Federal Reserve Bank of Cleveland Working Paper Series

Tariffs and Goods-Market Search Frictions

Pawel M. Krolikowski and Andrew H. McCallum

Working Paper No. 25-03

January 2025

Suggested citation: Krolikowski, Pawel M. and Andrew H. McCallum. 2025. "Tariffs and Goods-Market Search Frictions." Working Paper No. 25-03. Federal Reserve Bank of Cleveland. https://doi.org/10.26509/frbc-wp-202503.

Federal Reserve Bank of Cleveland Working Paper Series
ISSN: 2573-7953

Tariffs and Goods-Market Search Frictions *

Pawel M. Krolikowski † Andrew H. McCallum ‡

January 13, 2025

Abstract

We study uniform tariffs in a general equilibrium dynamic model with search frictions between heterogeneous exporting producers and importing retailers. We analytically characterize unilateral import tariffs that maximize domestic welfare. Search frictions lower these tariffs because of market thickness effects, which reinforce aggregate production nonconvexities. A calibration using 2016 U.S. and Chinese data suggests that optimal U.S. unilateral and Nash equilibrium tariffs with baseline search frictions are 10 ppt. below those in a model with reduced search frictions. Changes in welfare in response to changes in tariffs are smaller in the model with baseline search frictions than in the model with reduced frictions. In the Nash equilibrium with baseline search frictions, U.S. (Chinese) tariffs are 17 (8) ppt. higher and welfare is 0.1 (0.9) percent lower relative to 2016 tariff levels.

JEL codes: C78, D62, D83, F12, F13.

Keywords: Optimal tariffs, trade policy, efficiency, search, welfare, social planner.

*We thank Dave Donaldson, Lukasz Drozd (discussant), Doireann Fitzgerald (editor), Andrei Levchenko, and Robert Staiger for helpful advice. We also thank seminar participants at two Board International Finance workshops, a Federal Reserve Bank of Cleveland workshop, the 2023 International Institute of Public Finance, and the 2023 International Industrial Organization Conference. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors, the Federal Reserve Bank of Cleveland, or the Federal Reserve System.

†pawel.krolikowski@clev.frb.org, Federal Reserve Bank of Cleveland, 1455 E 6th St, Cleveland, OH 44114

‡Corresponding author, andrew.h.mccallum@frb.gov, Board of Governors of the Federal Reserve System, Washington, DC 20551

1 Introduction

We know that trade policy can have large effects on welfare (Broda, Limao, and Weinstein, 2008; Ossa, 2011; Costinot and Rodríguez-Clare, 2014). For example, Ossa (2014) finds that unilateral optimal tariffs can raise U.S. welfare by 2.3 percent. We also know that building connections with overseas buyers is a prevalent firm-level search friction faced by exporters (Kneller and Pisu, 2011; Brancaccio, Kalouptsidi, and Papageorgiou, 2020; Krolikowski and McCallum, 2021; Eaton, Jinkins, Tybout, and Xu, 2022). Despite these important facts about trade, we know little about how trade policy interacts with international search frictions.

We study uniform tariffs in a general equilibrium dynamic model with search frictions between heterogeneous exporting producers and importing retailers. We characterize unilateral import tariffs that maximize domestic welfare when other countries follow passive policies. Search frictions introduce a new incentive to subsidize imports because of market thickness effects, which contribute to aggregate nonconvexities in production. Naturally, these incentives strengthen when unmatched rates are high and with the matching function's responsiveness to the number of searching retailers. Quantitative results using 2016 U.S. and Chinese data suggest that the optimal U.S. unilateral tariff with search frictions is about 10 percentage points below that in a model with lower search frictions. Changes in welfare in response to changes in tariffs are smaller in the model with baseline search frictions than in the model with reduced frictions. In the Nash equilibrium with baseline search frictions, U.S. (Chinese) tariffs are 17 (8) ppt. higher and welfare is 0.1 (0.9) percent lower relative to 2016 tariff levels.

We study optimal import tariffs in the model of Krolikowski and McCallum (2021), hereafter KM. Our main modeling contribution relative to KM is the explicit inclusion of import tariffs. KM's model is a Melitz-style general equilibrium dynamic model with goods-market frictions between importing retailers in destination country d and heterogeneous exporting producers in origin country o. We focus on an analytically tractable model with random search and Nash bargaining, as in Pissarides (2000, Ch. 1). In the steady state of this model, an endogenous fraction of exporters are actively looking for importing partners but are unmatched. These unmatched exporters alter the *levels* of aggregate variables and the *changes* in aggregate variables in response to shocks because when producers are unmatched their associated varieties cannot be traded. Aside from the goods-market frictions, our model nests Melitz (2003) and Chaney (2008).

Search frictions lead to a new source of aggregate nonconvexity in the production possibility frontier (PPF). Specifically, increasing country o's exports lowers search frictions in this market because of increased retailer entry and leads to a higher matched rate. The higher matched rate lowers the opportunity cost of exports. At the same time, increasing exports causes substitution away from local goods. This substitution lowers retailer entry and the local matched rate, and increases the cost of producing local goods. Both effects imply increasing returns to scale in the PPF. Naturally, these search-friction effects are larger when unmatched rates in the export (do) and local (oo) markets are high, and when the number of matches is more responsive to the number of searching retailers.

The production nonconvexity introduced by search frictions is similar to the one introduced by selection in Melitz (2003), as emphasized by Costinot, Rodríguez-Clare, and Werning (2020), henceforth CRW. In that model, as exports from country o rise, producer entry rises in this market, and local goods production falls, which lowers producer entry in the local market. Both of these effects lower the opportunity cost of exports in terms of local goods, which gives rise to increasing returns to scale. Nevertheless, we highlight that even without self-selection into production, search frictions deliver an aggregate nonconvexity in an economy with one industry This effect contrasts with the increasing returns to scale in Krugman (1980), whose model requires multiple industries.

This nonconvexity creates an incentive for country d to subsidize imports from country o, in addition to the selection effect. We characterize the optimal uniform tariff that maximizes domestic welfare when other countries follow passive trade policies. We show that this optimal tariff is lower with larger nonconvexities. Therefore, even if firms were homogeneous and there were no fixed exporting costs (as in Gros, 1987), the country social planner might choose to subsidize imports because of search frictions. International search frictions also increase the local consumption share, and this further reduces the optimal tariff relative to a model without search frictions. These results extend those in CRW to an environment that includes goods-market search frictions between importers and exporters.

We present numerical examples under the conditions required by our analytical results to help build intuition. We find that the optimal uniform tariff expression in CRW works well, even with search frictions, once we account for the effects of these frictions on the production nonconvexity and the local consumption share. Using this approximation, the optimal tariff in a model with search frictions is below that in a model without search frictions. The numerical examples suggest that setting tariffs according to a model without search frictions reduces U.S. welfare relative to the optimal prescription in a model with search frictions. The numerical examples also suggest that the optimal tariff in a model with any positive search frictions is below that in the model without them and that most of the decline in the optimal tariff occurs at relatively low levels of search frictions. Finally, the numerical examples suggest that about two-thirds of the difference in the optimal tariff between the models with and without search frictions is accounted for by the higher production nonconvexities in the search model. A higher local consumption share in this model accounts for most of the rest of the optimal tariff difference.

To obtain a quantitatively realistic environment, we use the approach in KM to simultaneously recover parameters of the model and solve for the accompanying equilibrium endogenous variables to match U.S. and Chinese data in 2016. These data include economic aggregates and trading partner separation rates, among other measures. To calibrate importing retailers' search costs, we use the fraction of U.S. (Chinese) firms exporting to China (the US), similar to Armenter and Koren (2014), Eaton, Eslava, Jinkins, Krizan, and Tybout (2014), and Eaton, Jinkins, Tybout, and Xu (2016). To calibrate domestic retailers' search costs, we use manufacturing capacity utilization rates in each country, as in Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2017), and Petrosky-Nadeau, Wasmer, and Weil (2021). As a whole, the calibration matches the data well and delivers a realistic economic environment for the United States and China.

In this calibration, search frictions affect optimal tariffs and welfare. The U.S. unilateral optimal tariff in a model with search frictions is about 10 percentage points below that in a model with international search frictions reduced to domestic levels (''reduced search frictions''). For both the United States and China, the respective optimal unilateral tariff with baseline search frictions would increase welfare by 0.1 percent relative to 2016 tariff levels. We also solve for the Nash equilibrium in the model with baseline and reduced search frictions. The optimal U.S. Nash tariff in the model with baseline search frictions is 1.25, below the optimal tariff with reduced search frictions, 1.35. In the Nash equilibrium of the model with baseline frictions, U.S. (Chinese) welfare is lower by 0.1 (0.9) percent. Changes in welfare in response to changes in tariffs are smaller in the model with baseline search frictions than in the model with reduced frictions, echoing the attenuating effect of search frictions on welfare discussed in KM.

Search frictions in international goods markets are motivated by direct evidence. For example, Kneller and Pisu (2011) find that ''identifying the first contact'' and ''establishing initial dialogue'' are more common obstacles to exporting than ''dealing with legal, financial and tax regulations overseas'' in a survey of UK firms. The broad relevance of search frictions is also motivated by a variety of contexts beyond labor markets. For example, Wasmer and Weil (2004) study search frictions in credit markets, Lagos and Wright (2005) study search frictions in monetary economics, and Piazzesi, Schneider, and Stroebel (2020) study search frictions in housing markets. Search frictions have also been studied in the contexts of marriage markets (Smith, 2006), insurance markets (Cebul, Rebitzer, Taylor, and Votruba, 2011), and goods markets (Drozd and Nosal, 2012), among others.

We also compare our results to past studies of tariff policy and efficiency in settings without and with search frictions. Particularly relevant is work by Demidova and Rodríguez-Clare (2009). That paper extends the optimal-tariff results in Gros (1987) to a small country Melitz model. Both models are special cases of the model studied in

Felbermayr, Jung, and Larch (2013), which characterizes optimal tariffs in cooperative and noncooperative games for two large countries with heterogeneous producers. CRW generalize these results beyond homogeneous firms and Pareto-distributed productivity, and to tariffs that vary with firm productivity. We use an approach similar to theirs to characterize the optimal import tariff in our model with search frictions. Finally, Brancaccio, Kalouptsidi, Papageorgiou, and Rosaia (2023) study efficiency in markets with search but focus on the international transportation sector.

The remainder of the paper is structured as follows. Section 2 outlines the model and its solution. Section 3 studies the country's social planner's problem and presents analytic results and numerical examples. Section 4 presents our calibration. Section 5 provides quantitative exercises. Section 6 concludes.

2 The model, aggregation, and steady-state equilibrium

2.1 Model

We use an extension of the continuous-time model of KM and outline it here, with additional details and equations included in Appendix A. The model features D countries. We index importing countries with d (destination) in the first index position and exporting countries with o (origin) in the second index. For example, imports by d from o are denoted IM_{do}. We allow for search frictions between producers and retailers in domestic and international goods markets, and we focus on the steady-state implications.

2.1.1 Consumers

A representative consumer in destination market d has Cobb-Douglas utility, U_d, over a homogeneous good, $q_d(1)$, and a second good that is a constant elasticity of substitution (CES) aggregate of differentiated varieties, $q_{do}(\omega)$, from all origins. The two goods are combined with exponents $1-\alpha$ and α, respectively. The differentiated goods are substitutable with constant elasticity, $\sigma > 1$, across varieties and destinations. Formally the consumer's problem is

$$\begin{aligned} \max_{q_d(1), q_{dk}(\omega)} \quad & q_d(1)^{1-\alpha} \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} q_{dk}(\omega)^{\left(\frac{\sigma-1}{\sigma}\right)} d\omega \right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)} \qquad (1) \\ s.t. \; C_d \; = \; & p_d(1) q_d(1) + \sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega) q_{dk}(\omega) d\omega, \end{aligned}$$

which results in the following demand for the homogeneous good and each differentiated variety, respectively

$$q_d(1) = \frac{(1-\alpha) C_d}{p_d(1)}, \qquad q_{do}(\omega) = \alpha C_d \frac{p_{do}(\omega)^{-\sigma}}{P_d^{1-\sigma}}. \tag{2}$$

We denote the value of total consumption as C_d in destination country d. For prices paid by final consumers, we denote the value of consumption of the differentiated good to destination d from origin o as C_{do}. The homogeneous good has price $p_d(1)$. Define P_d as the price index for the bundle of differentiated varieties and P_{do} as the price index for the bundle of varieties produced in country o and consumed in country d, which have price $p_{do}(\omega)$. The ideal price index, defined as Ξ_d, combines $p_d(1)$ and P_d. Details about the ideal price index, Ξ_d, and differentiated goods consumption in the do market, C_{do}, are in Appendix A.1 and we discuss the price index more in Section 2.3.5.

We have written a static consumer problem so that our model nests Melitz (2003) and Chaney (2008), but we present a dynamic problem for producers and retailers in Sections 2.1.3 and 2.1.4. To reconcile the two approaches, we can assume that the consumer is perfectly patient so that solving the static problem each instant solves their dynamic problem. A perfectly patient consumer would imply that the equilibrium interest rate is zero, $r = 0$, which is a feasible value in the retailer and producer problems. Because it is our main focus, we follow many search models and do not endogenize the interest rate.

It is convenient to express the subutility from consuming differentiated goods in destination d from origin o as

$$Q_{do} = \left[\int_{\omega \in \Omega_{do}} q_{do}(\omega)^{1/\mu} \, d\omega\right]^{\mu}, \tag{3}$$

in which $\mu = \sigma/(\sigma-1)$. Therefore, the utility of the representative consumer can be written as $U_d = q_d(1)^{(1-\alpha)} \left[Q_{dd}^{1/\mu} + Q_{do}^{1/\mu}\right]^{\alpha\mu}$, in which Q_{dd} and Q_{do} are the utility from consuming local and imported differentiated goods, respectively.

2.1.2 The matching function

A costly process of search governs how producers and retailers find one another, similar to that in Diamond (1982), Pissarides (1985), and Mortensen (1986). We assume that the flow number of relationships formed at any moment in time between searching retailers and producers is determined by a Cobb-Douglas matching function, with matching efficiency ξ and elasticity with respect to the number of searching producers η. As a result, market tightness—the ratio of the mass of searching retailers to the mass of searching producers, which we denote, $\kappa_{do} = v_{do}N_d^m / u_{do}N_o^x$—is sufficient to determine contact rates on both sides of each do search market. The Poisson rate at which retailers in country d contact producers in country o is given on the left, and the contact rate for producers is given on the right:

$$\chi(\kappa_{do}) = \xi \kappa_{do}^{-\eta}, \qquad \kappa_{do}\chi(\kappa_{do}) = \xi \kappa_{do}^{1-\eta}. \tag{4}$$

Only the number of vacancies matters in our model, not the number of retailers. Vacancies can originate from one retailing firm posting all vacancies, all retailers posting one vacancy each, or anything in between. Therefore, we interpret matches as one retailer to one producer, as in Pissarides (2000), and we refer to vacancies and retailers interchangeably. Details about the matching function are in Appendix A.2.1, with details about continuous time Poisson processes in Appendix A.2 of KM.

2.1.3 Producers

We assume that the homogeneous good is produced with one unit of labor under constant returns to scale in each country. So, the price of the homogeneous good must equal the wage in each country, $p_d(1) = w_d, \forall d$.

We index differentiated goods producers by their permanent productivity, φ. We assume this productivity is exogenous and has the same distribution in all countries: Pareto with cumulative distribution function $G(\varphi) = 1 - \varphi^{-\theta}$ so that $\varphi = 1$ is the minimum possible value of productivity. We assume that $\theta > \sigma - 1$.

There are two production costs for differentiated goods. First, producers face a variable cost indexed by productivity

$$v(q_{do}, w_o, \tau_{do}, \varphi) = q_{do} w_o \tau_{do} \varphi^{-1}. \tag{5}$$

This variable cost function implies a constant-returns-to-scale production function in which labor is the only input. w_o is the wage in the exporting (origin) country; $\tau_{do} \geq 1$ is an iceberg cost such that one unit of the differentiated good arrives in destination d when τ_{do} units are sent from origin o and $\tau_{do} - 1$ units are lost to physical destruction; and q_{do} is the amount traded. Second, producers face a fixed cost of production, $w_o f_{do}$, in which f_{do} is in labor units, so that the total production cost is $v(q_{do}, w_o, \tau_{do}, \varphi) + w_o f_{do}$.

At any instant in time, each producer is in one of three mutually exclusive states. First, the producer could be matched with a retailer with value $X_{do}(\varphi)$ defined by

$$rX_{do}(\varphi) = n_{do}q_{do} - v(q_{do}, w_o, \tau_{do}, \varphi) - w_o f_{do} + \lambda (U_{do}(\varphi) - X_{do}(\varphi)). \tag{6}$$

In this state, the flow payoff is the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable, $v(q_{do}, w_o, \tau_{do}, \varphi)$, and fixed cost of production, $w_o f_{do}$. The negotiated price, n_{do}, and the quantity traded, q_{do}, are determined through a bargaining process that we describe in Section 2.2. Matches end exogenously at rate λ, which leads to a capital loss as the producer becomes unmatched and the future is discounted at rate r.

Second, the producer could be unmatched but searching with value $U_{do}(\varphi)$ defined by

$$rU_{do}(\varphi) = -w_o l_{do} + \kappa_{do}\chi(\kappa_{do})(X_{do}(\varphi) - U_{do}(\varphi) - w_o s_{do}). \tag{7}$$

The producer pays a flow cost, $w_o l_{do}$, to generate contacts with retailers. At endogenous Poisson rate $\kappa_{do}\chi(\kappa_{do})$ the producer contacts a retailer and becomes matched, after paying the sunk cost, $w_o s_{do}$, of starting up the relationship.

Third, producers have the option of remaining idle and not expending resources to look for a retailer with value $I_{do}(\varphi)$ defined by

$$rI_{do}(\varphi) = w_o h_{do}. \tag{8}$$

Idle producers receive a constant flow payoff, $w_o h_{do}$. We include an idle state because without it, all producers would search in all markets, even if they expect to reject all contacts. Allowing producers to optimally choose not to search in each market is both more general and more intuitive. Appendix A.2.2 has more details about the producers' value functions.

2.1.4 Retailers

Each retailer is in one of two states. First, the retailer could be matched with a producer and receive value $M_{do}(\varphi)$ defined by,

$$rM_{do}(\varphi) = p_{do}q_{do} - t_{do}n_{do}q_{do} + \lambda(V_{do} - M_{do}(\varphi)). \tag{9}$$

In this state, the flow payoff is the revenue, $p_{do}q_{do}$, generated by selling q_{do} units of the differentiated good at a final sales price, p_{do}, paid by the consumer less the tariff-inclusive cost of acquiring these goods, $t_{do}n_{do}q_{do}$. The retailer pays the *ad valorem* tariff, t_{do}, on the imported value, $n_{do}q_{do}$, to the government. The tariff creates a potential wedge between producer revenue, $n_{do}q_{do}$, in Eq. (6) and retailer cost, $t_{do}n_{do}q_{do}$ in Eq. (9). Tariff revenues are rebated in a lump sum from the government to consumers in the destination country as discussed in Section 2.3. When the relationship is destroyed exogenously, at rate λ, the retailing firm loses the capital value of being matched. All retailers are identical before matching but have differential matched values because producers are heterogeneous in their productivity.

Second, a retailer could be unmatched with value V_{do} defined by

$$rV_{do} = -w_d c_{do} + \chi\left(\kappa_{do}\right) \int \left[\max\left\{V_{do}, M_{do}\left(\varphi\right)\right\} - V_{do}\right] dG\left(\varphi\right). \tag{10}$$

The flow search cost, $w_d c_{do}$, generates the search friction between producers and retailers. At endogenous Poisson rate $\chi(\kappa_{do})$, retailing firms meet a producer and, before consummating a match, learn the productivity of the producer. Retailers then choose between matching with that producer or continuing to search. Because they are uncertain about the productivity of the producer they might meet, retailers take the expectation over all productivities they might encounter when computing their continuation value of searching. There is an unbounded mass of potential retailers that could decide to search. Appendix A.2.3 has more details about the retailers' value functions.

2.2 Solving the partial-equilibrium search problem

Retailing and producing firms use backward induction to maximize their value. The second-stage solution results from jointly Nash bargaining over negotiated price, n_{do}, and quantity, q_{do}, after a retailer and producer meet. In the first stage, retailers and producers—taking the solution to the second-stage bargaining problem as given—choose whether to search for a business partner, or to remain idle. Appendix A.3 solves the search problem in detail.

2.2.1 Match surplus

Define the total private surplus as the value of the relationship to the retailer and the producer less their outside options:

$$S_{do}\left(\varphi\right) = X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) + M_{do}\left(\varphi\right) - V_{do}. \tag{11}$$

Importantly, $S_{do}\left(\varphi\right)$ excludes the government's value of collecting tariffs from each match and the government is passive during bargaining. Bargaining over quantity, q_{do}, will maximize total private surplus and bargaining over price, n_{do}, will divide the surplus between the producer and retailer. Appendix A.3.1 derives the surplus in terms of appropriately discounted profits. That appendix also derives the value of a relationship and discusses the expected duration of matches.

2.2.2 Bargaining over the negotiated price

Bargaining over the negotiated price, n_{do}, will divide the private surplus, $S_{do}\left(\varphi\right)$, between producers and retailers according to the "surplus sharing rule," which is:

$$X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) = \frac{\beta S_{do}\left(\varphi\right)}{\beta + t_{do}\left(1-\beta\right)}, \qquad M_{do}\left(\varphi\right) - V_{do} = \frac{\left(1-\beta\right) t_{do} S_{do}\left(\varphi\right)}{\beta + t_{do}\left(1-\beta\right)}. \tag{12}$$

in which β is producers' bargaining power. Eq. (12) nests the sharing rule in KM (Eq. 13) when $t_{do} = 1$. In addition, as the tariff rises, retailers receive a larger fraction of the surplus to account for their increased import costs: As $t_{do} \to \infty$, the fraction of the surplus received by retailers approaches 1.

The negotiated price that splits the surplus according to Eq. (12) when we assume free entry into retailer vacancies, $V_{do} = 0$, is

$$n_{do} = (1 - \gamma_{do}) \left(\frac{p_{do}}{t_{do}} \right) + \gamma_{do} \left(\frac{v\left(q_{do}, w_o, \tau_{do}, \varphi\right) + w_o f_{do} - w_o l_{do} - \kappa_{do} \chi\left(\kappa_{do}\right) w_o s_{do}}{q_{do}} \right), \quad (13)$$

in which $\gamma_{do} \equiv \dfrac{(r+\lambda)(1-\beta)}{r+\lambda+\beta\kappa_{do}\chi\left(\kappa_{do}\right)} \in [0,1]$. The equilibrium negotiated price, n_{do}, is a convex combination of the tariff-adjusted final sales price, p_{do}/t_{do}, and the average total production cost less producers' search costs. Appendix A.3.2 discusses bargaining over price in detail.

2.2.3 Bargaining over quantity

Bargaining over quantity implies that the quantity exchanged within matches equates the marginal revenue obtained by retailers from consumers with the marginal production cost inclusive of tariffs. Our assumptions about the utility and variable cost functions result in an equivalent definition for negotiated quantity in terms of the final consumer being a markup over marginal production and tariff costs:

$$p_{do}\left(\varphi\right) = t_{do} \mu w_o \tau_{do} \varphi^{-1}. \quad (14)$$

Negotiated quantity is obtained by substituting Eq. (14) into the demand curve, Eq. (2). Appendix A.3.3 discusses bargaining over quantity in detail.

2.2.4 Producers' search productivity threshold

In the first stage, producers, taking the solution to this second-stage bargaining problem from Eqs. (13) and (14) as given, choose whether to search for a business partner or to remain idle. Therefore, a zero-value condition, $U_{do}\left(\bar{\varphi}_{do}\right) - I_{do}\left(\bar{\varphi}_{do}\right) = 0$, which can be written as

$$\left(\frac{p_{do}\left(\bar{\varphi}_{do}\right)}{t_{do}} \right) q_{do}\left(\bar{\varphi}_{do}\right) - v\left(q_{do}\left(\bar{\varphi}_{do}\right), w_o, \tau_{do}, \bar{\varphi}_{do}\right) = F\left(\kappa_{do}\right), \quad (15)$$

determines the producers' minimum productivity threshold, $\bar{\varphi}_{do}$, that makes searching worthwhile. Eq. (15) equates tariff-adjusted variable profits from the match with the "effective entry cost." The latter is defined as

$$F\left(\kappa_{do}\right) \equiv w_o f_{do} + \left(\frac{r+\lambda}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)} \right) w_o l_{do} + \left(1 + \frac{r+\lambda}{\beta\kappa_{do}\chi\left(\kappa_{do}\right)} \right) w_o h_{do} + \left(\frac{r+\lambda}{\beta} \right) w_o s_{do}, \quad (16)$$

which is the sum of the fixed cost of production, $w_o f_{do}$, and the (appropriately discounted) flow cost of searching for a retailer, $w_o l_{do}$, the opportunity cost of remaining idle, $w_o h_{do}$, and the sunk cost of starting up a relationship, $w_o s_{do}$.

Solve Eq. (15) using our functional forms to get the threshold explicitly as

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d}\right) \left(\frac{F(\kappa_{do})}{C_d}\right)^{\frac{1}{\sigma-1}} t_{do}^{\mu}. \tag{17}$$

A detailed discussion of the threshold productivity is in Appendix A.3.4.

2.2.5 Retailer free entry and equilibrium market tightness

We assume free entry into the market of unmatched retailers so that $V_{do} = 0$ in Eq. (10), as in Pissarides (1985) and Shimer (2005). This assumption implies that

$$\frac{w_d c_{do}}{\chi(\kappa_{do})} = \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi). \tag{18}$$

This equation defines the equilibrium market tightness, κ_{do}, that equates the expected cost of being an unmatched retailer, on the left, with the expected benefit from matching, on the right.

To get intuition from Eq. (18), notice that as the expected benefit from retailing rises, free entry implies that retailers enter the search market. This entry raises market tightness, κ_{do}, and, through congestion effects, reduces the rate at which searching retailers contact searching producers, $\chi(\kappa_{do})$. This increases retailers' expected cost of search (the left-hand side) so that, with free entry into retailing, κ_{do} always satisfies Eq. (18) in equilibrium.

