) by $t_{do}q_{do}$ and use equation (A27) to get that,

$$t_{do}n_{do}(\varphi)q_{do}(\varphi) = \frac{[\mu(1 - \gamma_{do}) + \gamma_{do}]}{\mu} p_{do}(\varphi)q_{do}(\varphi) + \gamma_{do}t_{do}\delta_{do}, \quad (\text{A41})$$

in which δ_{do} is defined in equation (A10) and then the profits to each retailer are

$$p_{do}(\varphi)q_{do}(\varphi) - t_{do}n_{do}(\varphi)q_{do}(\varphi) = \gamma_{do}(\sigma^{-1}p_{do}(\varphi)q_{do}(\varphi) - t_{do}\delta_{do}). \quad (\text{A42})$$

Using equation (A42) for the integrand in equation (A39) delivers retailer profits as,

$$\Pi_d^r = \sum_{k=1}^D \gamma_{dk}\sigma^{-1}C_{dk} - \sum_{k=1}^D (1 - u_{dk} - i_{dk}) N_k^x \gamma_{dk}\delta_{dk}t_{dk}, \quad (\text{A43})$$

in which equation (A67) defines differentiated goods consumption, C_{dk} . Similarly, using equations (A41) and (A27), producer profits for each variety are,

$$n_{do}(\varphi)q_{do}(\varphi) - v(q_{do}, w_o, \tau_{do}, \varphi) = \frac{(1 - \gamma_{do})}{\sigma t_{do}} p_{do}(\varphi)q_{do}(\varphi) + \gamma_{do}\delta_{do}. \quad (\text{A44})$$

Integrating equation (A44) over varieties and summing over destination markets shows that producer profits can be written as,

$$\Pi_d^p = \sum_{k=1}^D \frac{(1 - \gamma_{kd})}{\sigma t_{kd}} C_{kd} + \sum_{k=1}^D (1 - u_{kd} - i_{kd}) N_d^x \gamma_{kd}\delta_{kd}. \quad (\text{A45})$$

Under this location-based ownership structure, total flow variable profits earned in d are therefore the sum of equations (A43) and (A45), $\Pi_d = \Pi_d^r + \Pi_d^p$.

A.4.4.2 Profits from vertically integrated production

Assume that consumers in country d own retailers in country d and also own all the producers in the potentially many origin k markets that serve market d . This assumption implies that total profits in country d are profits from retailers in country d selling products from potentially many origin k markets and profits that producers in k countries earn from selling to country d but not other countries, in which $k = 1, \dots, D$.

This vertically-integrated ownership structure and the location-base ownership structure in Section A.4.4.1 imply the same retailer profits defined in equation (A39) and resulting equation (A43).

Producer profits, however, differ in a very simple way between these two approaches. Vertically-integrated profits change the order of the indexes in the sum so that the destination country is fixed at d but the origin country is indexed so that $\Pi_d^p = \sum_{k=1}^D \Pi_{dk}^p$ in contrast to the location-base profits $\Pi_d^p = \sum_{k=1}^D \Pi_{kd}^p$. Because the summing index, dk , for retailers and producers is the same under vertically-integrated ownership, total profits will be $\Pi_d = \sum_{k=1}^D \Pi_{dk}^r + \sum_{k=1}^D \Pi_{dk}^p$. Changing the index of equation (A45) and then adding to

equation (A43) gives,

$$\Pi_d = \Pi_d^r + \Pi_d^p = \sum_{k=1}^D \left[\frac{t_{dk}\gamma_{dk} + 1 - \gamma_{dk}}{\sigma t_{dk}} \right] C_{dk} + \sum_{k=1}^D (1 - u_{dk} - i_{dk}) N_k^x \gamma_{dk} \delta_{dk} (1 - t_{dk}). \quad (\text{A46})$$

One could alternatively derive equation (A46) by changing the indexes in the integrand of equation (A40) and then adding to the integrand of equation (A39) to get the profits of each retailer and producer that sell and produce each variety,

$$\pi_{do}^v(\varphi) = p_{do}(\varphi) q_{do}(\varphi) - t_{do} n_{do}(\varphi) q_{do}(\varphi) + n_{do}(\varphi) q_{do}(\varphi) - v(q_{do}, w_o, \tau_{do}, \varphi). \quad (\text{A47})$$