2.3 Aggregation

2.3.1 Fraction of unmatched producers

Because of search frictions, in the steady state there exists a set of unmatched producers (mass of unmatched product varieties) that are actively looking for a retail partner. This fraction of unmatched producers is given by

$$\frac{u_{do}}{1 - i_{do}} = \frac{\lambda}{\lambda + \kappa_{do}\chi(\kappa_{do})}, \tag{19}$$

in which u_{do} is the fraction of producers that are unmatched and searching and $u_{do}/(1 - i_{do})$ is the fraction of active producers that are unmatched adjusted by the fraction of producers that will ever search, $1 - i_{do}$. The fraction of idle producers, i_{do}, that choose not to search is defined by the steady-state productivity threshold, $\bar{\varphi}_{do}$, and the exogenous productivity distribution:

$$i_{do} = \int_1^{\bar{\varphi}_{do}} dG(\varphi) = G(\bar{\varphi}_{do}) = 1 - \bar{\varphi}_{do}^{-\theta}. \tag{20}$$

The unmatched producers characterized by Eq. (19) imply associated unmatched varieties that cannot be consumed and are therefore absent from imports, the indirect utility (welfare) function, and all other aggregates.

We can move from indexing over an unordered set of varieties that enter utility to indexing using a distribution of productivities using the steps in Appendix A.11.1 of KM. Specifically, if an unordered set of varieties, Ω_o, has measure $N_o^x = |\Omega_o|$, then the set of varieties above the threshold has measure $(1 - G(\bar{\varphi}_{do})) N_o^x = (1 - i_{do}) N_o^x$ and the set of matched varieties that are above the threshold has measure $(1 - u_{do}/(1 - i_{do})) N_o^x$. The measure of goods consumed will feature prominently in any aggregate quantity in the model.

2.3.2 Resource constraint

The aggregate resource constraint using the expenditure approach and the measure of matched varieties can be written as

$$\begin{aligned} Y_d = & \underbrace{p_d(1)\, q_d(1) + \sum_{k=1}^{D} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi)\, q_{dk}(\varphi)\, dG(\varphi)}_{\text{Consumption } (C_d)} \\ & \underbrace{+ N_d^x w_d e_d^x + \sum_{k=1}^{D} \kappa_{dk} u_{dk} N_k^x w_d c_{dk} + u_{kd} N_d^x \left(w_d l_{kd} + w_d s_{kd} \kappa_{kd} \chi(\kappa_{kd})\right) + (1 - u_{kd} - i_{kd}) N_d^x w_d f_{kd}}_{\text{Investment } (I_d)}. \end{aligned} \tag{21}$$

Consumption expenditure, C_d, is defined in Eq. (1). Investment expenditure, I_d, is the resources devoted to creating producers, to creating retailer-producer relationships, and to paying for the per-period fixed costs of production. We define investment costs as those that must be paid before producing the first unit of output and that do not scale with output. Notice that government expenditure, G_d, which is total tariffs that are levied on retailers at the negotiated price, is implicitly included in consumption expenditure because final sales prices, $p_{do}(\varphi)$, include the tariff (Eq. 14). The government budget is balanced by rebating tariff revenue to (taxing subsidy cost from) consumers as income, as discussed in Section 2.3.4. Government payments to idle producers are financed by a lump-sum tax on consumption so that they cancel out on the expenditure (right) side of the aggregate resource constraint. We impose balanced trade:

$$\sum_{k=1}^{D} IM_{kd} = \sum_{k=1}^{D} IM_{dk}, \tag{22}$$

so that net exports do not appear in Eq. (21). Finally, we assume that the number of producers is exogenous, as in CRW. Appendix A.4.1 contains details about the resource constraint.

2.3.3 Labor market clearing

Labor demand in country d, LD_d, is defined by

$$LD_d = \frac{I_d}{w_d} + q_d\left(1\right) + \sum_o \left(1 - \frac{u_{od}}{1 - i_{od}}\right) N_d^x \int_{\bar{\varphi}_{od}} q_{od}\left(\varphi\right) \tau_{od} \varphi^{-1} dG\left(\varphi\right), \tag{23}$$

with details in Appendix A.4.2. Labor demand is the sum of three terms. First, the cost to create firms, pay fixed costs, and form matches captured by the investment term, I_d, from Eq. (21). (Investment must be divided by the wage to yield units of labor.) Second, the labor used to produce the homogeneous good. Third, the labor used to produce local and exported differentiated goods. In contrast to tariffs, whose resources are reallocated but not lost, iceberg costs τ_{od} are lost to physical destruction. Labor supply is immobile and equal to a country's labor endowment, L_d.

2.3.4 Profits

Firms make profits in our framework because of a fixed and exogenous number of producers. Total resources paid to labor are defined by $Y_d = w_d L_d + \Pi_d + G_d$, in which L_d is the exogenous labor endowment, w_d is the equilibrium wage, Π_d are profits, and G_d is income raised by tariffs and rebated to consumers. To determine profits, Π_d, we discuss five ownership structures of firms in Appendix D.1.3: 1) Consumers in country d own retailers and producers in country d (profits attributed by location); 2) they own retailers in country d and all producers in country o that serve them (upstream vertical integration); 3) they own producers in country d and retailers that sell these goods in country o (downstream vertical integration); 4) they own producers in country o that serve country d and retailers in country o that sell products sourced from country d (inverted ownership structure); or 5) they own shares of a global mutual that collects all retailer and producer profits and then redistributes them in π proportion to the value of the labor endowment in each country, $w_d L_d$. In our analysis we use the first approach. Therefore, profits are defined by

$$\Pi_d = \Pi_d^r + \Pi_d^p = \sum_k \Pi_{dk}^r + \sum_k \Pi_{kd}^p = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk} + \sum_k IM_{kd} - \sum_k \frac{C_{kd}}{\mu t_{kd}}, \tag{24}$$

in which Π_{dk}^r is the total retailer profits from all varieties sold by retailers in country d who source their products from country k and Π_{kd}^p is the total producer profits from all varieties sold by producers in country k from country d.

2.3.5 Price index

Using the optimal final sales price from Eq. (14) and the other assumptions in Sections 2.1.1 through 2.2, we derive the price index for differentiated goods in country d:

$$P_d = \lambda_2 C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \rho_d, \qquad \rho_d \equiv \left(\sum_{k=1}^{D} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) N_k^x \left(w_k \tau_{dk}\right)^{-\theta} F_{dk}^{-\left[\frac{\theta}{\sigma-1}-1\right]} t_{dk}^{1-\mu\theta} \right)^{-\frac{1}{\theta}}, \tag{25}$$

in which $\lambda_2 \equiv \left(\theta / \left(\theta - (\sigma - 1)\right)\right)^{-\frac{1}{\theta}} \left(\sigma/\alpha\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}} \mu$. More details appear in Appendix A.4.3, which also defines other relevant price indices. To conserve on notation, sometimes we refer to $F\left(\kappa_{do}\right)$ as F_{do}. The ideal price index, Ξ_d, that minimizes expenditure to obtain utility level $U_d = 1$ combines the differentiated and homogeneous goods prices as $\Xi_d = \left[p_d\left(1\right) / \left(1-\alpha\right)\right]^{1-\alpha} \left[P_d/\alpha\right]^{\alpha}$.

2.3.6 Imports

The gravity equation gives total imports by destination d from origin o in the differentiated goods sector, which is the total value of all imported varieties evaluated at negotiated prices, $n_{do}q_{do}$. As we show in Appendix A.4.4.1, imports are:

$$IM_{do} = \left(1 - \frac{u_{do}}{1-i_{do}}\right) \left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right) \alpha N_o^x C_d \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} t_{do}^{-\mu\theta}, \tag{26}$$

in which the fraction of matched exporters, $1 - u_{do}/\left(1-i_{do}\right)$, and the import markdown, $1 - b\left(\cdot\right)$ reduce imports relative to a model without search (KM).

The total amount paid by consumers in d for imports from o, C_{do}, equals the value of all imported varieties evaluated at final sales prices. We show that $C_{do} = t_{do} IM_{do} / \left(1 - b\left(\sigma, \theta, \gamma_{do}, \delta_{do}, F_{do}\right)\right)$ in Appendix A.4.4.2.

2.4 Steady-state general equilibrium

A steady-state general equilibrium consists of threshold productivities, market tightnesses, κ_{do}, $\bar{\varphi}_{do}$, $\forall do$, aggregate consumptions, C_d, and wages, w_d, $\forall d$. These will jointly satisfy producers' zero profit conditions (Eq. 17), retailers' free-entry conditions (Eq. 18), aggregate resource constraints (Eq. 21), and labor market clearing (Eq. 23). We elaborate on the definition of the steady-state general equilibrium in Appendix A.5.1.

The exogenous parameters are β, λ, r, η, ξ, θ, σ, α, e_d^x, L_d, t_{do}, c_{do}, f_{do}, h_{do}, l_{do}, and s_{do}, in which d and o vary by countries. Tariffs, t_{do}, are exogenous parameters to economic agents, except when they are chosen by a social planner, as discussed in Section 3.

The main difference between our model's equilibrium definition and the definitions in trade models without search is that we introduce market tightnesses, k_{do}. Our model nests trade models without search frictions if market tightnesses are infinite. Specifically, our model exactly reproduces Chaney (2008) if retailers' search costs are zero, $w_d = 1$ and $G_d = 0$, $\forall d$, and we impose the same parameter value restrictions that he does ($s_{do} = h_{do} = e_d^x = 0$, $\forall d$, and, $\forall o$), among other assumptions. We provide more details for this result in Appendix A.5.2.

Formally, the decentralized equilibrium solves a system of nonlinear equations in the equilibrium variables in which the equilibrium conditions are constraints, our calibration will determine the model parameters (Section 4), and the objective function is any constant, including zero. This problem solves:

$$\left(\boldsymbol{\kappa}^c, \bar{\boldsymbol{\varphi}}^c, \vec{C}^c, \vec{w}^c\right) = \arg\max_{\boldsymbol{\kappa}, \bar{\boldsymbol{\varphi}}, \vec{C}, \vec{w}} 0 \tag{27a}$$

subject to:

$$\frac{w_d c_{do}}{\chi\left(\kappa_{do}\right)} = \left(\frac{1}{r+\lambda}\right) \frac{\Pi_{do}^r\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{d*}, \vec{C}, \vec{w}, \vec{t}_{d*}\right)}{\left(1-u_{do}\left(\kappa_{do}\right)/\left(1-i_{do}\left(\bar{\varphi}_{do}\right)\right)\right) N_o^x} \ \forall do, \tag{27b}$$

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{d*}, \vec{C}, \pi, \vec{w}, \vec{t}_{d*}\right)}\right) \left(\frac{F\left(\kappa_{do}\right)}{C_d}\right)^{\frac{1}{\sigma-1}} t_{do}^{\mu} \ \forall do, \tag{27c}$$

$$w_d L_d + \Pi_d + G_d\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{*d}, \vec{C}, \vec{t}_{d*}\right) = C_d + I_d\left(\vec{\kappa}_{d*}, \vec{\kappa}_{*d}, \vec{\bar{\varphi}}_{*d}, \vec{C}\right) \ \forall d, \tag{27d}$$

$$w_d = \frac{I_d\left(\vec{\kappa}_{d*}, \vec{\kappa}_{*d}, \vec{\bar{\varphi}}_{*d}, \vec{C}\right) + (1-\alpha) C_d + \frac{1}{\mu}\left(\vec{C}_{*d}\left(\vec{\kappa}_{*d}, \vec{\bar{\varphi}}_{*d}, \vec{C}, w_d, \vec{t}_{*d}\right)/\vec{t}_{*d}\right)' \vec{\iota}}{L_d} \ \forall d, \tag{27e}$$

$$\vec{t}_{d*} = \vec{t}_{d*}^{\,c} \ \forall d. \tag{27f}$$

Eq. (27) is expressed as a function of only the endogenous variables and parameters. Eqs. (27d) and (27e) implicitly define a trade balance condition because we assume that net exports are zero. In Section 2.3 we discuss three ownership structures of firms, and in Appendix D.1.3 we show that one of these structures implies an explicit trade balance condition, as in Demidova and Rodríguez-Clare (2009), Felbermayr, Jung, and Larch (2013), and CRW.

We denote these solutions to the decentralized competitive equilibrium defined by Eq. (27) with "c" superscripts. We also define vectors as collections of the variables across subindexes and matrices are denoted in bold. For example, search market tightnesses are collected into the following

$$\vec{\kappa}_{*o} = \begin{pmatrix} \kappa_{1o} \\ \kappa_{2o} \\ \vdots \\ \kappa_{Do} \end{pmatrix}, \qquad \vec{\kappa}_{d*} = \begin{pmatrix} \kappa_{d1} & \kappa_{d2} & \dots & \kappa_{dD} \end{pmatrix}, \qquad \boldsymbol{\kappa} = \begin{bmatrix} \kappa_{11} & \dots & \kappa_{1D} \\ \vdots & \ddots & \vdots \\ \kappa_{D1} & \dots & \kappa_{DD} \end{bmatrix}, \tag{28}$$

so that rows index destinations and columns index origins. $\vec{\kappa_{d*}}$ is the dth row of $\boldsymbol{\kappa}$ and $\vec{\kappa_{*o}}$ is the oth column of $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}$ is a square matrix. The column vector of D aggregate consumption expenditures in each d economy is collected in $\vec{C}$, $\vec{\iota}$ is a $D \times 1$ column vector of ones, and $\vec{L}$ is a column vector of D labor endowments. Division of matrices is element by element. Tarriffs, $\boldsymbol{t}^c$, are exogenous in the decentralized competitive economy. Higher variable export costs to country d, $\vec{t}^c_{d*}$, directly lower importers' period profits, Π^m_{do}, and raise the price index, P_d (Eq. 25), but only affect investment, I_d, through other equilibrium variables.

3 Optimal uniform import tariffs

3.1 The unilateral country social planner's problem

This section considers a country social planner that unilaterally chooses import tariffs to maximize its own country's welfare without considering the welfare of other countries. While this country can set import tariffs, t_{d*}, we assume it cannot choose domestic taxes and the import tariffs of other countries. Rather, these are set to their competitive levels so that $t_{dd} = t^c_{dd}$ and $\vec{t}_{o*} = \vec{t}^c_{o*}$, $\forall o \neq d$. This country's social planner remains constrained by the decentralized retailer entry condition and the other equilibrium constraints in all countries defined in Eq. (27). Formally, this problem is given by

$$\left(\boldsymbol{\kappa}^u, \bar{\boldsymbol{\varphi}}^u, \vec{C}^u, \vec{w}^u, \vec{t}^u_{d*}\right) = \underset{\vec{t}_{d*}}{\arg\max} \left(\frac{C_d}{\Xi_d \left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{d*}, \vec{C}, \vec{w}, \vec{t}_{d*} \right)} \right) \tag{29a}$$

subject to: Eqs. (27b) through (27e),

$$\vec{t}_{o*} = \vec{t}^c_{o*} \ \forall o \neq d, \tag{29b}$$

$$t_{dd} = t^c_{dd} \ \forall d. \tag{29c}$$

We denote the solutions to the unilateral problem defined by Eq. (29) with "u" superscripts. Appendix A.6 shows that real consumption in country d is welfare in country d because preferences are homothetic.

3.2 Optimal unilateral tariffs with passive trade policies: Analytic results

In this section we present results about the optimal import tariff chosen by country d's social planner, t^u_{do}, if other trade policies are passive, so that $t_{jk} = 1 \forall jk \neq do$. We show that search frictions lead to a new source of aggregate nonconvexity in the production possibility frontier (PPF). This nonconvexity creates an incentive to subsidize imports, in addition to the selection effect emphasized by CRW, under simplifying assumptions. In particular, import subsidies increase contact rates and reduce the threshold productivity for exporting in the do market. Contact rates rise with import subsidies because the value of being a matched retailer in the *do* market rises so that tightness rises. Subsidies also encourage lower-productivity producers to enter the export market. Both of these effects imply that import subsidies can lower import prices in the destination country by raising exports from country o if country o's PPF is not convex. Lower import prices reduce the price index in the destination country, making consumers better off and raising welfare.

We define how the slope of the PPF changes as the level of exports changes.

Definition 1. *Define ϵ_o^u as the elasticity of the MRT with respect to exports in country o at the optimal tariff:*

$$\epsilon_o^u = \frac{\partial \ln MRT_o^u \left(Q_{do}, Q_{oo}\left(Q_{do}\right)\right)}{\partial \ln Q_{do}}, \tag{30}$$

otherwise known as the elasticity of transformation (EoT).

The superscript "u" denotes variables evaluated at the unilaterally optimal uniform import tariff, which we discuss below. The EoT is negative (positive) for a PPF that is bowed into (out from) the origin because the MRT is falling (rising). A negative (positive) EoT implies increasing (decreasing) returns to scale. The EoT is zero for a linear PPF.

This simplifying assumption helps yield analytic solutions.

Assumption 1. *Assume that 1) $D = 2$ so that there are two countries, d and o; 2) there are no tariffs or subsidies except for tariffs in the do market; 3) the consumer's optimization problem yields an interior solution; and 4) the homogeneous good does not enter the utility function ($\alpha = 1$).*

We show in Appendix B.1 that if we use Assumption 1, then the unilateral tariff that maximizes welfare in country d, t_{do}^u, satisfies the approximation

$$t_{do}^u \approx 1 + \frac{1 + \sigma x_{oo}^u \epsilon_o^u}{(\sigma - 1)\, x_{oo}^u}, \tag{31}$$

in which $x_{oo} \equiv C_{oo} / \left(C_{oo} + C_{do}/t_{do}\right) \geq 0$, σ is the elasticity of substitution between differentiated varieties, and ϵ_o^u is defined in Definition 1. The approximation rests on similar markdowns ($1 - b_{do}(\cdot)$ from Eq. 26) in international markets, and small responses of ratios of these markdowns to exports and imports. Appendix B.1.4 shows that we recover the optimal uniform tariff expression in CRW exactly if we make the same assumptions as they do in a model without search frictions. Eq. (31) suggests that as the EoT falls, and the PPF in country o gets more bowed into the origin, the optimal tariff falls. Because x_{oo} can be interpreted as the share of differentiated goods consumption in country o devoted to local goods under simplifying assumptions (Appendix B.1.5), we refer to x_{oo} as the local consumption share in country o.

We highlight that computing the optimal tariff in Eq. (31) requires a calibrated structural model, except in special cases. In particular, the right-hand side (RHS) of Eq. (31) requires the local expenditure share and the EoT in country o to be evaluated at the optimal tariff. In general, these values cannot be calculated without a fully specified general equilibrium model because they are equilibrium objects that depend on the level of tariffs. Specifically, observed data do not yield the local consumption share at the optimal tariff. A structural model also allows us to consider and quantify welfare in counterfactuals. We pursue such quantitative exercises in Sections 3.3, 3.4, and 5. Only in some special cases is the RHS of Eq. (31) a constant and not a function of the import tariff, for example, if country d is a small open economy so that $x_{oo} = 1$ and productivity is distributed Pareto (Demidova and Rodríguez-Clare, 2009).

The following proposition characterizes the effects of goods-market tightness on the EoT in country o, ϵ_o^u. We show that search frictions lower this EoT, which creates an incentive to subsidize imports.

Proposition 1. *Use Assumption 1 and also assume that $l_{ko} = -h_{ko}$ for $k = d, o$ so that $F(\kappa_{ko})$ are parameters. Then*

$$\epsilon_o^u = \underbrace{-\left[\frac{\theta - (\sigma-1)}{[\sigma\theta - (\sigma-1)]\, x_{oo}^u}\right]}_{\text{Selection}} \underbrace{-(1-\eta)\left[\frac{(\sigma-1)}{\sigma\theta - (\sigma-1)}\right]\left[\left(\frac{u_{do}^u}{1-i_{do}^u}\right)\frac{\partial \ln \kappa_{do}^u}{\partial \ln Q_{do}} - \left(\frac{u_{oo}^u}{1-i_{oo}^u}\right)\frac{\partial \ln \kappa_{oo}^u}{\partial \ln Q_{do}}\right]}_{\text{Search frictions}}. \tag{32}$$

Proof. See Appendix B.2. □

The EoT in our model is the usual EoT without search frictions and the additional effect of exports on goods-market tightness. The first term in Eq. (32) is the EoT in a model with a Pareto distribution for firm productivity and constant fixed exporting cost, but without search frictions, as in Felbermayr, Jung, and Larch (2013). This EoT is negative because the PPF is bowed into the origin. This nonconvexity in the PPF arises because of self-selection into exporting, as discussed in Costinot, Rodríguez-Clare, and Werning (2016) and CRW. Specifically, as Q_{do} rises, there is more producer entry in the *do* market, as well as fewer local goods, Q_{oo}, which lowers producer entry in the *oo* market. Both of these effects lower the opportunity cost of exports in terms of local goods. As the productivity distribution has a thinner right tail ($\theta \to \infty$), this selection channel becomes more relevant and the PPF becomes more bowed into the origin. This EoT term would be zero in a model with homogeneous firms and no fixed exporting costs, as in Gros (1987).

The second term in Eq. (32) captures the effects of changes in goods-market tightness on the EoT. The first term in the second square parentheses captures the effects of exports on tightness in the *do* market. This term is positive because an increase in country o's exports increases retailer entry and tightness in the *do* market, which increases the matched rate and decreases the differentiated-goods price index. Simply put, by increasing exports, country o lowers search frictions, which lowers the opportunity cost of exports. The second term in the square parentheses is negative because higher exports imply substitution away from the local market, which lowers the local matched rate and increases the cost of producing local goods. These two effects imply that the second term in Eq. (32) is negative, so that search frictions introduce a new downward force on the EoT and the PPF becomes more nonconvex. Naturally, these search friction effects are larger when unmatched rates in the *do* and *oo* markets are high and when the number of matches is more responsive to the number of searching retailers, that is, when $(1-\eta)$ is closer to one.

Even if firms were homogeneous and there were no fixed exporting costs, the country social planner might choose to subsidize imports because of search frictions. Without search frictions, the optimal tariff in this environment would be $1+1/\left[(\sigma-1)\,x_{oo}^{u}\right]\geq 1$ because the PPF would be linear so that the EoT would be zero ($\epsilon_{o}^{u}=0$). Search frictions make the EoT more negative ($\epsilon_{o}^{u}\leq 0$) and the social planner would choose to lower import tariffs. This result is similar to those in Melitz and Redding (2015), who make the point that micro moments matter for welfare changes. And it is also similar to the point made by KM that search-related moments matter for welfare. Analogously, we show in this paper how search friction moments enter into and change the optimal tariff expression from CRW.

The effect of search frictions on the local consumption share, x_{oo}, further reduces the optimal tariff relative to a model without search frictions. In Appendix B.3 we show that the local consumption share rises when international search costs in the *do* market rise. This result is intuitive. For example, if search frictions in the import market are extremely high, then there are few retailers in that market, and almost all expenditure is directed toward local goods. This effect reduces the optimal tariff even further through the direct effect on the optimal tariff in Eq. (31).

As shown in Costinot, Rodríguez-Clare, and Werning (2016, pg. 24), the optimal tariff without search frictions and a Pareto productivity distribution equals

$$t_{do}^{u,ns}=1+\frac{1+\sigma x_{oo}^{u}\epsilon_{o}^{u}}{(\sigma-1)\,x_{oo}^{u}}=1+\frac{1}{(\mu\theta-1)\,x_{oo}^{u}}, \tag{33}$$

in which we use only the ‘‘selection’’ term of the EoT from Eq. (32) in Eq. (31). Appendix B.5 discusses additional findings about optimal tariffs in the model without search frictions.

Different search market structures could result in different optimal tariffs and efficiency results. KM discuss the efficiency properties of a Melitz-style model with exogenous wages in detail (Appendix A.16). It is well known that under constant elasticity of substitution preferences and without goods-market frictions, the decentralized equilibrium in the Melitz model is efficient (Dhingra and Morrow, 2019). However, the decentralized equilibrium in our model with random search and Nash bargaining does not attain the efficient market tightness in general because of the standard matching externalities in search models; namely, retailers and producers do not internalize how searching affects equilibrium matching probabilities. In addition, our model also has participation and output externalities because the threshold producer does not internalize their effect on average match productivity, as in Albrecht, Navarro, and Vroman (2010) and Julien and Mangin (2017). As a result, the Hosios (1990) condition, which sets producers' bargaining power, β, equal to the matching elasticity, η, does not ensure efficiency in our context. Alternatively, competitive search models—those in which some agents can post prices and other agents direct their search to the most attractive alternatives—typically yield efficient market tightness in the decentralized economy, both in static and in dynamic environments (Acemoglu and Shimer, 1999; Rogerson, Shimer, and Wright, 2005). Nevertheless, we know little about the interactions between optimal tariffs and optimal market tightness in search models. We leave formally characterizing the efficiency properties of alternative search market structures to future work.

3.3 Optimal unilateral tariffs with passive trade policies: Numerical results

In this section we present numerical examples of the results in Section 3.2. The results are based on a symmetric country example with simple parameter values, as discussed in Appendix B.4. We assume that the homogeneous good does not enter the utility function ($\alpha = 1$); that all tariffs are one, except for the U.S. import tariff; and that the Chinese wage serves as the numeraire. We solve for the competitive equilibrium for various U.S. import tariffs. The U.S. country social planner chooses the tariff that maximizes U.S. welfare. These numerical examples provide intuition for the quantitative results in Section 3.2. Solving the model numerically and verifying the optimal uniform import tariff in previous work are contributions in and of themselves.

Of course, the value of all parameters, for example, the flow payment of remaining idle, h_{do}, influences optimal tariffs. In our numerical examples and calibration in Section 4, we set many nontariff parameters to simple values, such as setting $h_{do} = 0$, to sharpen the focus on tariffs. We emphasize, however, that our model and theoretical results can accommodate many parameter values.

3.3.1 Optimal tariffs without and with search frictions

U.S. welfare—alternatively, real consumption—is single-peaked in the model with no search frictions, and the optimal tariff satisfies Eq. (33). The left vertical axis of Fig. 1a is in units of real consumption. The right vertical axis of Fig. 1a is the percent change in welfare from the free-trade welfare level, in which $t_{uc} = 1$. The figure shows that U.S. welfare is concave and peaks when the import tariff is 1.39, which is similar to the optimal tariff in other papers without search frictions (Felbermayr, Jung, and Larch, 2013; Costinot and Rodríguez-Clare, 2014). The gain in U.S. welfare relative to free trade is slightly above 2.5 percent. Fig. 1b plots the right-hand side of Eq. (33) for various U.S. import tariffs (dashed blue line) and the 45-degree line (in black). This figure verifies that the optimal tariff satisfies Eq. (33): At the optimal tariff, the right-hand side of that equation evaluates to the optimal tariff. We present further details in Appendix B.5.

In the model with search frictions in the *uc* market only, the optimal tariff satisfies Eq. (31), and that tariff is substantially below that in the model without search frictions. With search frictions, U.S. welfare peaks when the import tariff is 1.24, as shown in Fig. 2a. Therefore, these numerical examples suggest that the optimal tariff in a model with search frictions is about 10 percentage points below that in a model without search frictions. The gain in U.S. welfare relative to free trade is slightly below 1 percent. Therefore, even though the optimal tariff is lower in the model with search frictions than in the model without them, the change in welfare is smaller in the former than in the latter. This result follows because the model with search frictions has other costs relative to the model without search frictions. Fig. 2b plots the right-hand side of Eq. (31) for various U.S. import tariffs (solid blue line) and the 45-degree line (in black). This figure verifies that the optimal tariff is well-approximated by Eq. (31) in this numerical example. That equation implies an optimal tariff—where the solid blue and black lines intersect—that is only slightly higher than the actual optimal tariff (dotted black line).

We make two more observations about Figs. 1a and 2a. First, welfare is substantially lower for all shown tariff levels in the model with search frictions than in the model without them. For example, at the optimal tariff in the two models, U.S. welfare is about 6 percent lower in the model with search frictions than in the model without them. This reduced welfare reflects the expenditure required to form relationships in the model with search frictions—captured by investment in Eq. (21)—and the equilibrium fraction of unmatched varieties discussed in Section 2.3.1. Second, determining optimal tariffs using a model without search frictions but applying those tariffs in a model with search frictions results in lower welfare. In particular, compared to the maximum welfare in the model with search frictions, implementing the optimal tariff from the model without search frictions reduces welfare by about 0.2 percent.

U.S. tariffs on Chinese exports reduce Chinese welfare in the model without and with search frictions, as shown in Figs. 3a and 3b, respectively. Higher U.S. tariffs lower Chinese welfare. As with U.S. welfare, changes in tariffs have a larger effect on China's welfare in a model without search frictions than in a model with them. For example, moving from free trade to 100 percent U.S. tariffs ($t_{uc} = 2$) reduces welfare in China by about 8 percent in a model without search frictions (Fig. 3a), but only by about 4 percent in a model with search frictions (Fig. 3b).

3.3.2 How optimal tariffs vary with search frictions

Fig. 4 shows how the optimal unilateral tariff, t^u_{uc}, denoted with a solid blue line, varies with search frictions, as measured by the expected costs to retailers in the uc market of searching for a producer, $w_u c_{uc}/\chi(\kappa_{uc})$. The optimal tariff for each level of search costs is obtained by solving the U.S. country social planner's problem in Eq. (29). For this exercise, we set search costs to zero in all other markets except the uc market. So, the vertical axis intercept denotes the optimal tariff in a model without search frictions, when search costs are zero in all markets (see Section 3.3.1 and Fig. 1a)

We make three observations about Fig. 4. First, the optimal tariff in a model with any positive search frictions is below that in the model without them (solid blue line versus dashed blue line). Second, most of the decline in the optimal tariff occurs at relatively low levels of search frictions. For example, if expected search costs are only 0.05, the optimal tariff is substantially lower that in the model without search costs.

Third, the domestic consumption share has a small effect on the optimal tariff relative to the EoT. A higher local consumption share in the model with search frictions than in the model without them contributes to the lower optimal tariff in the former than in the latter, as discussed in Section 3.2 and shown in Appendix B.3. The dotted purple line in Fig. 4 uses the formula for the optimal tariff without search (Eq. 33), but uses the local consumption share from the model with search frictions for each search cost level. Therefore, the difference between the dashed blue line and the dotted purple line isolates the effect of a different local consumption share between the two models. The results suggest that the local consumption share explains a small fraction of the difference between the optimal tariff with and without search. Similarly, the dashed-dotted red line uses the formula for the optimal tariff without search but uses the EoT from the model with search frictions. The results suggest that about two-thirds of the difference in the optimal tariff between the two models is accounted for by the lower EoT in the search model. A higher local consumption share in this model accounts for the rest of the optimal tariff difference. The difference between the solid green line and the solid blue line captures the size of the approximation used in Eq. (31), which omits the markdown terms, as discussed in Appendix B.1.4. The results suggest that this approximation error is relatively small in this numerical example.

3.4 Optimal tariffs with retaliation

3.4.1 The Nash equilibrium

This section considers optimal unilateral tariffs in a strategic environment. We define and solve for a D-country pure-strategy Nash equilibrium in which countries choose import tariffs. We assume countries cannot choose domestic taxes, which are set to their competitive levels. The Nash equilibrium import tariffs are defined by tariffs that maximize each country's welfare, subject to the equilibrium conditions and the Nash tariffs set by other countries. Formally, this problem is given by

$$\text{Find } \left\{\boldsymbol{\kappa}^n, \bar{\boldsymbol{\varphi}}^n, \vec{C}^n, \vec{w}^n, \boldsymbol{t}^n\right\} \text{ subject to} \tag{34a}$$

$$\left\{\boldsymbol{\kappa}^n, \bar{\boldsymbol{\varphi}}^n, \vec{C}^n, \vec{w}^n, \vec{t}^{\,n}_{d*}\right\} = \arg\max_{\vec{t}_{d*}} \left(\frac{C_d}{\Xi_d\left(\vec{\kappa}_{d*}, \vec{\bar{\varphi}}_{d*}, \vec{C}, \vec{w}, \vec{t}_{d*}\right)}\right) \ \forall d, \tag{34b}$$

subject to: Eqs. (27b) through (27e),

$$\vec{t}_{o*} = \vec{t}^{\,n}_{o*} \ \forall o \neq d, \tag{34c}$$

$$t_{dd} = t^c_{dd} \ \forall d. \tag{34d}$$

We denote the solutions to the Nash equilibrium defined by Eq. (34) with "n" superscripts. Appendix B.6 describes how to solve for the Nash equilibrium using the Nikaidô-Isoda function (Nikaidô and Isoda, 1955).

Intuition for the mechanisms for optimal tariffs will remain largely the same as in Section 3.2. However, instead of one fixed point equation that determines the optimal tariff (Eq. 31), there exist similar equations for each country. In these equations, the equilibrium variables on the right-hand side are functions of tariffs. The optimal tariffs satisfy all the equations simultaneously and imply no incentive to deviate for any country.

3.4.2 Optimal tariffs with retaliation: Numerical results

We use the same numerical example as in Section 3.3, in which $D = 2$. Fig. 5a depicts the optimal U.S. import tariff for each Chinese import tariff (dashed blue line) and the optimal Chinese tariff for each U.S. import tariff (dashed red line). The intersection of these two best response curves identifies the Nash equilibrium import tariffs for which neither country has an incentive to deviate. In the model without search frictions, the Nash equilibrium import tariff is 1.35 in both the United States and China because the countries are symmetric in this numerical example. This U.S. tariff is lower than the optimal U.S. unilateral import tariff (1.39) in Section 3.3.1 because of strategic considerations. That is, if the United States raises import tariffs, China lowers import tariffs (and vice versa).

Fig. 5b depicts the best responses for the model with search frictions in the *uc* market. The U.S. best response line shifts down relative to the model without search frictions. The U.S. Nash equilibrium import tariff with search frictions is 1.16, which is below the Nash equilibrium import tariff without search frictions (1.35) and also below the optimal unilateral import tariff with search frictions (1.24) in Section 3.3. The Chinese best response is little changed from the model without search frictions, as is the Nash equilibrium Chinese import tariff.

In summary, optimal import tariffs are lower in a model with search frictions than in a model without them, even with strategic considerations. In the numerical example, U.S. optimal import tariffs are highest without search frictions and without strategic considerations (1.39) and lowest with search frictions and with strategic considerations (1.16).

4 Calibration

We use data for China and the United States in 2016 to calibrate our model, as in KM, but we can generalize our approach to include more trading partners or a different time period. The calibration proceeds in two steps. First, we externally calibrate parameters that can be normalized or that are standard in the literature. Second, we internally calibrate the remaining parameters by minimizing the distance between moments in the data and the decentralized model (Eq. 27) with search frictions subject to that model's equilibrium constraints. Formally, this minimization is accomplished by solving a mathematical program with equilibrium constraints (MPEC) following Dubé, Fox, and Su (2012) and Su and Judd (2012).

We present the calibrated parameters in Table 1, with a discussion of our calibration and intuition for identification of each internally calibrated parameter in Appendix C. In short, to calibrate retailers' flow search costs, c_{do}, we use the fraction of firms that export and manufacturing capacity utilization rates. Log-linear estimates of the trade elasticity inform the elasticity of matches with respect to the number of searching producers, η. The average duration of a Chinese and U.S. trading relationship informs our separation parameter, λ. Otherwise, our calibration is relatively standard.

Table 2 presents the moments from the model and shows that the model matches the data well. The calibrated model matches log-linear estimates of the import elasticity, fractions of exporting firms, manufacturing capacity utilization rates, economic aggregates, such as GDP, consumption, and international trade flows, as well as business failure rates, fixed foreign trade costs, and separation rates among trading partners. The model provides a realistic economic environment for general equilibrium exercises, a topic we pursue in the next section.

5 Quantitative results

5.1 Optimal tariffs with and without international search frictions

The calibrated model suggests that U.S. and Chinese welfare would increase by about 0.1 percent with optimal unilateral import tariffs and baseline search frictions, as shown in Table 3. Column 1 shows the calibrated values of the U.S. and Chinese tariffs along with the equilibrium value of trade from Table 2 and the welfare (real consumption) in both countries.

Column 2 (3) shows the optimal unilateral tariff that the U.S. (Chinese) country social planner would choose if China (United States) sets tariffs passively at the baseline values from column 1. The U.S. optimal unilateral tariff is 27 percent, 19 ppt. higher than tariffs in 2016, as shown in column 2. This tariff would reduce the value of trade between the United States and China by about $100 billion, or about 30 percent of the baseline value. U.S. welfare would rise by 0.1 percent and Chinese welfare would fall by slightly over 1 percent. The Chinese optimal unilateral tariff is 29 percent, 12 ppt. higher than tariffs in 2016, as shown in column 3. This tariff would reduce the value of trade between the United States and China by about $80 billion. Chinese welfare would rise by 0.11 percent and U.S. welfare would fall by about 0.18 percent.

The Nash equilibrium in the presence of search frictions implies that both countries would raise tariffs to 25 percent, as shown in column 4. These higher tariffs reduce the value of imports by about 40 percent, to $235 billion. U.S. (Chinese) welfare would fall by about 0.1 (0.9) percent.

With international search frictions reduced to domestic levels, the calibrated model suggests that U.S. (Chinese) welfare would increase by about 0.4 (0.2) percent with optimal unilateral import tariffs, as shown in Table 4. Column 1 shows the value of trade between the United States and China and welfare in both countries if we reduce international search frictions to the level of domestic search frictions but otherwise retain the rest of the baseline calibration. That is, we only reduce retailers' flow search costs in international markets to their domestic levels, $c_{do} = c_{dd}$. As a result, the value of trade more than doubles from the baseline search calibration in Table 3 and welfare rises by 1.7 (8.3) percent in the United States (China) (not shown in the tables). Starting with these lower search frictions, the optimal U.S. unilateral import tariff would raise U.S. welfare by 0.38 percent and reduce Chinese welfare by about 3.6 percent, as shown in column 2. Column 3 suggests that the optimal Chinese unilateral import tariff would raise Chinese welfare by 0.2 percent and reduce U.S. welfare by 0.4 percent. The Nash equilibrium outcome with these lower search costs would raise U.S. welfare by about 0.1 percent and lower Chinese welfare by about 3.4 percent, as shown in column 4. Finally, for the US at least, Nash equilibrium tariffs are lower in the model with baseline search frictions (25 percent) than in the model with reduced search frictions (35 percent).

These results are qualitatively similar to our much simpler numerical examples in Sections 3.3 and 3.4 despite allowing four search markets instead of just one. For example, the United States sets higher unilateral import tariffs with lower search frictions (27 percent with baseline but 37 percent with lower search frictions). And both levels of search costs have Nash equilibria with import tariffs that are lower than those the country social planner sets unilaterally for both countries.

5.2 Search frictions attenuate welfare effects of tariffs

Higher search frictions attenuate the welfare response to tariffs, as shown in Figs. 6a and 6b. The figures show the percent change in U.S. welfare from the baseline calibration as a function of U.S. import tariffs with reduced or baseline search costs. In both figures, the Chinese import tariff is set to its respective Nash equilibrium value. In the model with reduced search costs, Fig. 6a shows that varying tariffs from 1 to 2 yields welfare that is at most 0.3 percent above, and at most 0.3 percent below, the baseline level. In comparison, in the model with search frictions, varying tariffs from 1 to 2 yields welfare that is at most 0.1 percent above, and at most 0.25 percent below, the baseline level. That is, higher search frictions render tariffs less potent for welfare changes. Intuitively, higher search frictions imply fewer matched varieties and tariffs mainly affect aggregates through the intensive margin of matched varieties. This result is consistent with our findings in Section 3.3.1 and echoes one of the main conclusions of KM.

6 Conclusion

We study optimal import tariffs in an environment with search frictions between exporting producers and importing retailers. We analytically characterize unilateral import tariffs that maximize domestic welfare when other countries follow passive policies. Search frictions introduce a new incentive to subsidize imports because of market thickness effects, which contribute to aggregate nonconvexities in production. Naturally, these incentives strengthen when unmatched rates are high and with the matching function's responsiveness to the number of searching retailers.

Quantitative results using 2016 U.S. and Chinese data suggest that the optimal U.S. unilateral tariff with search frictions is about 10 percentage points below that in a model with lower search frictions. Changes in welfare in response to changes in tariffs are smaller in the model with baseline search frictions than in the model with reduced frictions. In the Nash equilibrium with baseline search frictions, U.S. (Chinese) tariffs are 17 (8) ppt. higher and welfare is 0.1 (0.9) percent lower relative to 2016 tariff levels.

Our study points to at least two directions for future research. First, empirical work could use variation in trade flows between countries to identify the parameters of the matching function following logic explained in Appendix C.2. Related, our calibration could be extended beyond two countries using moments on the fraction of exporting firms and capacity utilization rates. Also, the existence of search frictions could rationalize the pervasiveness of trade promotion programs, such as the State Trade Expansion Program (STEP, 2024) and the Trade Promotion Authority (TPA, 2024). Similarly, international search frictions could vary over time, as they do over the business cycle in labor markets, for example. If variation in search frictions over space and time is important, the framework in this paper would imply that optimal tariffs should vary in these dimensions as well.

Second, future work could explore the interactions between efficient levels of market tightness and optimal tariffs. It is well known that the efficiency properties of search models vary with the search structure (Rogerson, Shimer, and Wright, 2005). For example, in models with price posting and directed search (‘‘competitive search''), the decentralized equilibrium typically attains the efficient market tightness. And with random search and Nash bargaining, the decentralized equilibrium does not typically attain the social planner's market tightness. But we know little about the joint determination and interactions between optimal tariffs and optimal market tightness. Our framework provides a baseline for analyzing the welfare implications of trade policy in the presence of goods-market search frictions.

References

Acemoglu, Daron and Robert Shimer (1999). ‘‘Holdups and Efficiency with Search Frictions.’’ *International Economic Review*, 40(4), pp. 827--849. doi:https://doi.org/10.1111/1468-2354.00043.

Albrecht, James, Lucas Navarro, and Susan Vroman (2010). ‘‘Efficiency in a search and matching model with endogenous participation.’’ *Economics Letters*, 106(1), pp. 48--50. doi:10.1016/j.econlet.2009.09.022.

Anderson, James and Eric van Wincoop (2004). ‘‘Trade costs.’’ *Journal of Economic Literature*, 42(3), pp. 691--751. doi:10.1257/0022051042177649.

Armenter, Roc and Miklós Koren (2014). ‘‘A balls-and-bins model of trade.’’ *American Economic Review*, 104(7), pp. 2127--51. doi:10.1257/aer.104.7.2127.

BEA (2016a). ‘‘Gross domestic product [GDPA].’’ `https://fred.stlouisfed.org/series/GDPA`. Accessed: March 12, 2019.

BEA (2016b). ‘‘Personal consumption expenditures [PCECA].’’ `https://fred.stlouisfed.org/series/PCECA`. Accessed: March 12, 2019.

Brancaccio, Giulia, Myrto Kalouptsidi, and Theodore Papageorgiou (2020). ‘‘Geography, transportation, and endogenous trade costs.’’ *Econometrica*, 88(2), pp. 657--691. doi:10.3982/ECTA15455. URL `https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA15455`.

Brancaccio, Giulia, Myrto Kalouptsidi, Theodore Papageorgiou, and Nicola Rosaia (2023). ‘‘Search Frictions and Efficiency in Decentralized Transport Markets.’’ *The Quarterly Journal of Economics*, 138(4), pp. 2451--2503. doi:10.1093/qje/qjad023.

Broda, Christian, Nuno Limao, and David E. Weinstein (2008). ‘‘Optimal tariffs and market power: The evidence.’’ *American Economic Review*, 98(5), pp. 2032--65. doi:10.1257/aer.98.5.2032. URL `https://www.aeaweb.org/articles?id=10.1257/aer.98.5.2032`.

CB (2016a). ‘‘2016 Statistics of U.S. Businesses Annual Data Tables by Establishment Industry, U.S. by 6-digit NAICS.’’ `https://www.census.gov/data/tables/2016/econ/susb/2016-susb-annual.html`. Accessed: March 12, 2019.

CB (2016b). ‘‘A profile of U.S. importing and exporting companies, exhibits 1a and 5a.’’ URL `https://www.census.gov/foreign-trade/Press-Release/edb/edbrel2016.pdf`.

Cebul, Randall D., James B. Rebitzer, Lowell J. Taylor, and Mark E. Votruba (2011). ‘‘Unhealthy Insurance Markets: Search Frictions and the Cost and Quality of Health Insurance.’’ *American Economic Review*, 101(5), p. 1842–71. doi:10.1257/aer.101.5.1842. URL `https://www.aeaweb.org/articles?id=10.1257/aer.101.5.1842`.

Chaney, Thomas (2008). ‘‘Distorted gravity: The intensive and extensive margins of international trade.’’ *American Economic Review*, 98(4), pp. 1707--21. doi:10.1257/aer.98.4.1707.

Costinot, Arnaud and Andrés Rodríguez-Clare (2014). ‘‘Chapter 4 - Trade theory with numbers: Quantifying the consequences of globalization.’’ In Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, editors, *Handbook of International Economics*, volume 4 of *Handbook of International Economics*, pp. 197--261. Elsevier. doi:10.1016/B978-0-444-54314-1.00004-5.

Costinot, Arnaud, Andrés Rodríguez-Clare, and Iván Werning (2016). ‘‘Micro to macro:

Optimal trade policy with firm heterogeneity.'' Working Paper 21989, National Bureau of Economic Research. doi:10.3386/w21989.

Costinot, Arnaud, Andrés Rodríguez-Clare, and Iván Werning (2020). ''Micro to macro: Optimal trade policy with firm heterogeneity.'' *Econometrica*, 88(6), pp. 2739--2776. doi:https://doi.org/10.3982/ECTA14763. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA14763.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum (2008). ''Global rebalancing with gravity: Measuring the burden of adjustment.'' *IMF Eighth Jacques Polak Annual Research Conference*, 55(3), pp. 511--540. URL https://www.jstor.org/stable/40377786.

Demidova, Svetlana and Andrés Rodríguez-Clare (2009). ''Trade policy under firm-level heterogeneity in a small economy.'' *Journal of International Economics*, 78(1), pp. 100 -- 112. doi:https://doi.org/10.1016/j.jinteco.2009.02.009. URL http://www.sciencedirect.com/science/article/pii/S0022199609000269.

Dhingra, Swati and John Morrow (2019). ''Monopolistic competition and optimum product diversity under firm heterogeneity.'' *Journal of Political Economy*, 127(1), pp. 196--232. doi:10.1086/700732.

Diamond, Peter A. (1982). ''Aggregate demand management in search equilibrium.'' *Journal of Political Economy*, 90(5), pp. 881--894. doi:10.1086/261099.

Drozd, Lukasz A. and Jaromir B. Nosal (2012). ''Understanding International Prices: Customers as Capital.'' *American Economic Review*, 102(1), pp. 364--395. doi:10.1257/aer.102.1.364.

Dubé, Jean-Pierre, Jeremy T. Fox, and Che-Lin Su (2012). ''Improving the Numerical Performance of Static and Dynamic Aggregate Discrete Choice Random Coefficients Demand Estimation.'' *Econometrica*, 80(5), pp. 2231--2267. doi:10.3982/ECTA8585.

Eaton, Jonathan, Marcela Eslava, David Jinkins, C. J. Krizan, and James Tybout (2014). ''A search and learning model of export dynamics.'' Mimeo, Pennsylvania State University. Available at http://www.davidjinkins.com/docs/EEJKT_02_26_2014.pdf.

Eaton, Jonathan, David Jinkins, James Tybout, and Daniel Xu (2016). ''Two-sided search in international markets.'' Mimeo, Pennsylvania State University. Available at https://ideas.repec.org/p/red/sed016/973.html.

Eaton, Jonathan, David Jinkins, James Tybout, and Daniel Xu (2022). ''Two-Sided Search in International Markets.'' NBER Working Paper 29684, National Bureau of Economic Research. doi:10.3386/w29684.

Eaton, Jonathan and Samuel Kortum (2002). ''Technology, geography, and trade.'' *Econometrica*, 70(5), pp. 1741--1779. doi:10.1111/1468-0262.00352.

Felbermayr, Gabriel, Benjamin Jung, and Mario Larch (2013). ''Optimal tariffs, retaliation, and the welfare loss from tariff wars in the Melitz model.'' *Journal of International Economics*, 89(1), pp. 13 -- 25. doi:https://doi.org/10.1016/j.jinteco.2012.06.001. URL http://www.sciencedirect.com/science/article/pii/S0022199612001195.

FRB (2020). ''Capacity utilization: Manufacturing (SIC).'' https://fred.stlouisfed.org/series/CUMFNS. Accessed: May 14, 2020.

Gros, Daniel (1987). ''A note on the optimal tariff, retaliation and the welfare loss from tariff wars in a framework with intra-industry trade.'' *Journal of International Economics*, 23(3), pp. 357 -- 367. doi:https://doi.org/10.1016/0022-1996(87)90061-4. URL http://www.sciencedirect.com/science/article/pii/0022199687900614.

Heise, Sebastian (2016). "Firm-to-Firm Relationships and Price Rigidity Theory and Evidence." Working Paper 6226, CESifo. doi:10.2139/ssrn.2898148.

Hosios, Arthur J. (1990). "On the efficiency of matching and related models of search and unemployment." *Review of Economic Studies*, 57(2), pp. 279--298. doi:10.2307/2297382.

Imbs, Jean and Isabelle Mejean (2015). "Elasticity optimism." *American Economic Journal: Macroeconomics*, 7(3), pp. 43--83. doi:10.1257/mac.20130231.

Julien, Benoit and Sephorah Mangin (2017). "Efficiency of job creation in a search and matching model with labor force participation." *Economics Letters*, 150, pp. 149-- 151. doi:10.1016/j.econlet.2016.10.042.

Kneller, Richard and Mauro Pisu (2011). "Barriers to exporting: What are they and who do they matter to?" *World Economy*, 34(6), pp. 893--930. doi:10.1111/j.1467-9701.2011.01357.x.

Krolikowski, Pawel M. and Andrew H. McCallum (2021). "Goods-market frictions and international trade." *Journal of International Economics*, 129, p. 103,411. doi:https://doi.org/10.1016/j.jinteco.2020.103411. URL `https://www.sciencedirect.com/science/article/pii/S0022199620301264`.

Krugman, Paul (1980). "Scale economies, product differentiation, and the pattern of trade." *The American Economic Review*, 70(5), pp. 950--959. URL `http://www.jstor.org/stable/1805774`.

Lagos, Ricardo and Randall Wright (2005). "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*, 113(3), pp. 463--484. doi:10.1086/429804.

Mas-Colell, Andreau, Michael D. Whinston, and Jerry R. Green (1995). *Microeconomic Theory*. Oxford Student edition. Oxford University Press. URL `https://books.google.com/books?id=KGtegVXqD8wC`.

Melitz, Marc J. (2003). "The impact of trade on intra-industry reallocations and aggregate industry productivity." *Econometrica*, 71(6), pp. 1695--1725. doi:10.1111/1468-0262.00467.

Melitz, Marc J. and Stephen J. Redding (2015). "New trade models, new welfare implications." *The American Economic Review*, 105(3), pp. 1105--1146. URL `http://www.jstor.org/stable/43495412`.

Michaillat, Pascal and Emmanuel Saez (2015). "Aggregate Demand, Idle Time, and Unemployment." *Quarterly Journal of Economics*, 130(2), pp. 507--569. doi:10.1093/qje/qjv006.

Monarch, Ryan and Tim Schmidt-Eisenlohr (2023). "Longevity and the value of trade relationships." *Journal of International Economics*, 145, p. 103,842. doi:10.1016/j.jinteco.2023.103842.

Mortensen, Dale T. (1986). "Job search and labor market analysis." In Orley C. Ashenfelter and Richard Layard, editors, *Handbook of Labor Economics*, volume 2, chapter 15, pp. 849--919. Elsevier Science. doi:10.1016/s1573-4463(86)02005-9.

Nash, John F., Jr. (1950). "The bargaining problem." *Econometrica*, 18(2), pp. 155--162. doi:10.2307/1907266.

NBSC (2016a). "China industrial capacity utilization rate." `https://www.ceicdata.com/en/china/capacity-utilization-rate`. Accessed: March 12, 2019.

NBSC (2016b). "China private consumption expenditure." `https://www.ceicdata.com/en/indicator/china/private-consumption-expenditure`.

Accessed: March 12, 2019.