Combining terms and using equation (A27) gives,

$$\pi_{do}^v(\varphi) = (1 - t_{do}) n_{do}(\varphi) q_{do}(\varphi) + \frac{(t_{do}\mu - 1)}{t_{do}\mu} p_{do}(\varphi) q_{do}(\varphi). \quad (\text{A48})$$

Multiply equation (A41) by $(1 - t_{do}) q_{do}/t_{do}$ to get,

$$(1 - t_{do}) n_{do}(\varphi) q_{do}(\varphi) = \frac{(1 - t_{do}) [\mu(1 - \gamma_{do}) + \gamma_{do}]}{t_{do}\mu} p_{do}(\varphi) q_{do}(\varphi) + (1 - t_{do}) \gamma_{do} \delta_{do}, \quad (\text{A49})$$

and then combine with equation (A48) to show that

$$\pi_{do}^v(\varphi) = \left[\frac{t_{do}\gamma_{do} + 1 - \gamma_{do}}{\sigma t_{do}} \right] p_{do}(\varphi) q_{do}(\varphi) + \gamma_{do} \delta_{do}(\varphi) (1 - t_{do}). \quad (\text{A50})$$

Integrating equation (A50) over traded varieties and summing over all origin countries yields equation (A46).

Vertically integrated ownership results in a simple trade balance condition. Integrating equation (A49) over traded varieties and summing over all origin countries yields,

$$\Pi_d = G_d + \sum_{k=1}^D \left(\frac{\sigma t_{dk} - \sigma}{\sigma t_{dk}} \right) C_{dk}, \quad (\text{A51})$$

in which we use the differentiated goods consumption and government revenue from equations (A67) and (A37), respectively. Setting the income and expenditure approaches to national accounting equal to each other implies that $w_d L_d + \Pi_d = C_d + I_d + G_d$. Using equation (A51) and the labor market clearing condition (equation 17) to substitute for $w_d L_d$, we obtain

$$\sum_o \frac{C_{od}}{t_{od}} = \sum_o \frac{C_{do}}{t_{do}}, \quad (\text{A52})$$

which resembles a trade balance condition: The after-tariff value of exports on the left hand side is equal to the after-tariff value of imports on the right hand side.

A.4.4.3 The global mutual fund

Assume that all retailers and producers are owned by a global mutual fund that collects all variable profits and rebates them to consumers. Global profits can be expressed in many

ways. One way is to sum equation (A51) across all countries d to obtain Π shown in (18). It is useful to proportion these profits to each country as a constant share of labor income so that $Y_d = w_d L_d (1 + \pi)$, in which π is defined in equation (18).

We can be sure that equation (18) is consistent with our model by using equation (16), written as $I_d + C_d = Y_d - G_d$ and $Y_d = w_d L_d (1 + \pi)$ in the equilibrium wage from equation (17) to obtain

$$w_d L_d = w_d L_d (1 + \pi) - G_d - \alpha C_d + \frac{1}{\mu} \sum_o \frac{C_{od}}{t_{od}}.$$

Now rearrange and sum across all destination countries d to get

$$\pi = \frac{\alpha C - \frac{1}{\mu} \sum_d \sum_o \frac{C_{od}}{t_{od}} + G}{\sum_d w_d L_d}. \quad (\text{A53})$$

The numerator in Equation (A53) matches the definition for Π from equation (18) in the main text. Notice that the dividend per unit value of labor, π , is proportional to the value of the global labor endowment and constant across countries. This definition matches Chaney (2008) equation (6) adjusted to include tariffs.

A.5 The ideal price index with our productivity distribution

The ideal price index is provided in Equation (A5) and is a function of the homogeneous good price and the differentiated goods price index price index in Equation (A4), which indexes over an unordered set of varieties. We can move from an unordered set of varieties to an index over a distribution of productivities using the steps in Appendix A.11.1 of Krolikowski and McCallum (2021) so that the differentiated goods price index is given by:

$$P_d = \left[\sum_{k=1}^O P_{dk}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left[\sum_{k=1}^O \left(1 - \frac{u_{dk}}{1 - i_{dk}} \right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} p_{dk}(\varphi)^{1-\sigma} dG(\varphi) \right]^{\frac{1}{1-\sigma}}, \quad (\text{A54})$$

in which $G(\cdot)$ is a Pareto cumulative density function from Section 2.2.2. Using the final consumer price from Equation (11) and the definition for the number of producers, N_o^x , from Equation (A38) in Equation (A54) gives,

$$P_d = \left[\sum_{k=1}^O \left(1 - \frac{u_{dk}}{1 - i_{dk}} \right) \left(\frac{C}{1 + \pi} \right) \frac{C_k}{C} \int_{\bar{\varphi}_{dk}}^{\infty} \left(\frac{t_{dk} \mu w_k \tau_{dk}}{\varphi} \right)^{1-\sigma} dG(\varphi) \right]^{\frac{1}{1-\sigma}}. \quad (\text{A55})$$