Nikaidô, Hukukane and Kazuo Isoda (1955). ‘‘Note on non-cooperative convex game.’’ *Pacific Journal of Mathematics*, 5(5), pp. 807--815. doi:10.2140/pjm.1955.5.807.

Osborne, Martin J. and Ariel Rubinstein (1990). *Bargaining and Markets*. London: Academic Press. Available at https://books.osborne.economics.utoronto.ca/.

Ossa, Ralph (2011). ‘‘A ‘‘new trade’’ theory of GATT/WTO negotiations.’’ *Journal of Political Economy*, 119(1), pp. 122--152. doi:10.1086/659371.

Ossa, Ralph (2014). ‘‘Trade wars and trade talks with data.’’ *American Economic Review*, 104(12), pp. 4104--46. doi:10.1257/aer.104.12.4104. URL https://www.aeaweb.org/articles?id=10.1257/aer.104.12.4104.

Petrosky-Nadeau, Nicolas and Etienne Wasmer (2017). *Labor, credit, and goods markets: The macroeconomics of search and unemployment*. MIT Press. Available at https://mitpress.mit.edu/books/labor-credit-and-goods-markets.

Petrosky-Nadeau, Nicolas, Etienne Wasmer, and Philippe Weil (2021). ‘‘When Hosios Meets Phillips: Connecting Efficiency and Stability to Demand Shocks.’’ Working Paper 2018-13, Federal Reserve Bank of San Francisco. doi:10.24148/wp2018-13.

Piazzesi, Monika, Martin Schneider, and Johannes Stroebel (2020). ‘‘Segmented Housing Search.’’ *American Economic Review*, 110(3), p. 720–59. doi:10.1257/aer.20141772. URL https://www.aeaweb.org/articles?id=10.1257/aer.20141772.

Pissarides, Christopher A. (1985). ‘‘Short-run equilibrium dynamics of unemployment, vacancies, and real wages.’’ *American Economic Review*, 75(4), pp. 676--690. URL https://www.jstor.org/stable/1821347.

Pissarides, Christopher A. (2000). *Equilibrium Unemployment Theory*. MIT Press, second edition. URL https://ideas.repec.org/b/mtp/titles/0262161877.html.

Rogerson, Richard, Robert Shimer, and Randall Wright (2005). ‘‘Search-theoretic models of the labor market: A survey.’’ *Journal of Economic Literature*, 43(4), pp. 959--988. doi:10.1257/002205105775362014.

Romalis, John (2007). ‘‘NAFTA’s and CUSFTA’s Impact on International Trade.’’ *Review of Economics and Statistics*, 89(3), pp. 416--435. doi:10.1162/rest.89.3.416.

Shimer, Robert (2005). ‘‘The cyclical behavior of equilibrium unemployment and vacancies.’’ *American Economic Review*, 95(1), pp. 25--49. doi:10.1257/0002828053828572.

Smith, Lones (2006). ‘‘The Marriage Model with Search Frictions.’’ *Journal of Political Economy*, 114(6), pp. 1124--1144. doi:10.1086/510440.

STEP (2024). ‘‘State Trade Expansion Program.’’ https://www.sba.gov/funding-programs/grants/state-trade-expansion-program-step Accessed: December 24, 2024.

Su, Che-Lin and Kenneth L. Judd (2012). ‘‘Constrained Optimization Approaches to Estimation of Structural Models.’’ *Econometrica*, 80(5), pp. 2213--2230. doi:10.3982/ECTA7925.

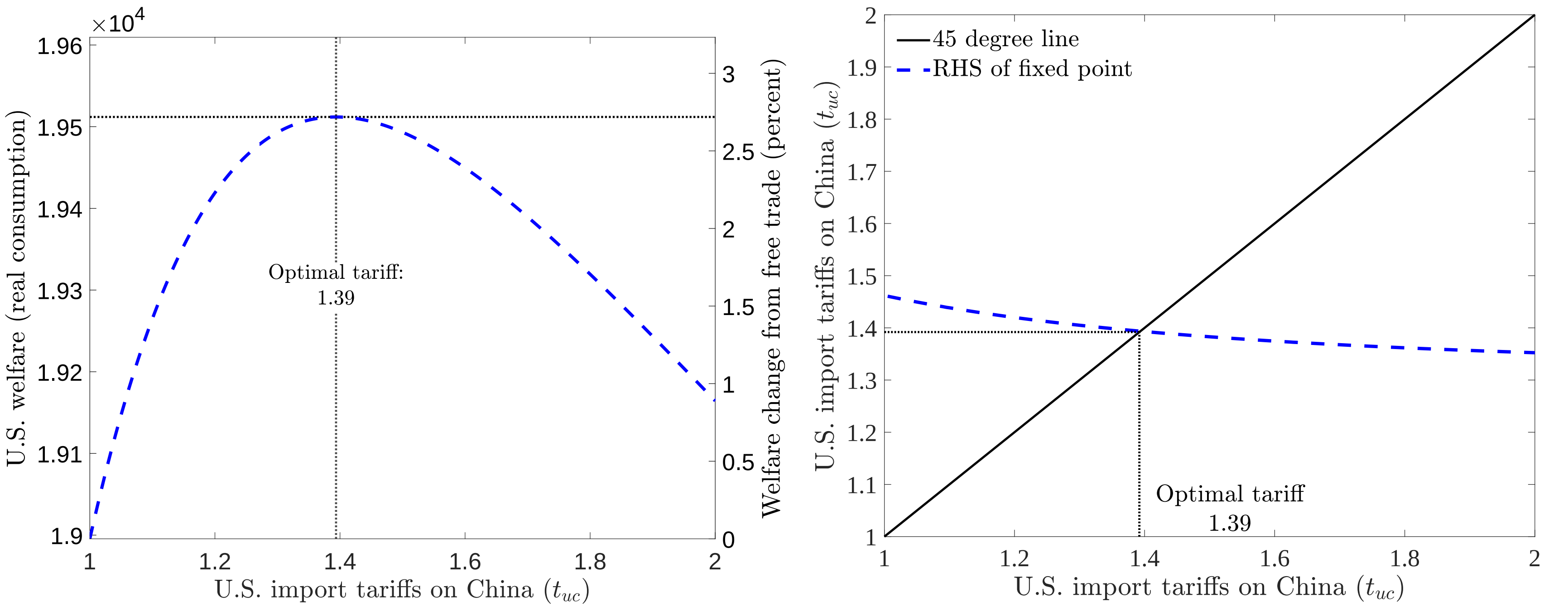
TPA (2024). ‘‘Trade Promotion Authority.’’ https://ustr.gov/trade-topics/trade-promotion-authority Accessed: December 24, 2024.

Wasmer, Etienne and Philippe Weil (2004). ‘‘The Macroeconomics of Labor and Credit Market Imperfections.’’ *American Economic Review*, 94(4), p. 944–963. doi:10.1257/0002828042002525.

WB (2016). ‘‘Gross domestic product for China [MKTGDPCNA646NWDB].’’

https://fred.stlouisfed.org/series/MKTGDPCNA646NWDB. Accessed: March 12, 2019.
WB (2018). "Enterprise surveys." http://www.enterprisesurveys.org/. Accessed: March 12, 2018.

Fig. 1: U.S. welfare and optimal tariffs without search frictions

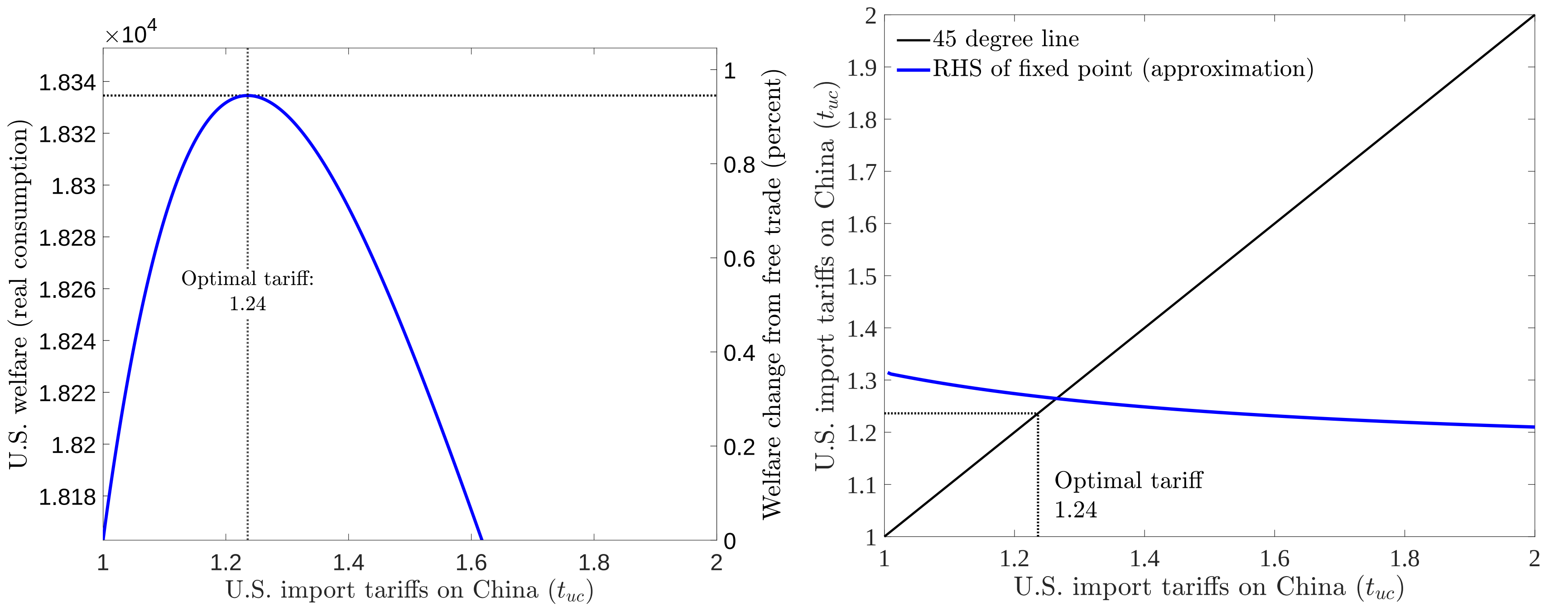


(a) U.S. welfare

(b) Solving for the optimal tariff

Note: Fig. 1a plots U.S. welfare as function of the U.S. import tariff, t_{uc}. Fig. 1b plots the left-hand side (LHS) and the right-hand side (RHS) of Eq. (33), which characterizes the optimal tariff. The solid black lines depicts the LHS of Eq. (33) and the dotted blue line depicts the RHS of Eq. (33) for various levels of the U.S. import tariff. The LHS is a 45-degree line. The intersection of the two lines identifies the optimal tariff, as described in Section 3.2. See Section 3.3.1 and Appendix B.5 for further details.

Fig. 2: U.S. welfare and optimal tariffs with search frictions

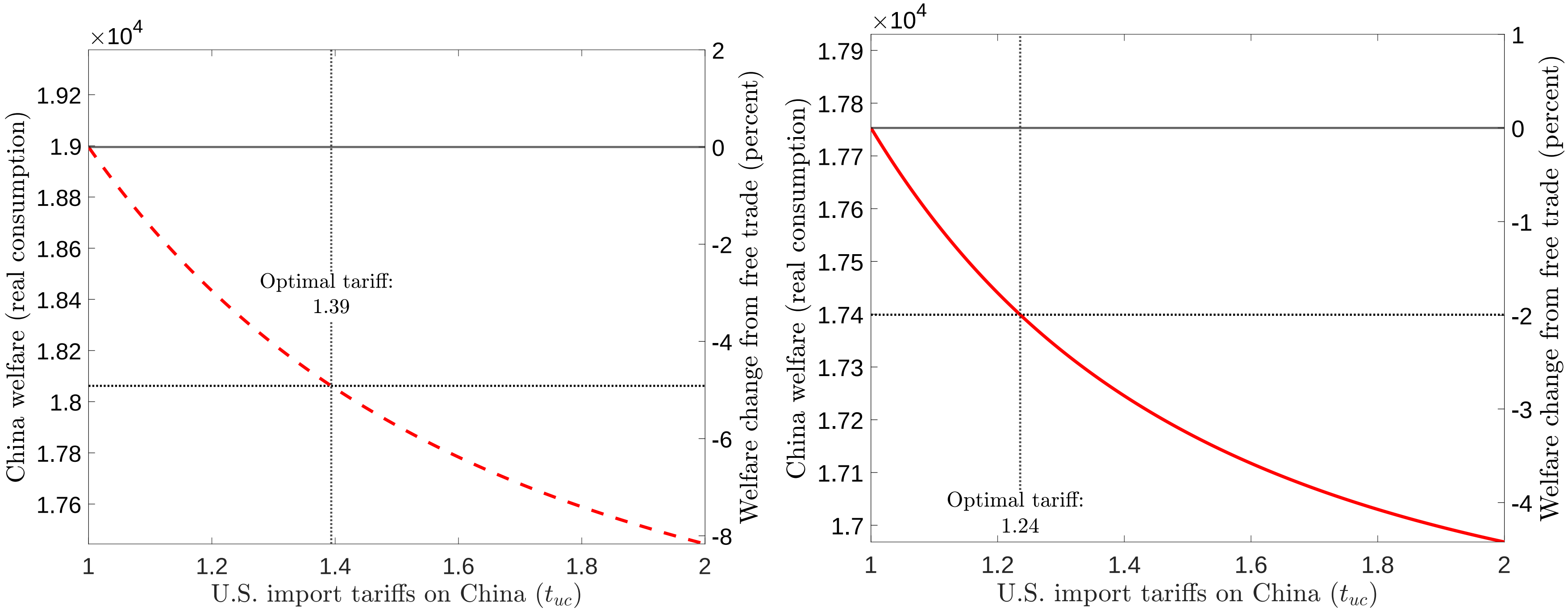


(a) U.S. welfare

(b) Solving for the optimal tariff

Note: Fig. 2a plots U.S. welfare as function of the U.S. import tariff, t_{uc}. Fig. 1b plots the left-hand side (LHS) and the right-hand side (RHS) of Eq. (31), which is an approximation to the optimal tariff. The solid black lines depicts the LHS of Eq. (31) and the solid blue line depicts the RHS of Eq. (31) for various levels of the U.S. import tariff. The LHS is a 45-degree line. The intersection of the two lines approximately identifies the optimal tariff, as described in Section 3.2. See Section 3.3.1 for further details.

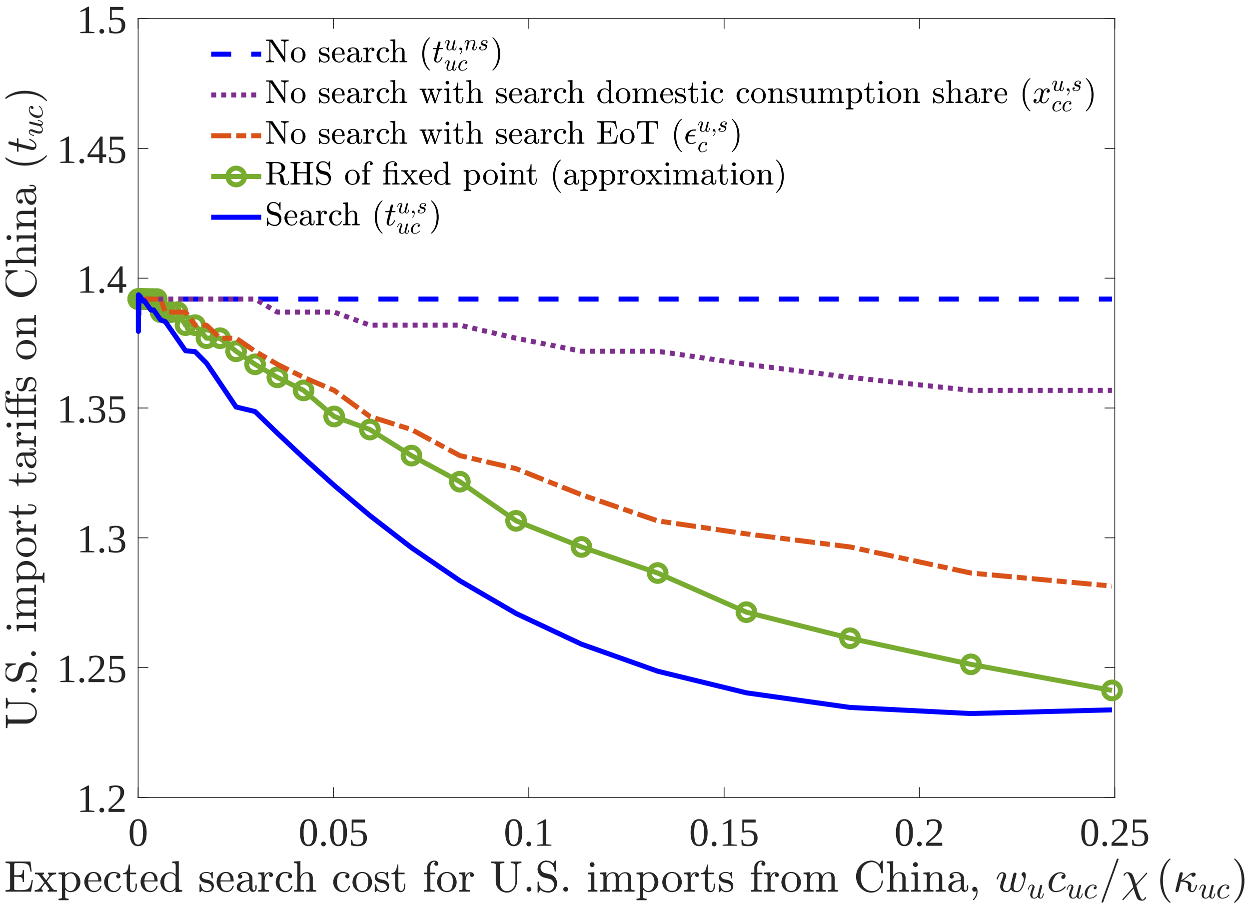
Fig. 3: Chinese welfare as a function of U.S. tariffs



(a) Without search frictions (b) With search frictions

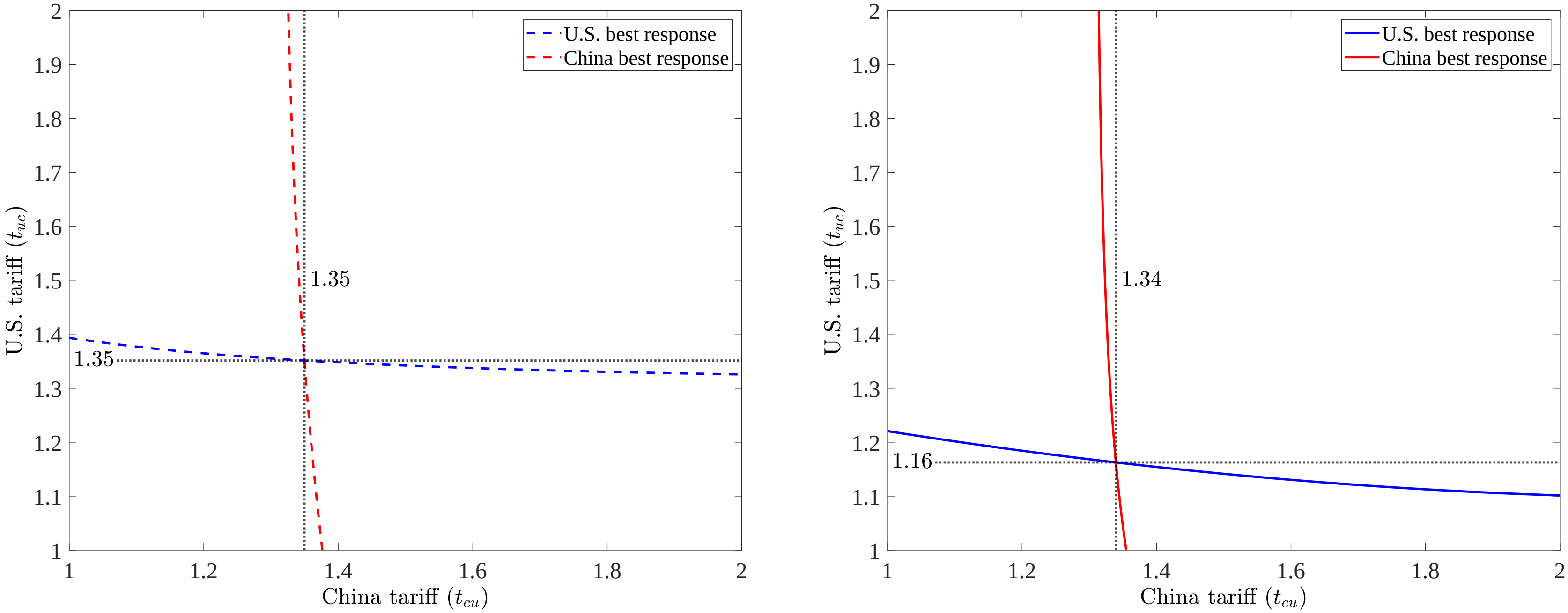
Note: Fig. 3a plots Chinese welfare as a function of the U.S. import tariff, t_{uc}, in the numerical example without search frictions. Fig. 3b plots Chinese welfare as a function of the U.S. import tariff, t_{uc}, in the numerical example with search frictions. See Section 3.3.1 for further details.

Fig. 4: Optimal import tariffs and search costs in the import market



Note: The optimal unilateral import tariff, t_{uc}^{u}, obtained by solving the U.S. country social planner's problem (Eq. 29) for different levels of search frictions, as measured by the expected costs to retailers in the uc market of searching for a producer in billions of U.S. dollars, $w_u c_{uc}/\chi\,(\kappa_{uc})$. For this exercise, we set search costs to zero in all other markets except the uc market. So, the vertical-axis intercept denotes the optimal tariff in a model with zero search costs in all markets (see Section 3.3.1 and Fig. 1a). See Section 3.3.2 for further details.

Fig. 5: Pure strategy Nash equilibrium: Numerical example

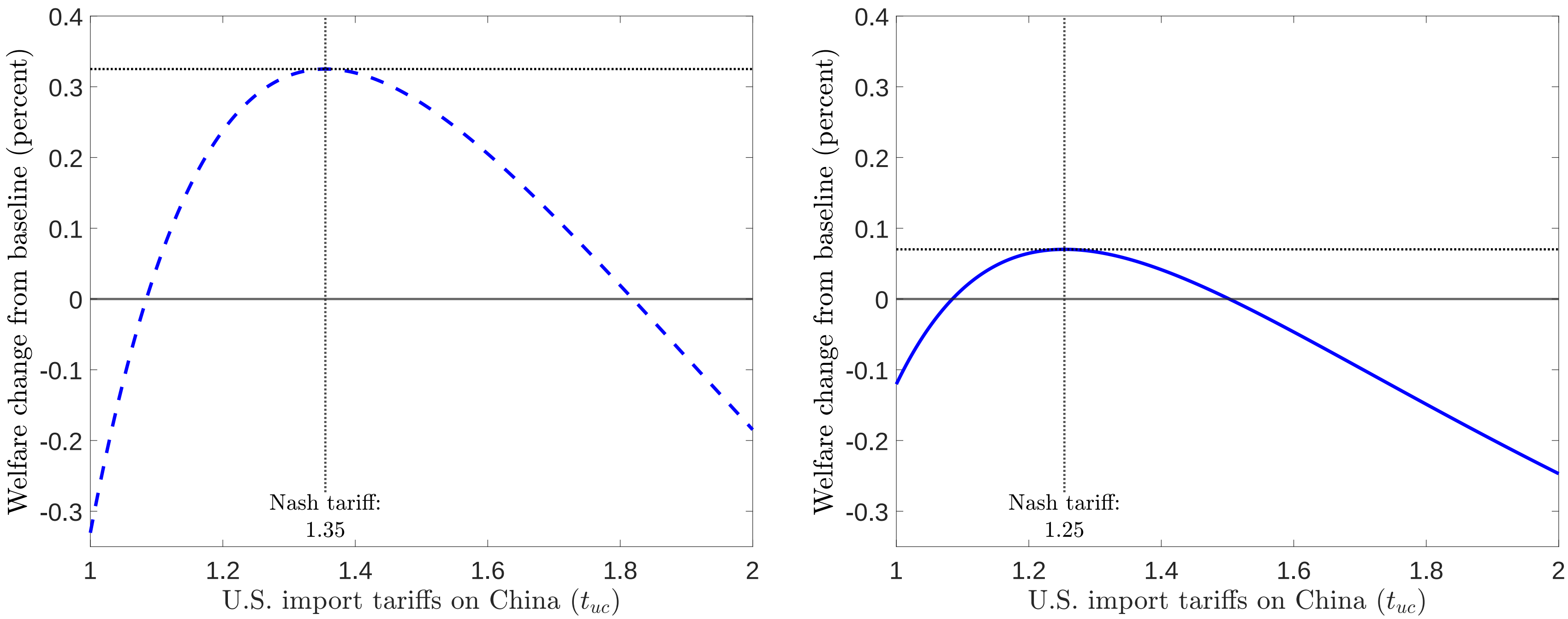


(a) Without search frictions

(b) With search frictions

Note: Fig. 5a plots best response functions of the US and China in the numerical example without search frictions. Fig. 5b plots best response functions of the US and China in the numerical example with search frictions. See Section 3.4.2 for further details.

Fig. 6: U.S. welfare effects attenuated by search frictions



(a) International search costs reduced to domestic levels

(b) With baseline search frictions

Note: Fig. 6a plots U.S. welfare as a function of U.S. import tariffs in the numerical example without international search frictions. Fig. 6b plots plots U.S. welfare as a function of U.S. import tariffs in the numerical example with search frictions. In both figures, China's import tariff is set to its respective Nash equilibrium solution. See Section 5.2 for further details.

Table 1: Calibrated model parameters

Parameter	Value	Unit	Reason
Panel A. Externally calibrated parameters			
Producers' bargaining power (β)	0.50	fraction	Benchmark
Risk-free rate (r)	0.05	percent	Interest rate
Separation rate (λ)	1.00	Poisson rate	Among trading partners
Elasticity of substitution (σ)	4.00	elasticity	Demand estimation
Pareto shape parameter (θ)	3.18	unitless	U.S. firm size distribution
Efficiency of matching function (ξ)	1.00	elasticity	Normalization
US domestic tax (t_{uu})	1.06	multiple	Sales tax rate
CH import tariff (t_{cu})	1.17	multiple	Import VAT rate
US import tariff (t_{uc})	1.08	multiple	Tariffs plus sales tax
CH domestic tax (t_{cc})	1.11	multiple	VAT rate
Internal distance US to US (distance_{uu})	1.44	kkm	Distance
Distance to CH from US (distance_{cu})	11.18	kkm	Distance
Distance to US from CH (distance_{uc})	11.18	kkm	Distance
Internal distance CH to CH (distance_{cc})	1.44	kkm	Distance
Panel B. Internally calibrated parameters			
US domestic search cost ($c_{uu} \times 10^3/\chi(\kappa_{uu})$)	2.14	labor	US mfg. capacity utilization
CH importers' search cost ($c_{cu} \times 10^2/\chi(\kappa_{cu})$)	1.18	labor	Percent of US firms exp. to CH
US importers' search cost ($c_{uc} \times 10/\chi(\kappa_{uc})$)	2.25	labor	Percent of CH firms exp. to US
CH domestic search cost ($c_{cc}/\chi(\kappa_{cc})$)	2.13	labor	CH mfg. capacity utilization
US domestic fixed cost ($f_{uu} \times 10^3$)	0.61	labor	US business failure rate
US export fixed cost ($f_{cu} \times 10^3$)	0.48	labor	CH-US exporter failure rate
CH export fixed cost (f_{uc})	0.27	labor	US-CH exporter failure rate
CH domestic fixed cost (f_{cc})	0.57	labor	CH business failure rate
Iceberg parameter (a_1)	9.89	multiple	Gravity equation
Effect of distance on iceberg (a_2)	0.11	elasticity	Gravity equation
US exploration cost ($e_u^x \times 10^3$)	11.91	labor	Investment expenditure
CH exploration cost (e_c^x)	57.08	labor	Investment expenditure
Labor endowment in US (L_u)	242.78	mn. people	Working age population
Labor endowment in CH (L_c)	940.40	mn. people	Working age population
Firm endowment in US ($N_u^x \times 10^{-3}$)	4.27	mn. varieties	Consumption level
Firm endowment in CH (N_x^c)	10.45	mn. varieties	Consumption level
Cobb-Douglas exponent (α)	0.50	fraction	Tradables consumption
Elasticity of matching function (η)	0.75	elasticity	Log-linear import elasticity

Note: The "Reason" column provides the reason for externally calibrated parameters and the main source of identification for internally calibrated parameters. The levels of the retailer search costs, c_{do}, do not have meaning because they depend on the normalization of the matching efficiency, ξ, as in Shimer (2005). Therefore, we report average retailer search costs, $c_{do}/\chi(\kappa_{do})$, which have intrinsic meaning. Parameters not shown are $h_{do} = l_{do} = s_{do} = 0\ \forall do$. Calibrated parameters of the model are at annual frequency. We discuss the calibration methodology in Section 4 and intuition for parameter identification in Appendix C.1. "CH" stands for China and "US" stands for the United States.

Table 2: Model fit

Moment description	Data	Model	Unit
Log-linear import	-6	-9	elasticity
US mfg. capacity utilization	75	66	percent
US firms exporting to CH	6	9	percent
CH firms exporting to US	21	30	percent
CH mfg. capacity utilization	74	65	percent
US business failure rate	20	20	percent
CH-US exporter failure rate	50	47	percent
US-CH exporter failure rate	40	38	percent
CH business failure rate	20	20	percent
US absorption of domestic prod. (IM_{uu})	2,839	5,330	\$ bn.
CH imports from US (IM_{cu})	463	377	\$ bn.
US imports from CH (IM_{uc})	463	377	\$ bn.
CH absorption of domestic prod. (IM_{cc})	2,715	2,335	\$ bn.
US working age population	214	243	mn. people
CH working age population	988	940	mn. people
US wage	52	59	\$ thsnd.
CH wage	11	11	\$ thsnd.
US GDP	18,707	16,191	\$ bn.
CH GDP	11,191	11,731	\$ bn.
US consumption	12,727	12,800	\$ bn.
CH consumption	4,418	4,775	\$ bn.
US non-tradable consump. share	68	50	percent
CH non-tradable consump. share	57	32	percent
US dom. consump. share ($IM_{uu}/(IM_{uu}+IM_{uc})$)	86	93	percent
CH dom. consump. share ($IM_{cc}/(IM_{cc}+IM_{cu})$)	85	86	percent
PPP price ratio (Ξ_c/Ξ_u)	60	60	percent

Note: The model matches the empirical targets relatively well. The "Data" and "Model" columns present the value of the corresponding moment in the data and model at the calibrated parameter values given in Table 1. We discuss model fit in Section 4. "CH" stands for China, "US" for the United States, "GDP" for gross domestic product, and "PPP" for purchasing power parity.

Table 3: Optimal tariffs with baseline search frictions

	(1) Baseline	(2) US CSP	(3) CH CSP	(4) Nash
CH import tariff (pct.)	17	17	29	25
US import tariff (pct.)	8	27	8	25
US-CH trade (bn $)	377	273	294	235
US welfare (bn real $)	175	175	175	175
CH welfare (bn real $)	108	107	109	107
US welfare change (pct.)		0.1	-0.18	-0.06
CH welfare change (pct.)		-1.05	0.11	-0.92

Note: The value of trade and welfare in the baseline calibration, under unilateral optimal import tariffs, and in the Nash equilibrium. See Section 5.1 for further details.

Table 4: Optimal tariffs with international search costs reduced to domestic levels

	(1) Baseline	(2) US CSP	(3) CH CSP	(4) Nash
CH import tariff (pct.)	17	17	29	24
US import tariff (pct.)	8	37	8	35
US-CH trade (bn $)	892	612	724	547
US welfare (bn real $)	178	179	177	178
CH welfare (bn real $)	117	113	117	113
US welfare change (pct.)		0.38	-0.4	0.09
CH welfare change (pct.)		-3.56	0.2	-3.38

Note: The value of trade and welfare in the baseline calibration, under unilateral optimal import tariffs, and in the Nash equilibrium. See Section 5.1 for further details.

Appendix to "Tariffs and Goods-Market Search Frictions"

Pawel M. Krolikowski* Andrew H. McCallum†

January 13, 2025

A Model appendix: Model details, model solution, and aggregation

A.1 Consumers

The homogeneous good has price $p_d(1)$. Define P_d as the price index for the bundle of differentiated varieties and P_{do} as the price index for the bundle of varieties produced in country o and consumed in country d:

$$P_d = \left[\sum_{k=1}^{O} \int_{\omega \in \Omega_{dk}} p_{dk}(\omega)^{1-\sigma} \, d\omega\right]^{\frac{1}{1-\sigma}} = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{A1}$$

The ideal price index including the homogeneous good that minimizes expenditure to obtain utility level $U_d = 1$ is

$$\Xi_d = \left[p_d(1) / (1-\alpha)\right]^{1-\alpha} \left[P_d/\alpha\right]^{\alpha}. \tag{A2}$$

We solve the consumer's utility maximization and expenditure minimization problems explicitly in KM, Appendix A.1.

The value of consumption of the differentiated good in the do market is defined as the integral over all varieties, ω, of the value of $q_{do}(\omega)$ units evaluated at final sales prices, $p_{do}(\omega)$:

$$C_{do} = \int_{\omega \in \Omega_{dk}}^{\infty} p_{do}(\omega) \, q_{do}(\omega) \, d\omega. \tag{A3}$$

A.2 Matching and value functions for producers and retailers

A.2.1 The matching function

The matching function, denoted by $m(u_{do}N_o^x, v_{do}N_d^m)$, gives the flow number of relationships formed at any moment in time as a function of the stock number of unmatched producers, $u_{do}N_o^x$, and unmatched retailers, $v_{do}N_d^m$, in the do market. N_o^x and N_d^m represent the total mass of producing firms in country o and retailing firms in country d regardless of their match status. The fraction of producers in country o looking for retailers in country d is u_{do}. The fraction of retailers that are searching for producing firms in this market is v_{do}.

As in many studies of the labor market (Pissarides, 1985; Shimer, 2005), we assume that the matching function takes a Cobb-Douglas form:

$$m(u_{do}N_o^x, v_{do}N_d^m) = \xi (u_{do}N_o^x)^{\eta} (v_{do}N_d^m)^{1-\eta}, \tag{A4}$$

in which ξ is the matching efficiency and η is the elasticity of matches with respect to the number of searching producers.

*pawel.krolikowski@clev.frb.org, Federal Reserve Bank of Cleveland, 1455 E 6th St, Cleveland, OH 44114

†Corresponding author, andrew.h.mccallum@frb.gov, Board of Governors of the Federal Reserve System, Washington, DC 20551

The matching function in Eq. (A4) is homogeneous of degree one. Therefore, market tightness, $\kappa_{do} = v_{do}N_d^m / u_{do}N_o^x$, which is the ratio of the mass of searching retailers to the mass of producers in a given market, is sufficient to determine contact rates on both sides of that market. In particular, the rate at which retailers in country d contact producers in country o, $\chi(\kappa_{do})$, is the number of matches formed each instant over the number of searching retailers:

$$\chi(\kappa_{do}) = \frac{m(u_{do}N_o^x, v_{do}N_d^m)}{v_{do}N_d^m} = \frac{\xi(u_{do}N_o^x)^{\eta}(v_{do}N_d^m)^{1-\eta}}{v_{do}N_d^m} = \xi\kappa_{do}^{-\eta}. \tag{A5}$$

Notice that retailers' contact rate falls with market tightness ($d\chi(\kappa_{do})/d\kappa_{do} < 0$) because with more retailers relative to producers, the search market becomes congested with retailers.

The rate at which producers in country o contact retailers in country d is the number of matches formed each instant over the number of searching producers, so that the producer contact rate is

$$\kappa_{do}\chi(\kappa_{do}) = \xi\kappa_{do}^{1-\eta}. \tag{A6}$$

Producers' contact rate rises with tightness ($d\kappa_{do}\chi(\kappa_{do})/d\kappa_{do} > 0$), also called a market thickness effect. Market tightness is defined from the perspective of producers so that the market is tighter when there are relatively more retailers than producers. Eqs. (A5) and (A6) are restated in Eq. (4) of the main text.

A.2.2 Producers' value functions

The value of a producer with productivity φ being matched to a retailer, $X_{do}(\varphi)$, can be summarized by a value function in continuous time defined in Eq. (6). That asset equation states that the flow return at the risk-free rate, r, from the value of producing must equal the flow payoff plus the expected capital gain from operating as an exporting producer. Each producer is indexed by exogenous productivity, φ. The flow payoff consists of $n_{do}q_{do}$, the revenue obtained from selling q_{do} units of the good at negotiated price n_{do} to retailers, less the variable, Eq. (5), and fixed costs of production, $w_o f_{do}$. The last term in Eq. (6) is the value from the dissolution of the match, which occurs at exogenous rate λ and leads to a capital loss of $U_{do}(\varphi) - X_{do}(\varphi)$ as the producer loses value $X_{do}(\varphi)$ but gains the value of being an unmatched producer, $U_{do}(\varphi)$. In writing Eq. (6), we explicitly write the value $X_{do}(\varphi)$ as a function of the producer's productivity, φ, but we conserve on notation by omitting this argument from the negotiated price, n_{do}, and traded quantity, q_{do}.

The value that an unmatched producer receives from looking for a retail partner without being in a business relationship is defined by Eq. (7). The flow search cost, $w_o l_{do}$, is what the producer pays when looking for a retailer, examples of which are the costs of maintaining foreign sales offices, sending sales representatives abroad, researching potential foreign buyers, and paying for a web presence. The second term captures the expected capital gain, in which $\kappa_{do}\chi(\kappa_{do})$ is the endogenous rate at which producing firms contact retailers, and $w_o s_{do}$ is the sunk cost of starting up the relationship. The producer considers the difference between being in a business relationship, $X_{do}(\varphi)$, and searching, $U_{do}(\varphi)$, rather than these quantities separately. Therefore, any additive term that enters both Eqs. (6) and (7) will not affect producers' decisions.

The producing firm also has the option of remaining idle and not expending resources to look for a retailer. For producers, the value of not searching, $I_{do}(\varphi)$, is given by Eq. (8).

The value to a producer of remaining idle can be interpreted, for example, as the value of the stream of payments after liquidation or the flow payoff from home production if these firms are viewed as entrepreneurs.

A.2.3 Retailers' value functions

The value of a retailing firm in a business relationship with a producer of productivity φ, is defined by the asset Eq. (9) The flow payoff from being in a relationship is the revenue generated by selling q_{do} units of the product to a representative consumer at a final sales price, p_{do},— determined in Appendix A.3.3—less the tariff-inclusive cost of acquiring these goods from producers at negotiated price n_{do}. As stated in the main text, tariff revenue is collected from the retailers by the government and rebated in a lump sum to consumers. When the relationship is destroyed exogenously, at rate λ, the retailing firm loses the capital value of being matched. All retailers are identical before matching but have differential matched values because producers are heterogeneous in their productivity.

Retailers do not use the product as an input in another stage of production but only facilitate the match between producers and consumers and collect tariffs that are paid to the government. In the event that the relationship undergoes an exogenous separation, at rate λ, the retailing firm loses the capital value of being matched, $V_{do} - M_{do}(\varphi)$.

The value of being an unmatched retailer, V_{do}, satisfies Eq. (10). Retailers need to pay a flow cost, $w_d c_{do}$, to search for a producing affiliate. At endogenous Poisson rate $\chi(\kappa_{do})$, retailing firms meet a producer of unknown productivity. Producers' productivities are ex-ante unknown to retailers so retailers take the expectation over all productivities they might encounter when computing the expected continuation value of searching. As a result, the value, V_{do}, is not a function of a producer's productivity, φ, but rather a function of the expected payoff. We assume that upon meeting, but before consummating a match, retailers learn the productivity of the producer. Depending on the producer's productivity, φ, retailers choose between matching with that producer, which generates value $M_{do}(\varphi)$, or continuing the search, which generates V_{do}. Hence, the capital gain to retailers from meeting a producer with productivity φ can be expressed as $\max\{V_{do}, M_{do}(\varphi)\} - V_{do}$. In an equilibrium with free entry into retailing, this approach is equivalent to retailers observing producers' productivity after matches are formed.

A.3 Solving the partial-equilibrium search problem

A.3.1 The surplus, value, and expected duration of a relationship

To derive the surplus in terms of model primitives, substitute Eqs. (6), (7), (9), and free entry for retailers, $V_{do} = 0$, into Eq. (11) to write the surplus as,

$$\left(r + \lambda + \frac{\beta \kappa_{do} \chi(\kappa_{do})}{\beta + t_{do}(1-\beta)}\right) S_{do}(\varphi) = p_{do} q_{do} + n_{do} q_{do}(1 - t_{do}) - v(q_{do}, w_o, \tau_{do}, \varphi) - \delta_{do}, \tag{A7}$$

in which we define

$$\delta_{do} \equiv w_o f_{do} - w_o l_{do} - \kappa_{do} \chi(\kappa_{do}) w_o s_{do}. \tag{A8}$$

Now substitute the negotiated price from Eq. (13) into Eq. (A7) and use the definition of

$$\gamma_{do} \equiv \frac{(r+\lambda)(1-\beta)}{r + \lambda + \beta \kappa_{do} \chi(\kappa_{do})} \tag{A9}$$

to write surplus as

$$S_{do}(\varphi) = \left(\frac{\beta + t_{do}(1-\beta)}{r + \lambda + \beta\kappa_{do}\chi(\kappa_{do})}\right)\left(\frac{p_{do}q_{do}}{t_{do}} - \upsilon(q_{do}, w_o, \tau_{do}, \varphi) - \delta_{do}\right). \tag{A10}$$

The surplus created by a match is the appropriately discounted after-tariff flow profit, with the search cost $w_o l_{do}$ and the sunk cost $w_o s_{do}$ also entering the surplus equation because being matched avoids paying these costs.

There are four things to notice about Eq. (A10). First, when $t_{do} = 1$, it becomes the surplus in Appendix A.3 Eq. (A33) of KM. Second, the surplus from a match is a function of productivity. We show in Appendix A.3.4.3 that matches that include a more productive exporting firm lead to greater surplus, that is, $S'_{do}(\varphi) > 0$. Third, the value of a relationship depends on aggregate endogenous quantities such as the price index, consumption, and finding rate $\kappa_{do}\chi(\kappa_{do})$, among others. Finally, surplus is greater than or equal to zero if and only if after-tariff total profits are. That is, when

$$\frac{p_{do}q_{do}}{t_{do}} - \upsilon(q_{do}, w_o, \tau_{do}, \varphi) - w_o f_{do} + w_o l_{do} + w_o s_{do}\kappa_{do}\chi(\kappa_{do}) \geq 0. \tag{A11}$$

The value of the relationship to the producer is, of course, $X_{do}(\varphi)$ and to the retailer $M_{do}(\varphi)$. Therefore, the total value of a matched relationship is

$$R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi). \tag{A12}$$

We can express Eq. (A12) in terms of surplus and then Eq. (11) with $V_{do} = 0$, by adding and subtracting Eq. (7), substituting in Eq. (12) for $X_{do}(\varphi) - U_{do}(\varphi)$ and then simplifying to get

$$R_{do}(\varphi) = \left[\frac{r(\beta + t_{do}(1-\beta)) + \beta\kappa_{do}\chi(\kappa_{do})}{r(\beta + t_{do}(1-\beta))}\right] S_{do}(\varphi) - \left[\frac{w_o l_{do} + \kappa_{do}\chi(\kappa_{do}) w_o s_{do}}{r}\right]. \tag{A13}$$

Eq. (A13) can be expressed in terms of model primitives using (A10) and the definitions for those functions provided in Eqs. (14), (5), and (2). Relationships are destroyed at Poisson rate λ in the model, which implies the average duration of each match is $1/\lambda$. Because the destruction rate is exogenous and does not vary in our model, the average duration of each match is constant. The value of a relationship in product markets has been of recent interest in Monarch and Schmidt-Eisenlohr (2023) and Heise (2016). Finally, Eq. (A13) is the same as the $R_{do}(\varphi)$ Eq. on page 7 of Appendix A.3 of KM when $t_{do} = 1$ and $w_o s_{do} = 0$.

A.3.2 Bargaining over the negotiated price

Upon meeting, the retailer and producer bargain over the negotiated price, n_{do}, and quantity, q_{do}, simultaneously. We assume that these objects are determined by the generalized Nash bargaining solution, which, as shown by Nash (1950) and Osborne and Rubinstein (1990), is equivalent to maximizing the following Nash product:

$$\max_{q_{do}, n_{do}} \left[X_{do}(\varphi) - U_{do}(\varphi)\right]^{\beta} \left[M_{do}(\varphi) - V_{do}\right]^{1-\beta}, 0 \leq \beta < 1, \tag{A14}$$

in which β is producers' bargaining power. To solve Eq. (A14), first solve for $X_{do}(\varphi) - U_{do}(\varphi)$ by combining Eqs. (6) and (7) to get that:

$$X_{do}(\varphi) - U_{do}(\varphi) = \frac{n_{do}q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} + w_o l_{do} + \kappa_{do}\chi(\kappa_{do}) w_o s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})}. \quad \text{(A15)}$$

Next rearrange Eq. (9) to get that:

$$M_{do}(\varphi) - V_{do} = \frac{p_{do}\left(q_{do}\right) q_{do} - t_{do} n_{do} q_{do} - r V_{do}}{r + \lambda}. \quad \text{(A16)}$$

Substitute Eqs. (A15) and (A16) into (A14), then log and differentiate with respect to the n_{do} to get the relevant first-order condition:

$$\beta \frac{q_{do}/\left(r+\lambda\right)}{X_{do}(\varphi) - U_{do}(\varphi)} + (1-\beta) \frac{-t_{do}q_{do}/\left(r+\lambda\right)}{M_{do}(\varphi) - V_{do}} = 0. \quad \text{(A17)}$$

We do not need to calculate the partial derivative with respect to κ_{do}, w_o, or other endogenous variables, because we assume individual varieties are too small to influence aggregate values. Hence, when they meet, the firms bargain taking everything but n_{do} and q_{do} as given. Furthermore, the partial derivative of the value of a vacancy is $\partial V_{do}/\partial n_{do} = 0$, because bargaining takes place over each variety, φ, individually. As long as the distribution of varieties is continuous, $\partial V_{do}/\partial n_{do}$ does not have an effect on the expectation in the continuation value in Eq. (10).

For any variety that is traded, $q_{do} > 0$, Eq. (A17) can be written as

$$\beta \left(M_{do}(\varphi) - V_{do}\right) = (1-\beta)\, t_{do} \left(X_{do}(\varphi) - U_{do}(\varphi)\right). \quad \text{(A18)}$$

Using Eqs. (A18) and (11) delivers the surplus sharing rule, Eq. (12). To find the negotiated price shown in Eq. (13), use the equilibrium free-entry condition $V_{do} = 0$ and substitute (A15) and (A16) into (A18), then solve for n_{do}.

A.3.3 Bargaining over the quantity

Substitute Eqs. (A15) and (A16) into (A14), then log and differentiate with respect to q_{do} to get the relevant first-order condition for quantity:

$$\beta \frac{\left(n_{do} - \dfrac{\partial v\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}\right)}{\left[X_{do}(\varphi) - U_{do}(\varphi)\right](r+\lambda)} + (1-\beta) \frac{\left(p_{do}\left(q_{do}\right) + \dfrac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}} q_{do} - t_{do} n_{do}\right)}{\left[M_{do}(\varphi) - V_{do}\right](r+\lambda)} = 0, \quad \text{(A19)}$$

in which we use the same reasoning for $\partial V_{do}/\partial n_{do} = 0$ as in Appendix A.3.2.

Considering only solutions with positive values for Eq. (A16) and $q_{do} > 0$, we plug Eq. (A18) into (A19) and rearrange to get:

$$p_{do}\left(q_{do}\right) + \frac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}} q_{do} = t_{do} \frac{\partial v\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}. \quad \text{(A20)}$$

This expression says that the quantity produced and traded equates the marginal revenue earned from consumers to the tariff-inclusive marginal production cost paid by producers. Eq. (A20) is the same in a model with or without search frictions, implying that search does not change the quantity traded within each match. In a model of search, parties agree upon a quantity that equates the marginal revenue and the marginal tariff-inclusive cost because that quantity maximizes surplus.

CES utility implies the consumer's price elasticity of demand from Eq. (2) is

$$\frac{\partial q_{do}}{\partial p_{do}}\frac{p_{do}}{q_{do}} = -\sigma. \tag{A21}$$

Indexing an individual variety by ω is equivalent to indexing by φ and we have treated these interchangeably when using Eq. (2) here. The equivalence of indexing variables contrasts with changing from a measure over a set of goods indexed by ω to a distribution of goods indexed by φ, which is subtle and discussed in detail in KM Appendix A.11.1.

Combining Eq. (A21) with the fact that $\partial p_{do}/\partial q_{do} = 1/\left(\partial q_{do}/\partial p_{do}\right)$, we can write Eq. (A20) as

$$p_{do}\left(q_{do}\right) + \frac{\partial p_{do}\left(q_{do}\right)}{\partial q_{do}}q_{do} = p_{do}\left(\frac{\sigma-1}{\sigma}\right) = t_{do}\frac{\partial v\left(q_{do}, w_o, \tau_{do}, \varphi\right)}{\partial q_{do}}. \tag{A22}$$

Rearranging Eq. (A22) and computing marginal costs from Eq. (5) gives Eq. (14).

Finally, setting the price equal to the average total cost (ATC) gives zero profit for any variety. As a result, Eq. (14) is always at least as high as the ATC for all traded varieties above the threshold defined in Eq. (17) and therefore defines the equilibrium price.

A.3.4 Producers' search productivity threshold

There are two productivity thresholds to consider. First, there is a productivity threshold, $\bar{\varphi}_{do}$, that makes the producer indifferent between searching and remaining idle defined by, $U_{do}\left(\bar{\varphi}_{do}\right) - I_{do}\left(\bar{\varphi}_{do}\right) = 0$. Second, there is a weakly lower productivity threshold, $\underline{\varphi}_{do}$, which makes that producer indifferent between consummating a relationship upon contacting a retailer and continuing to search defined by $X_{do}\left(\underline{\varphi}_{do}\right) - U_{do}\left(\underline{\varphi}_{do}\right) = 0$. We derive these two thresholds and show in Appendix A.3.4.2 that the binding threshold is defined by $\bar{\varphi}_{do}$, because $\bar{\varphi}_{do} \geq \underline{\varphi}_{do}$ if and only if $w_o l_{do} + w_o h_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) w_o s_{do} \geq 0$.

The productivity threshold nests the threshold from KM (Eq. 18) and we compare it to productivity thresholds in other models in Appendix A.6.4 of the same paper.