The relevant moment is,

$$\int_{\bar{\varphi}_{dk}}^{\infty} z^{\sigma-1} dG(z) = \frac{\theta \bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta - \sigma + 1}, \quad (\text{A56})$$

and the threshold from Equation (14) raised to the relevant exponent is,

$$\bar{\varphi}_{do}^{\sigma-1-\theta} = P_d^{\theta-(\sigma-1)} \mu^{\sigma-1-\theta} \left(\frac{\sigma}{\alpha} \right)^{1-\frac{\theta}{\sigma-1}} (w_o \tau_{do})^{\sigma-1-\theta} \left(\frac{F_{do}}{C_d} \right)^{1-\frac{\theta}{\sigma-1}} t_{do}^{\sigma-\mu\theta}. \quad (\text{A57})$$

Because the threshold is a function of P_d , Equation (A55) is itself a function of P_d too. Using Equation (A57) in Equation (A55) and simplifying gives,

$$P_d = P_d^{1-\frac{\theta}{\sigma-1}} \times \left[\left(\frac{\theta}{\theta - (\sigma - 1)} \right) \left(\frac{\sigma}{\alpha} \right)^{1-\frac{\theta}{\sigma-1}} \mu^{-\theta} \left(\frac{C}{1+\pi} \right) C_d^{\frac{\theta}{\sigma-1}-1} \sum_{k=1}^O \left(1 - \frac{u_{dk}}{1-i_{dk}} \right) \frac{C_k}{C} (w_k \tau_{dk})^{-\theta} F_{dk}^{1-\frac{\theta}{\sigma-1}} t_{dk}^{1-\mu\theta} \right]^{\frac{1}{1-\sigma}}. \quad (\text{A58})$$

Solving for P_d and rearranging gives,

$$P_d = \left(\frac{\theta}{\theta - (\sigma - 1)} \right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi} \right)^{-\frac{1}{\theta}} \times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \times \left(\sum_{k=1}^O \left(1 - \frac{u_{dk}}{1-i_{dk}} \right) \frac{C_k}{C} (w_k \tau_{dk})^{-\theta} F_{dk}^{-[\frac{\theta}{\sigma-1}-1]} t_{dk}^{-(\mu\theta-1)} \right)^{-\frac{1}{\theta}}. \quad (\text{A59})$$

We define,

$$\lambda_2 \equiv \left(\frac{\theta}{\theta - (\sigma - 1)} \right)^{-\frac{1}{\theta}} \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu \left(\frac{C}{1+\pi} \right)^{-\frac{1}{\theta}}, \quad (\text{A60})$$

and the ‘‘multilateral resistance term’’ as,

$$\rho_d \equiv \left(\sum_{k=1}^O \left(1 - \frac{u_{dk}}{1-i_{dk}} \right) \frac{C_k}{C} (w_k \tau_{dk})^{-\theta} F_{dk}^{-[\frac{\theta}{\sigma-1}-1]} t_{dk}^{1-\mu\theta} \right)^{-\frac{1}{\theta}}. \quad (\text{A61})$$

These definitions deliver final expression of the differentiated goods price index presented in Equation (19).

The price index in our model closely resembles the price index in Chaney (2008), equation (8). In that model, the price index is an equilibrium object in wages, GDP, iceberg and fixed entry costs, whereas in our model it is an equilibrium object in wages, total consumption expenditure, market tightness (through u_{dk} and $F_{dk}(\kappa_{dk})$), iceberg and fixed entry costs, and also tariffs.

We can also show that

$$P_{do} = \left(\frac{\theta}{\theta - \sigma + 1} \right)^{\frac{1}{1-\sigma}} \left(\frac{1 - u_{do} - i_{do}}{1 - i_{do}} \right)^{\frac{1}{1-\sigma}} (N_o^x)^{\frac{1}{1-\sigma}} (\mu w_o \tau_{do} t_{do}) \bar{\varphi}_{do}^{\frac{\sigma-\theta-1}{1-\sigma}}. \quad (\text{A62})$$

A.6 The gravity equation with search frictions

A.6.1 Deriving the gravity equation

The total amount paid by the consumers in d for imports from o have to sum up to the following three terms:

$$C_{do} = IM_{do} + \Pi_{do}^r - G_{do},$$

in which C_{do} is defined in Equation (A3), Π_{do}^r is defined in Equation (??), and G_{do} is defined in Equation (A37). Rearranging gives

$$IM_{do} = C_{do} - \Pi_{do}^r + G_{do}$$

so that

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}(\varphi) q_{do}(\varphi) dG(\varphi) \quad (\text{A63})$$

is the value of total imports.

We need to integrate over the varieties to get the total value of imports going into the domestic market, before tariffs are applied. Demand for a variety, φ , in the differentiated goods sector is given in equation (A2). Given this demand, monopolistic competition, and constant returns-to-scale production imply that producers set optimal prices according to equation (11). For notational simplicity, define $B_{do} \equiv \alpha (t_{do} \mu w_o \tau_{do})^{-\sigma} C_d P_d^{\sigma-1}$ and combine the optimal price with the demand curve to get $q_{do}(\varphi) = B_{do} \varphi^\sigma$. Evaluated at final prices, the value of sales of each variety is $p_{do}(\varphi) q_{do}(\varphi) = t_{do} \mu w_o \tau_{do} B_{do} \varphi^{\sigma-1}$ and the variable cost to produce $q_{do}(\varphi)$ units of this variety is $v_{do}(\varphi) = w_o \tau_{do} B_{do} \varphi^{\sigma-1}$. Using the negotiated price in Equation (10), the value of total imports is

$$n_{do} q_{do} = [1 - \gamma_{do}] \left(\frac{p_{do} q_{do}}{t_{do}} \right) + \gamma_{do} [v(q_{do}, w_o, \tau_{do}, \varphi) + \delta_{do}].$$

Using the functional forms assumptions from above, we obtain

$$n_{do} q_{do} = \left[\frac{\mu(1 - \gamma_{do}) + \gamma_{do}}{\mu} \right] w_o \tau_{do} \left(\frac{p_{do}}{t_{do}} \right) q_{do} + \gamma_{do} \delta_{do}.$$

We assume productivity, φ , has a Pareto distribution over $[1, +\infty)$ with cumulative density function $G[\tilde{\varphi} < \varphi] = 1 - \varphi^{-\theta}$ and probability density function $g(\varphi) = \theta \varphi^{-\theta-1}$. The Pareto parameter and the elasticity of substitution are such that $\theta > \sigma - 1$, which ensures that the moment of productivity distribution in Equation (A56) is bounded. Using this moment and substituting $n_{do} q_{do}$ into the integral gives

$$\left(\frac{\sigma - \gamma_{do}}{\sigma - 1} \right) w_o \tau_{do} B_{do} \left(\frac{\theta \bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta - \sigma + 1} \right) + \gamma_{do} \delta_{do} \bar{\varphi}_{do}^{-\theta}.$$

Substitute the export productivity threshold into this expression and simplify to get

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \left[(\sigma - \gamma_{do}) \frac{\theta}{\theta - \sigma + 1} + \gamma_{do} \frac{\delta_{do}}{F_{do}} \right] F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} A_{do}^{\frac{\theta}{\sigma-1}} t_{do}^{-\mu\theta}, \quad (\text{A64})$$

in which $A_{do} = \mu^{-\sigma} \alpha (w_o \tau_{do})^{1-\sigma} C_d P_d^{\sigma-1} [\mu - 1]$. Next, utilize the assumption that the number of producers in the origin market is proportional to output in that market (Equation

A38) to write

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left(\frac{C}{1 + \pi}\right) \frac{C_o}{C} \left[(\sigma - \gamma_{do}) \frac{\theta}{\theta - \sigma + 1} + \gamma_{do} \frac{\delta_{do}}{F_{do}} \right] \\ \times F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} (\mu^{-\sigma} \alpha (w_o \tau_{do})^{1-\sigma} C_d P_d^{\sigma-1} [\mu - 1])^{\frac{\theta}{\sigma-1}} t_{do}^{-\mu\theta}.$$