A.3.4.1 Solving for the binding productivity threshold

Combine Eqs. (7) and (8) to get

$$U_{do}\left(\varphi_{do}\right) - I_{do}\left(\varphi_{do}\right) = \frac{-w_o l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right)\left(X_{do}\left(\varphi\right) - U_{do}\left(\varphi\right) - w_o s_{do}\right) - w_o h_{do}}{r} \tag{A23}$$

The threshold productivity, $\bar{\varphi}_{do}$, is given by $U_{do}\left(\bar{\varphi}\right) - I_{do}\left(\bar{\varphi}_{do}\right) = 0$ so evaluate Eq. (A23) at $\bar{\varphi}_{do}$, set the left-hand side to zero, and rearrange to get

$$X_{do}\left(\bar{\varphi}_{do}\right) - U_{do}\left(\bar{\varphi}_{do}\right) = \frac{w_o l_{do} + w_o h_{do}}{\kappa_{do}\chi\left(\kappa_{do}\right)} + w_o s_{do}. \tag{A24}$$

Substitute Eq. (A15) into Eq. (A24) and suppress $\bar{\varphi}_{do}$ for simplicity to derive

$$\frac{n_{do}q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} + w_o l_{do} + \kappa_{do}\chi(\kappa_{do}) w_o s_{do}}{r + \lambda + \kappa_{do}\chi(\kappa_{do})} = \frac{w_o l_{do} + w_o h_{do}}{\kappa_{do}\chi(\kappa_{do})} + w_o s_{do}$$

$$\Rightarrow n_{do}q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} + w_o l_{do} + \kappa_{do}\chi(\kappa_{do}) w_o s_{do} = (r + \lambda + \kappa_{do}\chi(\kappa_{do}))\frac{w_o s_{do}\kappa_{do}\chi(\kappa_{do}) + w_o l_{do} + w_o h_{do}}{\kappa_{do}\chi(\kappa_{do})}$$

$$\Rightarrow n_{do}q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} + w_o l_{do} + \kappa_{do}\chi(\kappa_{do}) w_o s_{do} = (r+\lambda) w_o s_{do} + w_o s_{do}\kappa_{do}\chi(\kappa_{do}) + (r+\lambda)\frac{w_o l_{do} + w_o h_{do}}{\kappa_{do}\chi(\kappa_{do})} + w_o l_{do} + w_o h_{do}$$

$$\Rightarrow n_{do}q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} = (r+\lambda) w_o s_{do} + (r+\lambda)\frac{w_o l_{do} + w_o h_{do}}{\kappa_{do}\chi(\kappa_{do})} + w_o h_{do}$$

$$\Rightarrow n_{do}q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} = \left(\frac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}\right) w_o l_{do} + \left(1 + \frac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}\right) w_o h_{do} + (r+\lambda)\, w_o s_{do}.$$

Now, use the negotiated price, n_{do}, from Eq. (13), to get

$$\begin{aligned}(1-\gamma_{do})\left(\frac{p_{do}}{t_{do}}\right) q_{do} + \gamma_{do}\left(v\left(q_{do}, w_o, \tau_{do}, \varphi\right) + w_o f_{do} - w_o l_{do} - \kappa_{do}\chi(\kappa_{do}) w_o s_{do}\right) - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} \\ = \left(\frac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}\right) w_o l_{do} + \left(1 + \frac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}\right) w_o h_{do} + (r+\lambda)\, w_o s_{do}.\end{aligned}$$

which can be rearranged to obtain

$$\begin{aligned}&\left(\frac{p_{do}}{t_{do}}\right) q_{do} - v\left(q_{do}, w_o, \tau_{do}, \varphi\right) - w_o f_{do} \\ &= (1-\gamma_{do})^{-1}\left[\left(\frac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}\right) w_o l_{do} + \left(1 + \frac{r+\lambda}{\kappa_{do}\chi\left(\kappa_{do}\right)}\right) w_o h_{do} + (r+\lambda)\, w_o s_{do} + \gamma_{do}\left[w_o l_{do} + \kappa_{do}\chi\left(\kappa_{do}\right) w_o s_{do}\right]\right].\end{aligned}$$

Further simplification of the terms on the right-hand side with γ_{do} delivers Eq. (15) in the main text.

Using the price charged to consumers by retailers from Eq. (14) we can write retailer revenue as proportional to variable production costs, Eq. (5):

$$p_{do}\left(\varphi\right) q_{do}\left(\varphi\right) = \left(t_{do}\mu w_o \tau_{do}\varphi^{-1}\right) q_{do}\left(\varphi\right) = t_{do}\mu v\left(q_{do}, w_o, \tau_{do}, \varphi\right). \tag{A25}$$

Then Eq. (A25) implies that after-tariff variable profits are,

$$\left(\frac{p_{do}\left(\varphi_{do}\right)}{t_{do}}\right) q_{do}\left(\varphi_{do}\right) - v\left(q_{do}, w_o, \tau_{do}, \varphi_{do}\right) = \frac{p_{do}\left(\varphi\right) q_{do}\left(\varphi\right)}{\sigma t_{do}}. \tag{A26}$$

Or alternatively,

$$\left(\frac{p_{do}\left(\varphi_{do}\right)}{t_{do}}\right) q_{do}\left(\varphi_{do}\right) - v\left(q_{do}, w_o, \tau_{do}, \varphi_{do}\right) = (\mu - 1)\, v\left(q_{do}, w_o, \tau_{do}, \varphi\right). \tag{A27}$$

Substitute Eqs. (14) and (2) into Eq. (A26) and then substitute the resulting expression into the left hand side of Eq. (15) to get that:

$$\frac{\alpha}{\sigma} C_d P_d^{\sigma-1}\left(\mu w_o \tau_{do}\right)^{1-\sigma} t_{do}^{-\sigma}\bar{\varphi}_{do}^{\sigma-1} = F\left(\kappa_{do}\right). \tag{A28}$$

Solving this expression for $\bar{\varphi}_{do}$ gives Eq. (17).

Finally, all matches must have positive surplus so we can check that $S_{do}\left(\bar{\varphi}_{do}\right) \geq 0$ by using Eqs. (12) and (A24) to write

$$S_{do}\left(\bar{\varphi}_{do}\right)=\left(\frac{\beta+t_{do}\left(1-\beta\right)}{\beta}\right)\left(\frac{w_o l_{do}+w_o h_{do}}{\kappa_{do}\chi\left(\kappa_{do}\right)}+w_o s_{do}\right). \tag{A29}$$

Eq. (A29) puts restrictions on the parameters because they must be such that $S_{do}\left(\bar{\varphi}_{do}\right)\geq 0$. For example, $w_o h_{do}$ cannot be so negative as to make Eq. (A29) negative.

A.3.4.2 Solving for the nonbinding productivity threshold

The threshold productivity that is indifferent between matching and not, $\underline{\varphi}_{do}$, is defined by

$$X_{do}\left(\underline{\varphi}_{do}\right)-U_{do}\left(\underline{\varphi}_{do}\right)=0. \tag{A30}$$

We can be sure that $X_{do}\left(\bar{\varphi}_{do}\right)-U_{do}\left(\bar{\varphi}_{do}\right)\geq X_{do}\left(\underline{\varphi}_{do}\right)-U_{do}\left(\underline{\varphi}_{do}\right)$ as long as $\left(w_o l_{do}+w_o h_{do}\right)/\kappa_{do}\chi\left(\kappa_{do}\right)+w_o s_{do}\geq 0$. This result implies that as long as $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)$ is increasing in φ, then $\bar{\varphi}_{do}\geq\underline{\varphi}_{do}$. In Appendix A.3.4.3, we show the very general conditions under which $X_{do}\left(\varphi\right)-U_{do}\left(\varphi\right)$ is increasing in φ. The binding productivity threshold defining the mass of producers that have retail partners is the greater of these two thresholds and hence $\bar{\varphi}_{do}$. In other words, the productivity necessary to induce a producer to search for a retail partner is greater than the productivity necessary to consummate a match after meeting a retailer due to the costs that are incurred while searching. Similarly, the productivity necessary to form a match is greater than the productivity necessary to maintain one already in place. Note that $\bar{\varphi}_{do}>\underline{\varphi}_{do}$ if $\left(w_o l_{do}+w_o h_{do}\right)/\kappa_{do}\chi\left(\kappa_{do}\right)+w_o s_{do}>0$, which is true if and only if the cost of forming a relationship is positive: $w_o l_{do}+w_o h_{do}+\kappa_{do}\chi\left(\kappa_{do}\right)w_o s_{do}>0$.

A.3.4.3 The value of importing is strictly increasing in productivity

Here we show that the value of importing, $M_{do}(\varphi)$, is strictly increasing with the producer's productivity level, φ. This result leads to three implications. First, it allows us to replace the integral of the max over V_{do} and $M_{do}\left(\varphi\right)$ in Eq. (10) with the integral of $M_{do}\left(\varphi\right)$ from the threshold from Eq. (17). Second, in equilibrium, because $M'_{do}(\varphi)>0$, Eq. (12) implies that $S'_{do}\left(\varphi\right)>0$ and therefore that $X'_{do}\left(\varphi\right)-U'_{do}\left(\varphi\right)>0$. Third, it allows us to show that $\bar{\varphi}_{do}\geq\underline{\varphi}_{do}$, as we did in Appendix A.3.4.2.

Starting with Eq. (9) and $V_{do}=0$, substituting in negotiated prices from Eq. (13), and using the relationship between retailer revenue and variable costs from Eq. (A25) we can write

$$M_{do}\left(\varphi\right)=\frac{\sigma^{-1}\gamma_{do}p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)-t_{do}\gamma_{do}\delta_{do}}{r+\lambda} \tag{A31}$$

Remember that δ_{do} from Eq. (A8) and γ_{do} from Eq. (A9) are functions of tightness, κ_{do}, but not productivity, φ. It is clear from the integral in the creation of the import relationship, Eq. (18), that κ_{do} is not a function of φ. Given these facts, we can prove our result by differentiating both sides of Eq. (A31) with respect to φ and showing that $M'_{do}\left(\varphi\right)=\left(\partial M_{do}\left(\varphi\right)/\partial q_{do}\left(\varphi\right)\right)\cdot\left(\partial q_{do}\left(\varphi\right)/\partial\varphi\right)>0$. Using demand from Eq. (2), first write inverse demand $p_{do}\left(q_{do}\left(\varphi\right)\right)$ then

$$M'_{do}\left(\varphi\right)=\frac{\sigma^{-1}\gamma_{do}}{r+\lambda}\left(p_{do}\left(q_{do}\left(\varphi\right)\right)+\frac{\partial p_{do}\left(\varphi\right)}{\partial q_{do}\left(\varphi\right)}q_{do}\right)\frac{\partial q_{do}\left(\varphi\right)}{\partial\varphi}. \tag{A32}$$

The partial derivative in parentheses is marginal revenue, which we know in equilibrium will be equal to marginal cost times the tariff as shown in Eq. (A20). Using this fact and applying the chain rule to $\partial q_{do}(\varphi)/\partial\varphi$ leads to our final expression:

$$M'_{do}(\varphi) = \frac{\sigma^{-1}\gamma_{do}}{r+\lambda}\left(t_{do}\frac{\partial v(q_{do}, w_o, \tau_{do}, \varphi)}{\partial q_{do}(\varphi)}\right)\frac{\partial q_{do}(\varphi)}{\partial p_{do}(\varphi)}\frac{\partial p_{do}(\varphi)}{\partial\varphi}. \tag{A33}$$

As long as $\gamma_{do} > 0$ (which holds for finite κ_{do} and $\beta < 1$), marginal cost is positive, demand is downward sloping, and higher productivity varieties cost less, then $M'_{do}(\varphi) > 0$. These general conditions are satisfied for the functional forms of our model.

We can use the fact that $M'_{do}(\varphi) > 0$ to demonstrate the way in which many other important quantities depend on the producer's productivity level, φ. The surplus sharing rule, Eq. (12), can be rewritten as

$$\beta M_{do}(\varphi) = (1-\beta)\, t_{do}\left(X_{do}(\varphi) - U_{do}(\varphi)\right), \tag{A34}$$

We know that in equilibrium, because $M'_{do}(\varphi) > 0$, it must be that $X'_{do}(\varphi) - U'_{do}(\varphi) > 0$. Differentiating both sides of Eq. (7) gives $rU'_{do}(\varphi) = \kappa_{do}\chi(\kappa_{do})\left(X'_{do}(\varphi) - U'_{do}(\varphi)\right) > 0$. We can combine these facts to show $X'_{do}(\varphi) > U'_{do}(\varphi) > 0$. Using the definition of the joint surplus of a match, Eq. (11), we get $S'_{do}(\varphi) > 0$. Likewise, the value of a relationship, $R_{do}(\varphi) = X_{do}(\varphi) + M_{do}(\varphi)$, has $R'_{do}(\varphi) > 0$.

A.4 Aggregation

A.4.1 Resource constraint

We present extensive details about the aggregate resource constraint in Appendix D.

A.4.2 Labor market clearing country by country

Wages are determined endogenously in each country by setting labor demand equal to labor supply. Labor demand is given by Eq. (23). The costs to create firms, pay fixed costs, and form matches are included in the investment term, I_d. (Investment must be divided by the wage to yield units of labor.) Demand for $q_d(1)$ is given by Eq. (2), in which $p_d(1) = w_d$. Labor demand also includes all labor used to produce the differentiated goods for the domestic and all foreign markets. Rearranging Eq. (A25) gives

$$\frac{p_{od}(\varphi)\, q_{od}(\varphi)}{\mu w_d t_{od}} = q_{od}(\varphi)\,\tau_{od}\varphi^{-1}, \tag{A35}$$

which implies that the labor used to produce the differentiated good can be written as

$$\left(1 - \frac{u_{od}}{1-i_{od}}\right) N_d^x \int_{\bar{\varphi}_{od}} q_{od}(\varphi)\,\tau_{od}\varphi^{-1} dG(\varphi) = \frac{C_{od}}{\mu w_d t_{od}}. \tag{A36}$$

Using these facts, Eq. (23) can be written as

$$LD_d = \frac{I_d}{w_d} + \frac{(1-\alpha)\, C_d}{w_d} + \frac{1}{\mu}\sum_o \frac{C_{od}}{w_d t_{od}}.$$

Because labor is immobile and the homogeneous good is not traded, the equilibrium wage in country d is determined by substituting labor demand, LD_d, with labor supply, L_d, and then rearranging slightly:

$$w_d = \frac{I_d + (1-\alpha) C_d + \frac{1}{\mu} \sum_o \frac{C_{od}}{t_{od}}}{L_d}. \tag{A37}$$

We choose country o's wage to be the numeraire so that $w_o = 1$.

A.4.3 The ideal price index with our productivity distribution

The ideal price index is provided in Eq. (A2) and is a function of the homogeneous good price and the differentiated goods price index in Eq. (A1), which indexes over an unordered set of varieties. We can move from an unordered set of varieties to an index over a distribution of productivities using the steps in Appendix A.11.1 of KM so that the differentiated goods price index is given by:

$$P_d = \left[\sum_{k=1}^{O} P_{dk}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \left[\sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} p_{dk}(\varphi)^{1-\sigma} \, dG(\varphi)\right]^{\frac{1}{1-\sigma}}, \tag{A38}$$

in which $G(\cdot)$ is a Pareto cumulative density function from Section 2.1.3. Using the final consumer price from Eq. (14) gives

$$P_d = \left[\sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} \left(\frac{t_{dk}\mu w_k \tau_{dk}}{\varphi}\right)^{1-\sigma} dG(\varphi)\right]^{\frac{1}{1-\sigma}}. \tag{A39}$$

The relevant moment is

$$\int_{\bar{\varphi}_{dk}}^{\infty} z^{\sigma-1} dG(z) = \frac{\theta \bar{\varphi}_{dk}^{\sigma-\theta-1}}{\theta - \sigma + 1}, \tag{A40}$$

and the threshold from Eq. (17) raised to the relevant exponent is

$$\bar{\varphi}_{do}^{\sigma-1-\theta} = P_d^{\theta-(\sigma-1)} \mu^{\sigma-1-\theta} \left(\frac{\sigma}{\alpha}\right)^{1-\frac{\theta}{\sigma-1}} (w_o \tau_{do})^{\sigma-1-\theta} \left(\frac{F_{do}}{C_d}\right)^{1-\frac{\theta}{\sigma-1}} t_{do}^{\sigma-\mu\theta}. \tag{A41}$$

Because the threshold is a function of P_d, Eq. (A39) is itself a function of P_d too. Using Eq. (A41) in Eq. (A39) and simplifying gives

$$\begin{aligned} P_d &= P_d^{1-\frac{\theta}{\sigma-1}} \\ &\times \left[\left(\frac{\theta}{\theta-(\sigma-1)}\right)\left(\frac{\sigma}{\alpha}\right)^{1-\frac{\theta}{\sigma-1}} \mu^{-\theta} C_d^{\frac{\theta}{\sigma-1}-1} \sum_{k=1}^{O} \left(1 - \frac{u_{dk}}{1-i_{dk}}\right) N_k^x (w_k \tau_{dk})^{-\theta} F_{dk}^{1-\frac{\theta}{\sigma-1}} t_{dk}^{1-\mu\theta}\right]^{\frac{1}{1-\sigma}}. \end{aligned} \tag{A42}$$

Solving for P_d and rearranging gives

$$\begin{aligned} P_d &= \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu \\ &\times C_d^{\frac{1}{\theta}-\frac{1}{\sigma-1}} \\ &\times \left(\sum_{k=1}^{O}\left(1-\frac{u_{dk}}{1-i_{dk}}\right)N_k^x\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}^{-\left[\frac{\theta}{\sigma-1}-1\right]}t_{dk}^{-(\mu\theta-1)}\right)^{-\frac{1}{\theta}}. \end{aligned} \tag{A43}$$

We define

$$\lambda_2 \equiv \left(\frac{\theta}{\theta-(\sigma-1)}\right)^{-\frac{1}{\theta}}\left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}\mu, \tag{A44}$$

and the "multilateral resistance term" as

$$\rho_d \equiv \left(\sum_{k=1}^{O}\left(1-\frac{u_{dk}}{1-i_{dk}}\right)N_k^x\left(w_k\tau_{dk}\right)^{-\theta}F_{dk}^{-\left[\frac{\theta}{\sigma-1}-1\right]}t_{dk}^{-(\mu\theta-1)}\right)^{-\frac{1}{\theta}}. \tag{A45}$$

These definitions deliver the final expression of the differentiated goods price index presented in Eq. (25).

The price index in our model closely resembles the price index in Chaney (2008), Eq. (8). In that model, the price index is an equilibrium object in wages, GDP, iceberg and fixed entry costs, whereas in our model it is an equilibrium object in wages, the number of producers, market tightness (through u_{dk} and $F_{dk}(\kappa_{dk})$), iceberg and fixed entry costs, and also tariffs.

We can also show that

$$P_{do} = \left(\frac{\theta}{\theta-\sigma+1}\right)^{\frac{1}{1-\sigma}}\left(\frac{1-u_{do}-i_{do}}{1-i_{do}}\right)^{\frac{1}{1-\sigma}}\left(N_o^x\right)^{\frac{1}{1-\sigma}}\left(\mu w_o\tau_{do}t_{do}\right)\bar{\varphi}_{do}^{\frac{\sigma-\theta-1}{1-\sigma}}. \tag{A46}$$

In addition to this price index faced by consumers, which includes tariffs, we define the price index faced by producers, which includes subsidies, $w_o s_{do}$, but excludes tariffs, t_{do}:

$$\tilde{P}_{do} = \left[\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x\int_{\bar{\varphi}_{do}}^{\infty}w_o\tilde{s}_{do}\left(\mu w_o\tau_{do}\varphi^{-1}\right)^{1-\sigma}dG(\varphi)\right]^{\frac{1}{1-\sigma}}. \tag{A47}$$

For most of the paper, we assume that $\tilde{s}_{do} = 1\ \forall do$. We also define the price index at the dock, which does not include tariffs or subsidies

$$\bar{P}_{do} = \left[\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x\int_{\bar{\varphi}_{do}}^{\infty}\left(\mu w_o\tau_{do}\varphi^{-1}\right)^{1-\sigma}dG(\varphi)\right]^{\frac{1}{1-\sigma}}. \tag{A48}$$

Notice that

$$\bar{P}_{do} = \frac{P_{do}}{t_{do}}, \tag{A49}$$

and $\tilde{P}_{do} = \bar{P}_{do}$ if $\tilde{s}_{do} = 1$. Finally, define the negotiated price index as

$$\bar{N}_{do} = \frac{(1 - b_{do})}{t_{do}} P_{do} \tag{A50}$$

so that

$$\bar{N}_{do} Q_{do} = IM_{do}, \tag{A51}$$

which implies that

$$\bar{P}_{do} = \frac{\bar{N}_{do}}{1 - b_{do}}. \tag{A52}$$

A.4.4 The gravity equation with search frictions

A.4.4.1 Deriving the gravity equation

The total amount paid by the consumers in d for imports from o has to sum up to the following three terms:

$$C_{do} = IM_{do} + \Pi^r_{do} + G_{do},$$

in which C_{do} is defined in Eq. (A3), Π^r_{do} is defined in Eq. (D99), and

$$G_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_o \int_{\bar{\varphi}_{do}}^{\infty} (t_{do} - 1)\, n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi), \tag{A53}$$

so that $G_d = \sum_{k=1}^{D} G_{dk}$. Rearranging gives

$$IM_{do} = C_{do} - \Pi^r_{do} - G_{do}$$

so that

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_o \int_{\bar{\varphi}_{do}}^{\infty} n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) \tag{A54}$$

is the value of total imports.

We need to integrate over the varieties to get the total value of imports going into the domestic market, before tariffs are applied. Demand for a variety, φ, in the differentiated goods sector is given in Eq. (2). Given this demand, monopolistic competition and constant returns-to-scale production imply that producers set optimal prices according to Eq. (14). For notational simplicity, define $B_{do} \equiv \alpha (t_{do}\mu w_o \tau_{do})^{-\sigma} C_d P_d^{\sigma-1}$ and combine the optimal price with the demand curve to get $q_{do}(\varphi) = B_{do}\varphi^{\sigma}$. Evaluated at final prices, the value of sales of each variety is $p_{do}(\varphi)\, q_{do}(\varphi) = t_{do}\mu w_o \tau_{do} B_{do} \varphi^{\sigma-1}$ and the variable cost to produce $q_{do}(\varphi)$ units of this variety is $v_{do}(\varphi) = w_o \tau_{do} B_{do} \varphi^{\sigma-1}$. Using the negotiated price in Eq. (13), the value of total imports is

$$n_{do} q_{do} = [1 - \gamma_{do}] \left(\frac{p_{do} q_{do}}{t_{do}}\right) + \gamma_{do} \left[v\left(q_{do}, w_o, \tau_{do}, \varphi\right) + \delta_{do}\right].$$

Using the functional forms assumptions from above, we obtain

$$n_{do} q_{do} = \left[\frac{\mu (1 - \gamma_{do}) + \gamma_{do}}{\mu}\right] w_o \tau_{do} \left(\frac{p_{do}}{t_{do}}\right) q_{do} + \gamma_{do}\delta_{do}.$$

We assume productivity, φ, has a Pareto distribution over $[1,+\infty)$ with cumulative density function $G[\tilde{\varphi}<\varphi]=1-\varphi^{-\theta}$ and probability density function $g(\varphi)=\theta\varphi^{-\theta-1}$. The Pareto parameter and the elasticity of substitution are such that $\theta>\sigma-1$, which ensures that the moment of productivity distribution in Eq. (A40) is bounded. Using this moment and substituting $n_{do}q_{do}$ into the integral gives

$$\left(\frac{\sigma-\gamma_{do}}{\sigma-1}\right) w_o\tau_{do}B_{do}\left(\frac{\theta\bar{\varphi}_{do}^{\sigma-\theta-1}}{\theta-\sigma+1}\right)+\gamma_{do}\delta_{do}\bar{\varphi}_{do}^{-\theta}.$$

Substitute the export productivity threshold into this expression and simplify to get

$$\begin{aligned} IM_{do} &= \left(1-\frac{u_{do}}{1-i_{do}}\right) N_o^x\left[\left(\sigma-\gamma_{do}\right)\frac{\theta}{\theta-\sigma+1}+\gamma_{do}\frac{\delta_{do}}{F_{do}}\right] \\ &\times\; F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\left(\mu^{-\sigma}\alpha\left(w_o\tau_{do}\right)^{1-\sigma}C_dP_d^{\sigma-1}\left[\mu-1\right]\right)^{\frac{\theta}{\sigma-1}}t_{do}^{-\mu\theta}. \end{aligned}$$

Substituting in for the price index using Eq. (25) gives

$$\begin{aligned} IM_{do} &= \left(1-\frac{u_{do}}{1-i_{do}}\right)\left[\left(\sigma-\gamma_{do}\right)\frac{\theta}{\theta-\sigma+1}+\gamma_{do}\frac{\delta_{do}}{F_{do}}\right] \\ &\times\; \left(\mu^{-\sigma}\alpha\left[\mu-1\right]\right)^{\frac{\theta}{\sigma-1}} N_o^x\lambda_2^\theta C_d\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta}F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}t_{do}^{-\mu\theta}. \end{aligned}$$

Substitute in for λ_2 using Eq. (A44), to get

$$IM_{do}=\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-\frac{\gamma_{do}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-\left(\sigma-1\right)\right)\right)\right)\alpha N_o^xC_d\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta}F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}t_{do}^{-\mu\theta}. \tag{A55}$$

Define the bundle of search parameters

$$b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)=\frac{\gamma_{do}}{\sigma\theta}\left(\theta-\frac{\delta_{do}}{F_{do}}\left(\theta-\left(\sigma-1\right)\right)\right) \tag{A56}$$

and substitute it into Eq. (A55) to obtain the gravity equation as

$$IM_{do}=\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)\alpha N_o^xC_d\left(\frac{w_o\tau_{do}}{\rho_d}\right)^{-\theta}F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}t_{do}^{-\mu\theta},$$

which is Eq. (26).

A.4.4.2 Consumption is after-tariff imports evaluated at final sales prices

The value of consumption of the differentiated good in the *do* market is defined as the integral over all varieties, ω, of the value of $q_{do}(\omega)$ units evaluated at final sales prices, $p_{do}(\omega)$, as shown in Eq. (A3). After moving from an unordered set of varieties to an index over a distribution of productivities this consumption is given by:

$$C_{do}=\left(1-\frac{u_{do}}{1-i_{do}}\right)N_o^x\int_{\bar{\varphi}_{do}}^{\infty}p_{do}\left(\varphi\right)q_{do}\left(\varphi\right)dG\left(\varphi\right). \tag{A57}$$

To evaluate this integral, notice that the value of imports in Eq. (A54) is a similar expression, but integrates $n_{do}(\varphi)\, q_{do}(\varphi)$ rather than $p_{do}(\varphi)\, q_{do}(\varphi)$. From Eq. (13), $p_{do}(\varphi) = t_{do} n_{do}(\varphi)$ if $\gamma_{do} = 0$. Therefore, to evaluate the right side of Eq. (A57) we can set $\gamma_{do} = 0$ in Eq. (A55) and multiply by t_{do}, which gives

$$C_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \alpha N_o^x C_d \left(\frac{w_o \tau_{do}}{\rho_d}\right)^{-\theta} F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)} t_{do}^{1-\mu\theta}. \tag{A58}$$

A.4.4.3 Government expenditure

Government expenditure in each *do* market is defined in Eq. (A53) and can be written as

$$G_{do} = (t_{do} - 1)\, IM_{do},$$

in which IM_{do} is defined in Eq. (26).

A.5 Steady-state general equilibrium

A.5.1 Defining the equilibrium

The equilibrium reduces to the following equations in the equilibrium variables.

1. The free-entry condition for retailers (Eq. 10):

$$\frac{w_d c_{do}}{\chi(\kappa_{do})} = \int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi), \tag{A59}$$

when market tightness, κ_{do}, is not directly chosen by a social planner. Notice that there are d times o markets and each market has an associated tightness. With our functional form assumptions, this equation can be simplified. Remember that with $V_{do} = 0$

$$M_{do}(\varphi) = \frac{p_{do} q_{do} - t_{do} n_{do} q_{do}}{r + \lambda}$$

so that

$$\begin{aligned}\int_{\bar{\varphi}_{do}} M_{do}(\varphi)\, dG(\varphi) &= \left(\frac{1}{r+\lambda}\right) \int_{\bar{\varphi}_{do}} p_{do}(\varphi)\, q_{do}(\varphi) - t_{do} n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi) \\ &= \left(\frac{1}{r+\lambda}\right) \left(1 - \frac{u_{do}}{1 - i_{do}}\right)^{-1} \left(\frac{1}{N_o^x}\right) \Pi_{do}^r,\end{aligned}$$

in which Π_{do}^r is defined in Eq. (D99). Plugging this expression into Eq. (18) gives

$$\kappa_{do} = \left(\frac{1}{r+\lambda}\right) (\lambda + \kappa_{do}\chi(\kappa_{do})) \left(\frac{1}{w_d c_{do} N_o^x}\right) \Pi_{do}^r, \tag{A60}$$

in which we used $\chi(\kappa_{do}) = \xi \kappa_{do}^{-\eta}$ and $[1 - u_{do}/(1 - i_{do})]^{-1} = (\lambda + \kappa_{do}\chi(\kappa_{do})/\kappa_{do}\chi(\kappa_{do}))$.

2. The expression that equates variable profits with the effective entry cost, which pins

down $\bar{\varphi}_{do}$ (Eq. 17):

$$\bar{\varphi}_{do} = \mu \left(\frac{\sigma}{\alpha}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_o \tau_{do}}{P_d}\right) \left(\frac{F(\kappa_{do})}{C_d}\right)^{\frac{1}{\sigma-1}} t_{do}^{\mu},$$

in which $F(\kappa_{do})$ is defined in Eq. (16), w_o is defined in Eq. (A37), and P_d is defined in Eq. (25).

3. National accounting/consumer's budget constraint pins down consumption C_d (Eq. 21):

$$C_d = Y_d - I_d,$$

in which I_d is defined in Eq. (21) and $Y_d = w_d L_d + \Pi_d + G_d$, in which w_d is defined in Eq. (A37) and Π_d is defined in Eq. (D.1.3.1).

4. Labor market clearing pins down w_d (Eq. A37):

$$w_d = \frac{I_d + (1-\alpha) C_d + \frac{1}{\mu} \sum_o \frac{C_{od}}{t_{od}}}{L_d},$$

in which I_d is defined in Eq. (21) and C_{od} is defined in Eq. (A58).

A.5.2 Nesting trade models without search frictions

Our model nests trade models without search frictions if retailers' search costs are zero, $c_{do} = 0$, $\forall do$, among other restrictions. The main difference between our model's equilibrium definition and the definitions in trade models without search is that we introduce market tightness, k_{do}. When search costs are zero, free entry into product vacancies leads to infinite market tightness and instantaneous matching for producers. Instantaneous matching implies that all producers are matched (Eq. 19), as in a standard trade model without search frictions.

In particular, our model exactly reproduces Chaney (2008) if retailers' search costs are zero, we make the same assumptions about the homogeneous good as he does (so that $w_d = 1$, $\forall d$), we assume the same profit redistribution (Appendix D.1.3.5), we assume that the number of producers is proportional to consumption ($N_o^x = C_o / (1+\pi)$) and that there are no tariffs ($G_d = 0 \forall d$), and we impose the same parameter value restrictions that he does ($t_{do} = s_{do} = h_{do} = e_d^x = 0$, $\forall d$, and, $\forall o$). We demonstrate this equivalence by showing that all equilibrium equations are the same. If retailers' search costs are zero, market tightness is infinite and the negotiated price (Eq. 13) attains the final sales price if $t_{do} = 1$, $\forall do$, which is given by Eq. (14). There is, in effect, no intermediate retailer; producers sell their goods directly to the final consumer at price p_{do}. Instant contacts for producers imply that the effective entry cost (Eq. 16) equals the fixed cost of production, $F_{do} = w_o f_{do}$, and our threshold productivity expression (Eq. 17) coincides with Chaney (2008, Eq. 7). With no search costs and $G_d = 0$, Eq. (21) implies that $Y_d = C_d + I_d$ and the only investment expenditure is the fixed cost of production. Total income is given by $Y_d = (1+\pi) w_d L_d$, which also matches Chaney (2008, Eq. 9). Assumptions about the homogeneous good as in Chaney (2008) would imply that $w_d = 1$ $\forall d$ and the percapita dividend is determined by Eq. (D113), as shown in KM. With the same equations defining the equilibrium variables, our ideal price index (Eq. 25) and gravity equation (Eq. 26) would coincide with Eqs. 8 and 10, respectively, in Chaney (2008).

A.6 Deriving aggregate welfare

Here we outline the steps to show that the indirect utility function (welfare) is C_d/Ξ_d, in which C_d is total consumption expenditure, p is the vector of prices for each good, and Ξ_d is the ideal price index. Assume that preferences are homothetic, which is defined in Mas-Colell, Whinston, and Green (1995), section 3.B.6, page 45. This means that they can be represented by a utility function that is homogeneous of degree one in quantities and that the corresponding indirect utility function is linear in total consumption expenditure. We can begin with the indirect utility function and then manipulate it as follows

$$\begin{aligned} W_d(p, C_d) &= W_d(p,1)\,C_d \\ W_d(p, e(p,u)) &= W_d(p,1)\,e(p,u) \\ u &= W_d(p,1)\,e(p,u) \\ 1 &= W_d(p,1)\,e(p,1) \\ \frac{1}{e(p,1)} &= W_d(p,1)\,, \end{aligned}$$

in which the first line comes from homothetic preferences; the second line follows by plugging in for consumption expenditure $C_d = e(p,u)$; the third line comes from Eq. (3.E.1) in MWG that says $W_d(p, e(p,u)) = u$ (also known as duality); and in the fourth line we plug in for utility level $u = 1$. The function $e(p,u)$ is the consumption expenditure function that solves the expenditure minimization problem. Using this result and the fact that the price index is defined as $e(p,1) \equiv \Xi_d$ we can show that

$$W_d(p, C_d) = W_d(p,1)\,C_d = \frac{1}{e(p,1)}C_d = \frac{C_d}{\Xi_d}.$$

Hence, as long as preferences are homothetic, we will always get welfare equal to consumption expenditure divided by the price index, $W_d(p, Y) = C_d/\Xi_d$.

We can prove this result directly in our setting. Plugging the optimal quantities in Eq. (2) into the utility function in Eq. (1) yields:

$$U(q_d(1), q_d(\varphi)) = \left(\frac{(1-\alpha)\,C_d}{p_d(1)}\right)^{1-\alpha}\left(\frac{\alpha C_d}{P_d^{1-\sigma}}\right)^{\alpha}\left[\sum_{k=1}^{O}\left(1-\frac{u_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}} p_{dk}(\varphi)^{1-\sigma}\,dG(\varphi)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)}. \tag{A61}$$

Use Eq. (A38) to obtain

$$P_d^{-\alpha\sigma} = \left[\sum_{k=1}^{O}\left(1-\frac{u_{dk}}{1-i_{dk}}\right)N_k^x\int_{\bar{\varphi}_{dk}}^{\infty} p_{dk}(\varphi)^{1-\sigma}\,dG(\varphi)\right]^{\alpha\left(\frac{\sigma}{\sigma-1}\right)}. \tag{A62}$$

Plugging this expression into Eq. (A61) and rearranging yields

$$U\left(q_d\left(1\right), q_d\left(\varphi\right)\right) = \frac{C_d}{\left(\frac{p_d\left(1\right)}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_d}{\alpha}\right)^{\alpha}}. \tag{A63}$$

The right-hand side of this expression is C_d/Ξ_d, in which Ξ_d is defined in Eq. (A2).

B Optimal unilateral uniform import tariffs

B.1 Optimal uniform tariffs with search frictions

We prove Eq. (31) in several steps.

B.1.1 Cost minimization problem: Producing domestic varieties

This problem takes the form:

$$\begin{aligned} L_d\left(Q_{dd},Q_{od}\right) \quad &\equiv \quad \min_{\tilde{q}_{dd}(\varphi),\tilde{q}_{od}(\varphi),N_d} I_d + N_d\left[\sum_{k=d,o}\left(1-\frac{u_{kd}}{1-i_{kd}}\right)\int_{\bar{\varphi}_{kd}} l_{kd}\left(\tilde{q}_{kd}\left(\varphi\right),\varphi\right)dG_d\left(\varphi\right)\right] \quad \text{(B64)} \\ s.t. \quad &\int_{\bar{\varphi}_{kd}} N_d\left(1-\frac{u_{kd}}{1-i_{kd}}\right)\left(q_{kd}\left(\varphi\right)\right)^{1/\mu}dG_d\left(\varphi\right) \geq Q_{kd}^{1/\mu}, k=d,o \end{aligned}$$

with the Lagrangian given by

$$\begin{aligned} \mathcal{L}_d \quad &= \quad I_d + N_d\left[\sum_{k=d,o}\left(1-\frac{u_{kd}}{1-i_{kd}}\right)\int_{\bar{\varphi}_{kd}} l_{kd}\left(\tilde{q}_{kd}\left(\varphi\right),\varphi\right)dG_d\left(\varphi\right)\right] \\ &+ \quad \sum_{k=d,o}\lambda_{kd}\left[Q_{kd}^{1/\mu}-\int_{\bar{\varphi}_{kd}} N_d\left(1-\frac{u_{kd}}{1-i_{kd}}\right)\left(\tilde{q}_{kd}\left(\varphi\right)\right)^{1/\mu}dG_d\left(\varphi\right)\right] \end{aligned}$$

in which $l_{do}\left(q,\varphi\right)=a_{do}\left(\varphi\right)q, q>0$. Solving the problem variety by variety, we get the quantity demanded:

$$\tilde{q}_{kd}\left(\varphi\right)=\begin{cases}\left(\dfrac{\mu a_{kd}\left(\varphi\right)}{\lambda_{kd}}\right)^{-\sigma} & ,\varphi\geq\bar{\varphi}_{kd} \\ 0 & ,otherwise\end{cases}$$

Using the constraint and plugging this in, we get the Lagrange multiplier:

$$\lambda_{kd}=\left[\int_{\bar{\varphi}_{kd}} N_d\left(1-\frac{u_{kd}}{1-i_{kd}}\right)\left(\mu a_{kd}\left(\varphi\right)\right)^{1-\sigma}dG_d\left(\varphi\right)\right]^{\frac{1}{1-\sigma}} Q_{kd}^{\frac{1}{\sigma}}. \quad \text{(B65)}$$

B.1.2 The social planner's solution

The goal of the social planner is to maximize $U_d\left(Q_{dd},Q_{do}\right)$ subject to the labor constraint. The maximization problem can be written as

$$\begin{gathered} \max_{Q_{dd},Q_{do},Q_{od}} U_d\left(Q_{dd},Q_{do}\right) \\ Q_{do}\leq Q_{do}\left(Q_{od}\right) \\ L_d\left(Q_{dd},Q_{od}\right)=L_d \end{gathered}$$

Computing first-order conditions and rearranging yields

$$\frac{\left(\dfrac{\partial U_d\left(Q_{dd},Q_{do}\right)}{\partial Q_{dd}}\right)}{\left(\dfrac{\partial U_d\left(Q_{dd},Q_{do}\right)}{\partial Q_{do}}\right)}=\frac{\left(\dfrac{\partial L_d\left(Q_{dd},Q_{od}\right)}{\partial Q_{dd}}\right)}{\left(\dfrac{\partial L_d\left(Q_{dd},Q_{od}\right)}{\partial Q_{od}}\right)}\left(\frac{\partial Q_{do}\left(Q_{od}\right)}{\partial Q_{od}}\right). \quad \text{(B66)}$$

Then use the balanced trade condition for country d $\bar{N}_{od}(Q_{od})\,Q_{od} = \bar{N}_{do}(Q_{od}, Q_{do})\,Q_{do}$. Rearrange slightly to get

$$\frac{\bar{N}_{od}(Q_{od})\,Q_{od}}{\bar{N}_{do}(Q_{od}, Q_{do})\,Q_{do}} = 1.$$

Incorporating this expression into Eq. (B66) yields the optimality condition

$$\frac{1}{H} = \frac{P(Q_{do}, Q_{od})}{\left(-\frac{dQ_{dd}}{dQ_{od}}\right)/MRS_d}, \tag{B67}$$

in which we define $P(Q_{do}, Q_{od}) = \bar{N}_{od}(Q_{od})\,/\bar{N}_{do}(Q_{od}, Q_{do})$ as the aggregate terms of trade in country d (price of exports over price of imports), $H = d\ln Q_{do}/d\ln Q_{od}$ is the elasticity of the offer curve in country o, $MRS_d = (\partial U_d(Q_{dd}, Q_{do})\,/\partial Q_{do})\,/\,(\partial U_d(Q_{dd}, Q_{do})\,/\partial Q_{dd})$ is the marginal rate of substitution in country d, and
$-\frac{dQ_{dd}}{dQ_{od}} = (\partial L_d(Q_{dd}, Q_{od})\,/\partial Q_{od})\,/\,(\partial L_d(Q_{dd}, Q_{od})\,/\partial Q_{dd})$.

B.1.3 Overall level of taxes

With uniform import tariffs and passive trade policies elsewhere, we obtain

$$t_{do} = \left(\frac{1-b_{do}}{1-b_{od}}\right)\frac{P(Q_{do}, Q_{od})}{MRT_d/MRS_d}. \tag{B68}$$

Notice that

$$P(Q_{do}, Q_{od}) = \frac{(1-b_{od})\,P_{od}t_{do}}{(1-b_{do})\,P_{do}},$$

which uses the definitions of P_{do} from Appendix A.4.3. At an interior solution $MRS_d = P_{do}/P_{dd}$ and $MRT_d = P_{od}/P_{dd}$. So

$$\frac{P(Q_{do}, Q_{od})}{MRT_d/MRS_d} = \left(\frac{1-b_{od}}{1-b_{do}}\right)t_{do},$$

which yields Eq. (B68).

B.1.4 Optimal uniform tariffs

We show that

$$t^u_{do} = \left(\frac{1-b^u_{do}}{1-b^u_{od}}\right)\frac{1}{H}\left(\left(-\frac{dQ^u_{dd}}{dQ^u_{od}}\right)/MRT^u_d\right) \tag{B69}$$

and solving for $1/H$ yields

$$t^u_{do} = \mathcal{B}^u_{do}\frac{\left[1+\frac{1}{\sigma}\left(\frac{1}{x^u_{oo}}-1\right)+\epsilon^u_o+\epsilon^u_{\mathcal{B}_{do},Q_{do}}\right]}{\left[1-\frac{1}{\sigma}+\epsilon^u_{\mathcal{B}_{od},Q_{od}}\right]}\left(\left(-\frac{dQ^u_{dd}}{dQ^u_{od}}\right)/MRT^u_d\right), \tag{B70}$$

in which $\mathcal{B}^u_{jk} = \left(1-b^u_{jk}\right)/\left(1-b^u_{kj}\right)$ and $\epsilon^u_{\mathcal{B}_{jk},Q_{jk}}$ is the elasticity of $\mathcal{B}_{jk}$ with respect to Q_{jk} evaluated at the optimal tariff. In our numerical examples in Section 3.3, we find that

$\mathcal{B}_{jk}^{u} = \left(1-b_{jk}^{u}\right)/\left(1-b_{kj}^{u}\right) \approx 1$, $\epsilon_{\mathcal{B}_{jk},Q_{jk}}^{u} \approx 0$, and $-\left(dQ_{dd}^{u}/dQ_{od}^{u}\right)/MRT_{d}^{u} \approx 1$, which yields Eq. (31) after simplifying.

Start with the necessary condition from the social planner's solution, Eq. (B67), and manipulate to obtain

$$\left(\frac{1-b_{do}}{1-b_{od}}\right)\frac{1}{H}\left(-\frac{dQ_{dd}}{dQ_{od}}/MRT_{d}\right) = \left(\frac{1-b_{do}}{1-b_{od}}\right)\frac{P\left(Q_{do},Q_{od}\right)}{MRT_{d}/MRS_{d}}.$$

Now plug in (B68) to obtain

$$t_{do}^{u} = \left(\frac{1-b_{do}^{u}}{1-b_{od}^{u}}\right)\frac{1}{H}\left(\left(-\frac{dQ_{dd}^{u}}{dQ_{od}^{u}}\right)/MRT_{d}^{u}\right). \tag{B71}$$

Let's solve for H. An interior solution implies that $MRS_{o}\left(Q_{od},Q_{oo}\left(Q_{do}\right)\right) = \bar{P}_{od}/\bar{P}_{oo}$ and $MRT_{o}\left(Q_{do},Q_{oo}\left(Q_{do}\right)\right) = \bar{P}_{do}/\bar{P}_{oo}$ so that

$$\frac{MRS_{o}\left(Q_{od},Q_{oo}\left(Q_{do}\right)\right)}{MRT_{o}\left(Q_{do},Q_{oo}\left(Q_{do}\right)\right)} = \frac{\bar{P}_{od}}{\bar{P}_{do}} = P\left(Q_{do},Q_{od}\right)\left(\frac{1-b_{do}}{1-b_{od}}\right). \tag{B72}$$

Trade balance implies that

$$P\left(Q_{do},Q_{od}\right)Q_{od} = Q_{do}.$$

Log, totally differntiate, and rearrange to yield

$$H = \frac{1+\rho_{od}}{1-\rho_{do}},$$

in which $\rho_{do} = \partial\ln P\left(Q_{od},Q_{do}\right)/\partial\ln Q_{do}$.

Use $P\left(Q_{do},Q_{od}\right) = \dfrac{MRS_{o}\left(Q_{od},Q_{oo}\left(Q_{do}\right)\right)}{MRT_{o}\left(Q_{do},Q_{oo}\left(Q_{do}\right)\right)}\left(\dfrac{1-b_{od}}{1-b_{do}}\right)$ to find that

$$\rho_{od} = -\frac{1}{\sigma} + \frac{\partial\ln\left(1-b_{od}\right)}{\partial\ln Q_{od}} - \frac{\partial\ln\left(1-b_{do}\right)}{\partial\ln Q_{od}},$$

in which we use that

$$MRS_{o} = \left(\frac{Q_{od}}{Q_{oo}\left(Q_{do}\right)}\right)^{-\frac{1}{\sigma}}. \tag{B73}$$

One can also show that

$$\rho_{do} = \frac{1}{\sigma}\frac{\partial\ln Q_{oo}\left(Q_{do}\right)}{\partial\ln Q_{do}} - \epsilon_{o} + \frac{\partial\ln\left(1-b_{od}\right)}{\partial\ln Q_{do}} - \frac{\partial\ln\left(1-b_{do}\right)}{\partial\ln Q_{do}},$$

in which we use Eq. (B73) and

$$MRT_{o} = \left(\frac{\lambda_{do}}{\lambda_{oo}}\right)\left(\frac{Q_{do}}{Q_{oo}\left(Q_{do}\right)}\right)^{-\frac{1}{\sigma}}$$

by applying the envelope theorem to the optimization problem in Eq. (B64).

Finally, notice that

$$\frac{\partial \ln Q_{oo}(Q_{do})}{\partial \ln Q_{do}} = \frac{dQ_{oo}(Q_{do})}{dQ_{do}} \frac{Q_{do}}{Q_{oo}(Q_{do})} = -\frac{\bar{P}_{do}Q_{do}}{\bar{P}_{oo}Q_{oo}(Q_{do})} \tag{B74}$$

so to ensure that $\frac{\partial \ln Q_{oo}(Q_{do})}{\partial \ln Q_{do}} = -\left(\frac{1}{x_{oo}} - 1\right)$, define

$$x_{oo} = \frac{C_{oo}}{C_{do}/t_{do} + C_{oo}}.$$

Using ρ_{od} and ρ_{do} in H and plugging into Eq. (B71) yields

$$t^u_{do} = \left(\frac{1-b^u_{do}}{1-b^u_{od}}\right) \frac{\left[1 + \frac{1}{\sigma}\left(\frac{1}{x^u_{oo}} - 1\right) + \epsilon^u_o + \frac{\partial \ln(1-b^u_{do})}{\partial \ln Q^u_{do}} - \frac{\partial \ln(1-b^u_{od})}{\partial \ln Q^u_{do}}\right]}{\left[1 - \frac{1}{\sigma} + \frac{\partial \ln(1-b^u_{od})}{\partial \ln Q^u_{od}} - \frac{\partial \ln(1-b^u_{do})}{\partial \ln Q^u_{od}}\right]} \left(\left(-\frac{dQ^u_{dd}}{dQ^u_{od}}\right)/MRT^u_d\right).$$

Notice that if there are no search frictions so that $b_{jk} = 0\ \forall jk$, this implies that we recover the optimal uniform tariff expression in CRW as long as $\left(-dQ^u_{dd}/dQ^u_{od}\right)/MRT^u_d = 1$.

B.1.5 x_{oo} as the local consumption share

Notice that Eq. (B74) can also be written as

$$\frac{\partial \ln Q_{oo}(Q_{do})}{\partial \ln Q_{do}} - \frac{\bar{N}_{do}Q_{do}}{\bar{N}_{oo}Q_{oo}(Q_{do})}\left(\frac{1-b_{oo}}{1-b_{do}}\right) = -\frac{IM_{do}}{IM_{oo}}\left(\frac{1-b_{oo}}{1-b_{do}}\right).$$

Using trade balance, we obtain that

$$x_{oo} = \frac{IM_{oo}(1-b_{do})}{IM_{od}(1-b_{oo}) + IM_{oo}(1-b_{do})}.$$

If $t_{do} = 1$ and $(1-b_{do}) = 1$ then we can write x_{oo} as

$$x_{oo} = \frac{C_{oo}}{C_{od} + C_{oo}}, \tag{B75}$$

which is the share of differentiated goods consumption in country o devoted to local goods.

B.2 Proof of proposition 1: The elasticity of transformation, ϵ_o

The cost minimization problem solved by country o (Eq. B64 for country o), together with the envelope theorem, implies that

$$MRT_o = \left(\frac{\lambda_{do}}{\lambda_{oo}}\right)\left(\frac{Q_{do}}{Q_{oo}}\right)^{-\frac{1}{\sigma}}, \tag{B76}$$

in which $\lambda_{ko} = \left[\int_{\bar{\varphi}_{ko}} N_o \left(1 - \frac{u_{ko}}{1 - i_{ko}}\right) (\mu a_{ko}(\varphi))^{1-\sigma} dG_o(\varphi)\right]^{\frac{1}{1-\sigma}} Q_{ko}^{\frac{1}{\sigma}}$ (Eq. B65 for country o). Plugging this expression into Eq. (B76) yields

$$MRT_o = \frac{\left[\left(1 - \frac{u_{do}}{1 - i_{do}}\right) \int_{\bar{\varphi}_{do}} \left(\frac{\tau_{do}}{\varphi}\right)^{1-\sigma} \varphi^{-\theta-1} d\varphi\right]^{\frac{1}{1-\sigma}}}{\left[\left(1 - \frac{u_{oo}}{1 - i_{oo}}\right) \int_{\bar{\varphi}_{oo}} \left(\frac{\tau_{oo}}{\varphi}\right)^{1-\sigma} \varphi^{-\theta-1} d\varphi\right]^{\frac{1}{1-\sigma}}}. \tag{B77}$$

We proceed to solve the integrals in closed form.

The definition of the threshold productivity (Eq. 15) implies that

$$(\mu - 1)\left(\frac{\tau_{do}}{\bar{\varphi}_{do}}\right)^{1-\sigma} \left(\frac{\mu}{\lambda_{do}}\right)^{-\sigma} = F(\kappa_{do}).$$

Solve this equation for $\bar{\varphi}_{do}$ and plug in for the definition of λ_{do} to yield

$$\bar{\varphi}_{do} = \frac{\tau_{do} \left(\frac{F(\kappa_{do})}{\mu - 1}\right)^{\frac{1}{\sigma-1}} Q_{do}^{\frac{1}{1-\sigma}}}{\left[N_o \theta \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \int_{\bar{\varphi}_{do}} \left(\frac{\tau_{do}}{\varphi}\right)^{1-\sigma} \varphi^{-\theta-1} d\varphi\right]^{\frac{\sigma}{(1-\sigma)(\sigma-1)}}}. \tag{B78}$$

Use Eq. (A40) and rearrange to solve for $\bar{\varphi}_{do}$:

$$\bar{\varphi}_{do} = \frac{\tau_{do}^{\frac{(\sigma-1)}{(\sigma-1)-\sigma\theta}} \left(\frac{F(\kappa_{do})}{\mu - 1}\right)^{\frac{1-\sigma}{(\sigma-1)-\sigma\theta}} Q_{do}^{\frac{\sigma-1}{(\sigma-1)-\sigma\theta}}}{\left[\frac{N_o \theta \left(1 - \frac{u_{do}}{1 - i_{do}}\right)}{\theta - \sigma + 1}\right]^{\frac{\sigma}{(\sigma-1)-\sigma\theta}}}. \tag{B79}$$

Rearrange Eq. (B78), plug in Eq. (B79), and simplify to obtain

$$\left[\int_{\bar{\varphi}_{do}} \left(1 - \frac{u_{do}}{1 - i_{do}}\right) \left(\frac{\tau_{do}}{\varphi}\right)^{1-\sigma} \varphi^{-\theta-1} d\varphi\right]^{\frac{1}{1-\sigma}}$$
$$= (N_o \theta)^{\frac{1}{\sigma-1}} \tau_{do}^{\frac{(\sigma-1)\theta}{\sigma\theta-(\sigma-1)}} \left(\frac{F(\kappa_{do})}{\mu - 1}\right)^{\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}} Q_{do}^{-\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}} \left(\frac{N_o \theta}{\theta - \sigma + 1}\right)^{-\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}} \left(1 - \frac{u_{do}}{1 - i_{do}}\right)^{-\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}}.$$

Plug this integral back into Eq. (B77) to obtain:

$$MRT_o = \left(\frac{\tau_{do}}{\tau_{oo}}\right)^{\frac{\theta(\sigma-1)}{\sigma\theta-(\sigma-1)}} \left(\frac{F(\kappa_{do})}{F(\kappa_{oo})}\right)^{\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}} \left(\frac{Q_{oo}}{Q_{do}}\right)^{\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}} \left[\frac{\left(1-\frac{u_{oo}}{1-i_{oo}}\right)}{\left(1-\frac{u_{do}}{1-i_{do}}\right)}\right]^{\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}}.$$

Therefore, $\epsilon_o = \partial \ln MRT_o (Q_{do}, Q_{oo}(Q_{do})) / \partial \ln Q_{do}$ is given by

$$\begin{aligned}\epsilon_o &= -\left[\frac{\theta-(\sigma-1)}{[\sigma\theta-(\sigma-1)]\,x_{oo}}\right] + \left[\frac{\theta-(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\left[\frac{\partial \ln F_{do}}{\partial \ln Q_{do}} - \frac{\partial \ln F_{oo}}{\partial \ln Q_{do}}\right] \\ &- (1-\eta)\left[\frac{(\sigma-1)}{\sigma\theta-(\sigma-1)}\right]\left[\left(\frac{u_{do}}{1-i_{do}}\right)\frac{\partial \ln \kappa_{do}}{\partial \ln Q_{do}} - \left(\frac{u_{oo}}{1-i_{oo}}\right)\frac{\partial \ln \kappa_{oo}}{\partial \ln Q_{do}}\right]\end{aligned}$$

because

$$\frac{\partial \ln Q_{oo}(Q_{do})}{\partial \ln Q_{do}} = -\left(\frac{1}{x_{oo}} - 1\right). \tag{B80}$$

Assuming that $h_{oo} = -l_{oo}$ and $h_{do} = -l_{do}$, so that F_{oo} and F_{do} are parameters so that $\frac{\partial \ln F_{do}}{\partial \ln Q_{do}} = \frac{\partial \ln F_{oo}}{\partial \ln Q_{do}} = 0$, yields Eq. (32).