Substituting in for the price index using Equation 19 gives

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left[(\sigma - \gamma_{do}) \frac{\theta}{\theta - \sigma + 1} + \gamma_{do} \frac{\delta_{do}}{F_{do}} \right] \\ \times (\mu^{-\sigma} \alpha [\mu - 1])^{\frac{\theta}{\sigma-1}} \left(\frac{C}{1 + \pi}\right) \lambda_2^\theta \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} t_{do}^{-\mu\theta}.$$

Substitute in for λ_2 using Equation (A60), to get

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left(1 - \frac{\gamma_{do}}{\sigma\theta} \left(\theta - \frac{\delta_{do}}{F_{do}} (\theta - (\sigma - 1))\right)\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} t_{do}^{-\mu\theta}. \quad (\text{A65})$$

Define the bundle of search parameters

$$b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}) = \frac{\gamma_{do}}{\sigma\theta} \left(\theta - \frac{\delta_{do}}{F_{do}} (\theta - (\sigma - 1))\right) \quad (\text{A66})$$

and substitute it into (A65) in order to write the gravity equation more compactly as

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) (1 - b(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do})) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} t_{do}^{-\mu\theta},$$

which is equation (20).

A.6.2 Consumption is after-tariff imports evaluated at final sales prices

The value of consumption of the differentiated good in the do market is defined as the integral over all varieties, ω , of the value of $q_{do}(\omega)$ units evaluated at final sales prices, $p_{do}(\omega)$, as shown in Equation (A3). After moving from an unordered set of varieties to an index over a distribution of productivities this consumption is given by:

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} p_{do}(\varphi) q_{do}(\varphi) dG(\varphi). \quad (\text{A67})$$

To evaluate this integral, notice that the value of imports in Equation (A63) is a similar expression, but integrates $n_{do}(\varphi) q_{do}(\varphi)$ rather than $p_{do}(\varphi) q_{do}(\varphi)$. From equation (10), $p_{do}(\varphi) = t_{do} n_{do}(\varphi)$ if $\gamma_{do} = 0$. As such, to evaluate the right side of Equation (A67) we can set $\gamma_{do} = 0$ in equation (A65) and multiply by t_{do} , which gives

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \alpha \left(\frac{C_o C_d}{C}\right) \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} t_{do}^{1-\mu\theta}. \quad (\text{A68})$$

A.6.3 Government expenditure

Government expenditure is defined in Equation (A37) and can be written as

$$G_{do} = (1 - t_{do}) IM_{do},$$

in which IM_{do} is defined in Equation (20). XXXX Add relationship to consumption as well
XXXX

A.7 Steady-state general equilibrium

A.7.1 Defining the equilibrium

The equilibrium reduces to the following equations in the equilibrium variables.

1. The free entry condition for retailers (Equation 7):

$$\frac{c_{do}}{\chi(\kappa_{do})} = \int_{\bar{\varphi}_{do}} M_{do}(\varphi) dG(\varphi), \quad (\text{A69})$$

when market tightness, κ_{do} , is not directly chosen by a social planner, as discussed in Section 3.1. Notice that there are d times o markets and each market has an associated tightness. With our functional form assumptions, this equation can be simplified.

Remember that with $V_{do} = 0$

$$M_{do}(\varphi) = \frac{p_{do}q_{do} - t_{do}n_{do}q_{do}}{r + \lambda}$$

so that

$$\begin{aligned} \int_{\bar{\varphi}_{do}} M_{do}(\varphi) dG(\varphi) &= \left(\frac{1}{r + \lambda} \right) \int_{\bar{\varphi}_{do}} p_{do}(\varphi) q_{do}(\varphi) - t_{do}n_{do}(\varphi) q_{do}(\varphi) dG(\varphi) \\ &= \left(\frac{1}{r + \lambda} \right) \left(1 - \frac{u_{do}}{1 - i_{do}} \right)^{-1} \left(\frac{1}{N_o^x} \right) \Pi_{do}^r, \end{aligned}$$

in which Π_{do}^r is defined in Equation (??). Plugging this expression into Equation (A69) gives

$$\kappa_{do} = \left(\frac{1}{r + \lambda} \right) (\lambda + \kappa_{do}\chi(\kappa_{do})) \frac{(1 + \pi)}{c_{do}C_o} \Pi_{do}^r, \quad (\text{A70})$$

in which we used $N_o^x = \frac{1}{1 + \pi} C_o$, $\chi(\kappa_{do}) = \xi \kappa_{do}^{-\eta}$, and

$[1 - u_{do}/(1 - i_{do})]^{-1} = (\lambda + \kappa_{do}\chi(\kappa_{do})/\kappa_{do}\chi(\kappa_{do}))$. An analogous expression to that in Equation (A43) for the do market provides an expression for Π_{do}^r and π is defined in Equation (18).

2. The expression that equates variable profits with the effective entry cost, which pins down $\bar{\varphi}_{do}$ (Equation 14):

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha} \right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d} \right) \left(\frac{F(\kappa_{do})}{C_d} \right)^{\frac{1}{\sigma-1}} t_{do}^\mu,$$

in which $F(\kappa_{do})$ is defined in Equation (13), w_o is defined in Equation (17), and P_d is defined in Equation (19).

3. National accounting/consumer's budget constraint pins down consumption C_d (Equation 16):

$$C_d = Y_d - I_d - G_d,$$

in which I_d and G_d are defined in Equation (16) and $Y_d = w_d L_d (1 + \pi)$, in which w_d is defined in Equation (17) and π is defined in Equation (18).

4. Labor market clearing pins down w_d (Equation 17):

$$w_d = \frac{I_d + (1 - \alpha) C_d + \frac{1}{\mu} \sum_o \frac{C_{od}}{t_{od}}}{L_d},$$

in which I_d is defined in Equation (16) and C_{od} is defined in Equation (A68).

5. The global mutual fund pins down π (Equation 18):

$$\pi = \frac{\Pi}{\sum_d w_d L_d},$$

in which w_d is defined in Equation (17) and Π is defined in Equation (18).

A.7.2 Nesting trade models without search frictions

Our model nests trade models without search frictions if retailers' search costs are zero, $c_{do} = 0, \forall do$, among other restrictions. The main difference between our model's equilibrium definition and the definitions in trade models without search is that we introduce market tightness, k_{do} . When search costs are zero, free entry into product vacancies leads to infinite market tightness and instantaneous matching for producers. Instantaneous matching implies that all producers are matched (equation 15), as in a standard trade model without search frictions.

In particular, our model exactly reproduces Chaney (2008) if retailers' search costs are zero, we make the same assumptions about the homogeneous good as he does (so that $w_d = 1, \forall d$), there are no tariffs ($G_d = 0 \forall d$), and we make the same parameter value restrictions that he does ($t_{do} = s_{do} = h_{do} = e_d^x = 0, \forall d$, and, $\forall o$). We demonstrate this equivalence by showing that all equilibrium equations are the same. If retailers' search costs are zero, market tightness is infinite and the negotiated price (Equation 10) attains the final sales price if $t_{do} = 1, \forall do$, which is given by equation (11). There is, in effect, no intermediate retailer; producers sell their goods directly to the final consumer at price p_{do} . Instant contacts for producers imply that the effective entry cost (equation 13) equals the fixed cost of production, $F_{do} = f_{do}$, and our threshold productivity expression (equation 14) coincides with Chaney (2008, equation 7). With no search costs and $G_d = 0$, Equation (16) implies that $Y_d = C_d + I_d$ and the only investment expenditure is the fixed cost of production. Total income is still given by $Y_d = (1 + \pi) w_d L_d$, which also matches Chaney (2008, equation 9). Assumptions about the homogeneous good as in Chaney (2008) would imply that $w_d = 1 \forall d$ and the per-capita dividend is determined by equation (18) (as shown

in [Krolikowski and McCallum, 2021](#)). With the same equations defining the equilibrium variables, our ideal price index (equation 19) and gravity equation (equation 20) would coincide with equations 8 and 10, respectively, in [Chaney \(2008\)](#).

A.8 Different search market structures

This appendix accompanies section 3.

A.8.1 Deriving aggregate welfare

Here we outline the steps to show that the indirect utility function (welfare) is C_d/Ξ_d , in which C_d is total consumption expenditure, p is the vector of prices for each good, and Ξ_d is the ideal price index. Assume that preferences are homothetic, which is defined in [Mas-Colell, Whinston, and Green \(1995\)](#), section 3.B.6, page 45. This means that they can be represented by a utility function that is homogeneous of degree one in quantities and that the corresponding indirect utility function is linear in total consumption expenditure. We can begin with the indirect utility function and then manipulate it as follows

$$\begin{aligned} W_d(p, C_d) &= W_d(p, 1) C_d \\ W_d(p, e(p, u)) &= W_d(p, 1) e(p, u) \\ u &= W_d(p, 1) e(p, u) \\ 1 &= W_d(p, 1) e(p, 1) \\ \frac{1}{e(p, 1)} &= W_d(p, 1), \end{aligned}$$

in which the first line comes from homothetic preferences; the second line follows by plugging in for consumption expenditure $C_d = e(p, u)$; the third line comes from equation (3.E.1) in MWG that says $W_d(p, e(p, u)) = u$ (also known as duality); and in the fourth line we plug in for utility level $u = 1$. The function $e(p, u)$ is the consumption expenditure function that solves the expenditure minimization problem. Using this result and the fact that the price index is defined as $e(p, 1) \equiv \Xi_d$ we can show that

$$W_d(p, C_d) = W_d(p, 1) C_d = \frac{1}{e(p, 1)} C_d = \frac{C_d}{\Xi_d}.$$

Hence, as long as preferences are homothetic, we will always get welfare equal to consumption expenditure divided by the price index, $W_d(p, Y) = C_d/\Xi_d$. The expenditure approach to accounting can be particularly useful for computing aggregate welfare in this setting because, $W_d(p, C_d) = \frac{C_d}{\Xi_d} = \frac{Y - I_d - G_d}{\Xi_d}$.

A.8.2 Proof of proposition 1

We show that there exists a bargaining parameter for each market such that the decentralized equilibrium coincides with the social planner's solution. In particular, we show that there exist β_{do}^c for all do such that the market tightnesses implied by the free entry condition in the competitive equilibrium, κ_{do}^c in Eq. (22b), equals the efficient market tightnesses, κ_{do}^e in Eq. (24), when we assume that $l_{do} = -h_{do}$ and $s_{do} = 0$ and the other endogenous variables are at their efficient values.

Assuming $l_{do} = -h_{do}$ and $s_{do} = 0$ implies that $F(\kappa_{do}) = f_{do} + h_{do}$ is not a function of β_{do} and implies that only for an arbitrary jk market does Eq. (22b) depend directly on β_{jk} . All

other $do \neq jk$ markets for Eqs. (22b), and Eqs. (22c) through (22f) for all markets, depend on β_{jk} but only through the values of endogenous variables.

Denote efficient market tightness in an arbitrary jk market as κ_{jk}^e and collect all market tightnesses for markets other than jk in κ_{-jk}^e so that we can define the efficient equilibrium as $\{\kappa^e, \bar{\varphi}^e, \vec{C}^e, \vec{w}^e, \pi^e\} = \{\kappa_{jk}^e, \kappa_{-jk}^e, \bar{\varphi}^e, \vec{C}^e, \vec{w}^e, \pi^e\}$.

Next we show that there exists a bargaining parameter for an arbitrary jk market β_{jk}^c , such that κ_{jk}^c implied by the free entry condition, Eq. (22b) in that jk market can attain any value, including the efficient market tightness, $\kappa_{jk}^c = \kappa_{jk}^e$ when all the other endogenous variables are also at their efficient levels.

A.9 Market tightness and the cost of search

Let's first prove that $\kappa_{do} < \infty$ if $c_{do} > 0$. To do this, let's prove the contrapositive: assume that $c_{do} = 0$ and show that $\kappa_{do} = \infty$. Rearrange equation (A69) slightly to get

$$0 = c_{do} = \chi(\kappa_{do}) \int_{\bar{\varphi}_{do}} M_{do}(\varphi) dG(\varphi).$$

We have shown that $M_{do}(\bar{\varphi}_{do}) \geq 0$ for any consummated match in equilibrium (Nash bargaining together with appendix A.3.4.2) and $M'_{do}(\varphi) > 0$ (appendix A.3.4.3). Therefore we know that $\int_{\bar{\varphi}_{do}} M_{do}(\varphi) dG(\varphi) > 0$. Thus, $\chi(\kappa_{do})$ must be zero. Because $\chi'(\kappa_{do}) < 0$ this is true if and only if $\kappa_{do} = \infty$.

To prove that if $c_{do} > 0$ then $\kappa_{do} < \infty$, let's use equation (A69) again. In particular, because $c_{do} > 0$ it must mean that $\chi(\kappa_{do}) \int_{\bar{\varphi}_{do}} M_{do}(\varphi) dG(\varphi) > 0$. As before, we know that $\int_{\bar{\varphi}_{do}} M_{do}(\varphi) dG(\varphi) > 0$ so it must be that $\chi(\kappa_{do}) > 0$ as well, which is true if and only if $\kappa_{do} < \infty$.