B.3 International search costs and the local consumption share

The next lemma characterizes how the local consumption share on differentiated goods, x_{oo}, varies with international search costs, c_{do} and c_{od}.

Lemma 1. *In addition to the assumptions in Proposition 1, assume that the consumer's optimization problem yields an interior solution, that changes in tightness in the do and od markets have small effects on the price index and local goods in the oo market so that* $\partial \ln (P_{oo}) / \partial \ln (\kappa_{ij}) = \partial \ln (Q_{oo}) / \partial \ln (\kappa_{ij}) = 0$ *for* $ij \in \{do, od\}$*, that effects on aggregate variables in country o from changes in do tightness are small, so that* $\frac{\partial \ln C_o}{\partial \ln \kappa_{do}} = 0$*, and that markdowns are small so that* $1 - b_{do}(\cdot) = 1$*. Then,*

$$\frac{\partial \ln x_{oo}}{\partial \ln c_{do}} = 0, \tag{B81}$$

and

$$\frac{\partial \ln x_{oo}}{\partial \ln c_{od}} = -(1-\eta)\left(\frac{u_{od}}{1-i_{od}}\right)(1-x_{oo})\frac{\partial \ln \kappa_{od}}{\partial \ln c_{od}} \geq 0. \tag{B82}$$

Proof. The local consumption share, x_{oo}, is defined under Eq. (31). Here we use the expression in Eq. (B75).

$$\frac{\partial \ln x_{oo}}{\partial \ln c_{do}} = \frac{\partial \ln P_{oo}}{\partial \ln \kappa_{do}}\frac{\partial \ln \kappa_{do}}{\partial \ln c_{do}} + \frac{\partial \ln Q_{oo}}{\partial \ln \kappa_{do}}\frac{\partial \ln \kappa_{do}}{\partial \ln c_{do}} - \frac{\partial \ln C_o}{\partial \ln \kappa_{do}}\frac{\partial \ln \kappa_{do}}{\partial \ln c_{do}}.$$

The assumptions stated in Lemma 1 yield Eq. (B81).

The definition of x_{oo}, together with the assumptions of the Lemma, implies that

$$\frac{\partial \ln x_{oo}}{\partial \ln c_{od}} = -\frac{\partial \ln C_o}{\partial \ln \kappa_{od}} \frac{\partial \ln \kappa_{od}}{\partial \ln c_{od}} = -\frac{\partial \ln \left(P_{oo}Q_{oo} + P_{od}Q_{od}\right)}{\partial \ln \kappa_{od}} \frac{\partial \ln \kappa_{od}}{\partial \ln c_{od}}. \tag{B83}$$

We can write

$$\frac{\partial \ln C_o}{\partial \ln \kappa_{od}} = \left(\frac{\partial \ln P_{od}}{\partial \ln \kappa_{od}} + \frac{\partial \ln Q_{od}}{\partial \ln \kappa_{od}}\right)\left(1 - x_{oo}\right), \tag{B84}$$

and therefore

$$\frac{\partial \ln C_o}{\partial \ln \kappa_{od}} = \left(1 - \eta\right)\left(\frac{u_{od}}{1 - i_{od}}\right)\left(1 - x_{oo}\right).$$

Plugging back into (B83) yields Eq. (B82). □

Increasing international search frictions does not reduce the local consumption share in country o. Eq. (B82) implies that the local consumption share increases if search costs, c_{od}, increase because tightness responds negatively to search costs, $\partial \ln \kappa_{od} / \partial \ln c_{od} \leq 0$.

This increase in the local consumption share reduces the optimal tariff in Eq. (31). Notice that the local consumption share enters the optimal tariff expression directly in Eq. (31) and also indirectly in the EoT expression in Eq. (32). These two effects work in opposite directions, but we show that the direct effect is stronger than the indirect effect. In particular,

$$\frac{\partial t^u_{do}}{\partial x^u_{oo}} = -\frac{1}{(\sigma - 1)} \frac{1}{\left(x^u_{oo}\right)^2} + \frac{\sigma}{(\sigma - 1)} \frac{\theta - (\sigma - 1)}{\left[\sigma\theta - (\sigma - 1)\right]} \frac{1}{\left(x^u_{oo}\right)^2},$$

and this is negative if $\sigma > 1$, which is true by assumption.

As a result, raising international search frictions reduces the optimal tariff in Eq. (31) by raising the local consumption share in country o. This effect on the local consumption share works in the same direction as the effect of search frictions on the EoT, which also implies a lower optimal import tariff in a model with search frictions than in a model without them.

B.4 Parameters for numerical examples

We present the parameter values for the numerical examples in Table A1. The first column shows the parameters in the model with search frictions and the second column has the unit of each parameter. The parameterization assumes that there are two symmetric countries. The only difference between the numerical example with and without search frictions is that the model without search has $c_{uc} = 0$.

B.5 Optimal tariffs without search frictions

As discussed in Appendix B.1.4, we recover the optimal uniform tariff without search frictions in Eq. (33) if $(-dQ^u_{dd}/dQ^u_{od}) = MRT^u_d$. In our numerical examples in Section 3.3, we find that this assumption does not hold with equality. In the numerical examples without search frictions in the main text, we adjust for the slight discrepancy. In particular, in Fig. 1b, we depict

$$t^{u,ns}_{do} = \left(1 + \frac{1 + \sigma x^u_{oo}\epsilon^u_o}{(\sigma - 1)\, x^u_{oo}}\right)\left(-\frac{dQ^u_{dd}}{dQ^u_{od}}\right) / MRT^u_d, \tag{B85}$$

as the dotted black line. Without this adjustment, the implied optimal tariff is slightly lower than the actual optimal tariff. Future research could explore under what conditions this assumption holds. Our conclusions about the effect of search frictions on optimal tariffs are unaffected.

B.6 Nash equilibrium and the Nikaidô-Isoda function

We solve for the Nash equilibrium using the Nikaidô-Isoda (NI) function (Nikaidô and Isoda, 1955) given by

$$\Psi\left(\boldsymbol{t}, \boldsymbol{\zeta}\right) = \sum_{d=1}^{D} \left[\mathcal{L}_d\left(\boldsymbol{t}, \boldsymbol{\zeta}\right) - \sup_{\hat{\vec{t}}_{d*}, \hat{\vec{\zeta}}_{d*}} \mathcal{L}_d\left(\hat{\boldsymbol{t}}, \hat{\boldsymbol{\zeta}}\right) \right], \tag{B86}$$

in which the Lagrangian, $\mathcal{L}_d\left(\boldsymbol{t}, \boldsymbol{\zeta}\right)$, is written as a function of the exogenous tariffs, $\boldsymbol{t}$, and exogenous matrix of Lagrange multipliers, $\boldsymbol{\zeta}$, corresponding to all of the constraints defined in Eq. (34). The Lagrangian is also a function of the endogenous variables—$\boldsymbol{\kappa}, \bar{\boldsymbol{\varphi}}, \vec{C}, \vec{w}$—that define the economy's equilibrium, but those are determined by satisfying the constraints in Eq. (34) for given values of $\boldsymbol{t}$ and $\boldsymbol{\zeta}$. Therefore, we do not write out the endogenous variables explicitly in Eq. (B86).

Intuitively, each summand of the NI function (B86) can be thought of as the difference in equilibrium welfare for a country d and that country's best response. When the summand for country d is zero, that country has no unilateral incentive to deviate. When the sum for all countries is zero, no country has a unilateral incentive to deviate. Hence, the Nash equilibrium is defined as $\Psi\left(\boldsymbol{t}^n, \boldsymbol{\zeta}^n\right) = 0$ because this is when no country can benefit by unilaterally changing its tariffs. It is possible to show that $\Psi\left(\boldsymbol{t}^n, \boldsymbol{\zeta}^n\right) = 0$ is a global maximum because $\Psi\left(\boldsymbol{t}, \boldsymbol{\zeta}\right) \leq 0$.

C Calibration appendix

C.1 Intuition for parameter identification

We use a calibration strategy similar to that in KM and include the details here. Table 1 summarizes the discussion and Table 2 presents the moments and model fit.

The search frictions in our model are governed by retailers' flow search cost, c_{do}. If the fraction of matched exporters is low, it implies that there are few searching retailers, market tightness is low, and international search costs are high. Consequently, we use the fact that 21 percent of Chinese firms export (WB, 2018) and that 6 percent of U.S. firms export to China (CB, 2016a,b) to identify c_{uc} and c_{cu}, respectively. Eaton et al. (2014) and Eaton et al. (2016) also use the fraction of firms that export to identify search model parameters. We use manufacturing capacity utilization to inform the level of domestic search frictions in goods markets, as in Michaillat and Saez (2015), Petrosky-Nadeau and Wasmer (2017), and Petrosky-Nadeau, Wasmer, and Weil (2021). We target 75 and 74 percent manufacturing capacity utilization in the United States and China in 2016, respectively, to inform c_{uu} and c_{cc} (FRB, 2020; NBSC, 2016a). We also assume that international search costs are larger than domestic search costs so that $c_{uc} \geq c_{uu}$ and $c_{cu} \geq c_{cc}$.

Targeting log-linear estimates of the trade elasticity informs the elasticity of matches with respect to the number of searching producers, η. This moment is informative because as the matching elasticity increases to one, producers' contact rate becomes unresponsive to changes in market tightness. Without an endogenous response in the producers' matched rate, trade becomes less responsive to variable trade costs and the trade elasticity increases. We target a trade elasticity of -6 based on a range of empirical estimates, which vary between -4 and -10 (Eaton and Kortum, 2002; Anderson and van Wincoop, 2004; Romalis, 2007; Imbs and Mejean, 2015).

The average duration of a Chinese and U.S. trading relationship is about one year (Monarch and Schmidt-Eisenlohr, 2023, Figure 9), so we set our separation parameter, $\lambda = 1$, because average match duration in the model is $1/\lambda$. This observed expected duration is also broadly consistent with survival probabilities among Colombian-U.S. trading relationships (Eaton et al., 2014).

We use business failure rates to inform the fixed costs of production, f_{do}. Fixed costs inform the productivity thresholds in the model. In turn, these define the idle rates. Business failure rates capture the fraction of firms that cease production, which helps inform the measure of firms in the model that take a productivity draw and do not produce.

Trade in both directions between China and the United States, together with the level of absorption of domestic production (IM_{uu} and IM_{cc}), as well as tariffs and distance, helps to identify the iceberg trade costs, τ_{do}. We define IM_{uu} and IM_{cc} as manufacturing value added minus merchandise exports plus merchandise imports similar to Dekle, Eaton, and Kortum (2008).

Working age population, L_c and L_u, is informed by the levels of gross domestic product (GDP), aggregate consumption, and the ratio of consumption to GDP in China and the US, as reported in the national accounts of each country (BEA, 2016a; WB, 2016; BEA, 2016b; NBSC, 2016b). Labor share of GDP per working age person informs the wage. The fraction of consumption on tradables informs α. The ratio of consumption at purchasing power parity informs the ratio of price indexes. The difference between consumption and GDP and the size of the working age population informs both the number of varieties and the exploration cost e_d^x.

C.2 Calibrating the matching elasticity

To calibrate our model to standard trade elasticities, we must first derive a log-linear estimating equation implied by our model that matches the specifications in the literature. We do this by rearranging the gravity Eq. (26) to collect similar indices of observation:

$$\begin{aligned}\ln\left(IM_{do}\right) &= \ln\left(\alpha\right)+\ln\left(\frac{C_d}{\rho_d^{-\theta}}\right)+\ln\left(N_o^x w_o^{-\theta}\right) \\ &+ \ln\left(\tau_{do}^{-\theta}\right)+\ln\left(t_{do}^{-\mu\theta}\right) \\ &+ \ln\left[\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\right].\end{aligned} \tag{C87}$$

Because the first three terms in Eq. (C87) are either a constant or only vary by destination or origin, we can simplify notation by writing

$$\phi=\ln\left(\alpha\right),\qquad \phi_d=\ln\left(\frac{C_d}{\rho_d^{-\theta}}\right),\qquad \text{and } \phi_o=\ln\left(N_o^x w_o^{-\theta}\right).$$

Also define the log of the terms that include search frictions as

$$z_{do}=\ln\left[\left(1-\frac{u_{do}}{1-i_{do}}\right)\left(1-b\left(\sigma,\theta,\gamma_{do},\delta_{do},F_{do}\right)\right)F_{do}^{-\left(\frac{\theta}{\sigma-1}-1\right)}\right].$$

Thus, the log-linear gravity equation from our model can be expressed as

$$\ln\left(IM_{do}\right)=\phi+\phi_d+\phi_o-\theta\left[\ln\left(\tau_{do}\right)+\mu\ln\left(t_{do}\right)\right]+z_{do}. \tag{C88}$$

Most gravity specifications in the literature omit search frictions, z_{do}, and instead parameterize the iceberg cost as a function of distance, for example, $\tau_{do}=a_1^o a_2^d \text{distance}_{do}^{a_3}$ and then estimate a specification similar to,

$$\ln\left(IM_{do}\right)=\zeta+\zeta_d+\zeta_o+\beta_1\ln\left(\text{distance}_{do}\right)-\hat{\theta}\mu\ln\left(t_{do}\right). \tag{C89}$$

If z_{do} was added as a covariate to Eq. (C89), then $\hat{\theta}=\theta$ would be an unbiased estimate (with externally calibrated μ). Instead, omitting $\ln\left(t_{do}\right)$ results in bias for estimates of θ characterized by

$$\mathbb{E}\left[-\hat{\theta}\mid\ln\left(t_{do}\right),\zeta,\zeta_d,\zeta_o\right]=-\theta+\beta_{z,t}. \tag{C90}$$

In this equation, $\beta_{z,t}$ is the coefficient from a regression of z_{do} on $\ln\left(t_{do}\right)$ and fixed effects;

$$z_{do}=\psi+\psi_d+\psi_o+\beta_{z,t}\mu\ln\left(t_{do}\right).$$

We use the moment defined in Eq. (C90) to inform the elasticity of matches with respect to the number of searching producers, η, because estimating Eq. (C89) when the true model is Eq. (C88) implies

$$\beta_{z,t}=\mathbb{E}\left[\left(\frac{u_{do}}{1-i_{do}}\right)\left(1-\eta\right)\frac{d\ln\kappa_{do}}{d\ln t_{do}}+\frac{d\ln\left(1-b_{do}\left(\cdot\right)\right)}{d\ln t_{do}}-\left(\frac{\theta}{\sigma-1}-1\right)\frac{d\ln F\left(\kappa_{do}\right)}{d\ln t_{do}}\mid\psi,\psi_d,\psi_o,\ln\left(t_{do}\right)\right].$$

See KM for more details.

D National accounting

In the National Income and Product Accounts (NIPA), gross domestic product (GDP) can be measured in three ways: 1) as the sum of income payments and other costs incurred in the production of goods and services (income approach), 2) as the sum of goods and services sold to final users (expenditure approach), 3) and as the sum of the value added at each stage of production. This appendix will explain how we use equating the incomes and expenditure approaches to define the resource constraint for consumers and solve for the equilibrium in our model.

D.1 Income approach to accounting

The income approach sums compensation of employees, $w_d L_d$, income from taxes or subsidies on production and imports, T_d, net operating surplus, Π_d^{nos}, and consumption of fixed capital (depreciation). Our model only has the first three of these components so that

$$GDI_d = w_d L_d + T_d + \Pi_d^{nos}. \tag{D91}$$

Government tax income is government expenditure, $T_d = G_d$, in our model because we assume the government runs a balanced budget.

D.1.1 Wage income

Wage income, $w_d L_d$, is derived from creating producers, Φ_d^e, the formation of relationships, Φ_d^r, fixed, Φ_d^f, and variable, Φ_d^v, costs of heterogeneous goods production, and the variables costs of homogeneous goods production, $w_d q_d(1)$ given by

$$w_d L_d = \Phi_d^e + \Phi_d^r + \Phi_d^f + \Phi_d^v + w_d q_d(1), \tag{D92}$$

in which

$$\Phi_d^e = N_d^x w_d e_d^x, \tag{D93a}$$

$$\Phi_d^r = \sum_k \kappa_{dk} u_{dk} N_k^x w_d c_{dk} + \sum_k u_{kd} N_d^x \left(w_d l_{kd} + w_d s_{kd} \kappa_{kd} \chi(\kappa_{kd}) \right), \tag{D93b}$$

$$\Phi_d^f = \sum_k (1 - u_{kd} - i_{kd}) N_d^x w_d f_{kd}, \text{ and} \tag{D93c}$$

$$\Phi_d^v = \sum_k \Phi_{kd}^v = \sum_k \left(1 - \frac{u_{kd}}{1 - i_{kd}}\right) N_d^x \int_{\bar{\varphi}_{kd}} v(q_{kd}, w_d, \tau_{kd}, \varphi) \, dG(\varphi). \tag{D93d}$$

With free entry of retailers as we assume, there can be no retailer entry cost in addition to search cost $w_o c_{od}$ as shown in KM. Therefore, the cost of creating retailers is the same as the cost of forming relationships, $\sum_k \kappa_{dk} u_{dk} N_k^x w_d c_{dk}$.

Using variable costs from Eq. (5), optimal final sales price from Eq. (14) written as $p_{do}(\varphi)/t_{do}\mu = w_o \tau_{do} \varphi^{-1}$, and demand from Eq. (2) we can write

$$\Phi_{do}^v = \frac{C_{do}}{\mu t_{do}} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}} v(q_{do}, w_o, \tau_{do}, \varphi) \, dG(\varphi). \tag{D94}$$

Dividing Eq. (D94) by w_d and summing across k destination markets gives

$$\frac{\Phi_d^v}{w_d} = \sum_k \frac{\Phi_{kd}^v}{w_d} = \sum_k \frac{C_{kd}}{\mu w_d t_{kd}} = \sum_k \left(1 - \frac{u_{kd}}{1 - i_{kd}}\right) N_d^x \int_{\bar{\varphi}_{kd}} q_{kd}(\varphi)\, \tau_{kd} \varphi^{-1} dG(\varphi)$$

which is one of the three additive terms in labor demand from Eq. (23).

Comparing investment, I_d, from Eq. (21) to Eq. (D93), we see that $I_d = \Phi_d^e + \Phi_d^r + \Phi_d^f$ providing another additive term from labor demand. The final term is just the labor used to produce the homogeneous good. So, Eq. (D92) is just a restatement of equilibrium labor market clearing from Eq. (23).

D.1.2 Government income

We assume that retailers in the do market pay a tariff, $t_{do} - 1$, on the value of imported differentiated goods, $n_{do}(\varphi)\, q_{do}(\varphi)$. Integrating over all the products and summing over all the origin countries yields total government tax income

$$T_d = \sum_{k=1}^{D} T_{dk} = \sum_{k=1}^{D} \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}}^{\infty} (t_{dk} - 1)\, n_{dk}(\varphi)\, q_{dk}(\varphi)\, dG(\varphi). \tag{D95}$$

$T_d = G_d$ in Eq. (21) in the main text. Notice that when $t_{dk} = 1 \forall k$ then $T_d = 0$ because there are no import tariffs or subsidies.

We define the value of total imports, IM_{do}, in Appendix A.4.4.1 (Eq. A54). It is trivial to show that government tax revenue in the do market is given by

$$T_{do} = (t_{do} - 1)\, IM_{do}. \tag{D96}$$

Similarly, because $C_{do} = t_{do} IM_{do} / (1 - b_{do})$, as shown in Appendix A.4.4.2,

$$T_{do} = \frac{(1 - b_{do})(t_{do} - 1)}{t_{do}} C_{do}, \tag{D97}$$

with b_{do} defined in Eq. (A56).

D.1.3 Profit income

In this appendix, we present five ownership structures for the profits earned by retailers and producers. Importantly, we assume that these alternative ownership structures do not affect optimal behavior of firms but rather only the apportionment of profits.

First, we discuss ownership of firms by location: Consumers in country d own retailers and producers in country d. Second, we discuss upstream (backward) vertical integration: Consumers in country d own retailers in country d and they own producers in the potentially many origin countries that produce for country d. Third, we discuss downstream (forward) vertical integration: Consumers in country d own producers in country d and they own the retailers that sell these goods in potentially many countries. Fourth, we discuss that consumers in country d own $w_d L_d$ shares of a global mutual fund that owns all retailers and producers as in Chaney (2008). The mutual fund redistributes profits derived anywhere proportionally to each country in the form of π dividends per share. Fifth, we discuss an inverted ownership structure in which consumers in country d own the producers that source country d and the retailers that sell country d goods. We implement the third ownership structure in this paper to facilitate comparisons with CRW, who take the same approach, but it would be straightforward to implement the other structures.

D.1.3.1 Profits attributed by location

Assume that consumers in country d own retailers and producers in country d. This assumption implies that total profits in country d are profits from retailers in country d selling products from potentially many origin k markets and profits from producers in country d selling to potentially many other k markets, in which $k = 1, \ldots, D$.

Retailer profits in country d can then be calculated from the retailer profits from each variety, which are

$$\pi^r_{do}(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do} n_{do}(\varphi)\, q_{do}(\varphi)\,. \tag{D98}$$

Integrating Eq. (D98) over varieties gives retailer profits earned in country d from selling products sourced from origin country o,

$$\Pi^r_{do} = C_{do} - t_{do} IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_o \int_{\bar{\varphi}_{do}} \pi^r_{do}(\varphi)\, dG(\varphi)\,. \tag{D99}$$

Total profits from retailers in country d selling products from potentially many origin k markets is then

$$\Pi^r_d = \sum_k \Pi^r_{dk} = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk}. \tag{D100}$$

Producer profits for each variety are

$$\pi^p_{do}(\varphi) = n_{do}(\varphi)\, q_{do}(\varphi) - v\left(q_{do}, w_o, \tau_{do}, \varphi\right). \tag{D101}$$

Integrating Eq. (D101) over varieties gives producer profits earned in country d from selling products from origin country o,

$$\Pi^p_{do} = IM_{do} - \Phi^v_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N^x_k \int_{\bar{\varphi}_{do}} \pi^p_{do}(\varphi)\, dG(\varphi)\,, \tag{D102}$$

in which Φ^v_{do} is the variable production costs paid by producers in o that are making products for destination d defined in Eq. (D93d). Total profits from producers in country d selling to potentially many other k markets is then

$$\Pi^p_d = \sum_k \Pi^p_{kd} = \sum_k IM_{kd} - \sum_k \Phi^v_{kd} = \sum_k IM_{kd} - \sum_k \frac{C_{kd}}{\mu t_{kd}} = \sum_k IM_{kd} - \Phi^v_d. \tag{D103}$$

in which we use the definition of Φ^v_{do} from Eq. (D94). Under this location-based ownership structure, total flow variable profits earned in d are therefore the sum of Eqs. (D100) and (D103)

$$\Pi_d = \Pi^r_d + \Pi^p_d = \sum_k \Pi^r_{dk} + \sum_k \Pi^p_{kd} = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk} + \sum_k IM_{kd} - \sum_k \frac{C_{kd}}{\mu t_{kd}}. \tag{D104}$$

Notice that Π^r_d and Π^p_d sum over different country indices so further simplification is not possible.

D.1.3.2 Upstream vertical integration

Assume that consumers in country d own retailers in country d and also own all the producers in the potentially many origin k markets that serve market d. This assumption implies that total profits in country d are profits from retailers in country d selling products from potentially many origin k markets and profits that producers in k countries earn from selling to country d but not other countries, in which $k = 1, \ldots, D$.

This upstream vertically integrated ownership structure and the location-based ownership structure in Appendix D.1.3.1 imply the same retailer profits defined in Eq. (D100).

Producer profits, however, differ in a very simple way between these two approaches. Location-based profits sum across destinations with fixed origin country at d so that $\Pi_d^p = \sum_{k=1}^{D} \Pi_{kd}^p$ using Eq. (D102). In contrast, upstream vertically integrated profits sum across origin countries with fixed destination country at d so that $\Pi_d^p = \sum_{k=1}^{D} \Pi_{dk}^p$ and also using Eq. (D102). Because the summing index, dk, for retailers' and producers' profits is the same under vertically integrated ownership, total profits are

$$\Pi_d = \Pi_d^r + \Pi_d^p = \sum_k \Pi_{dk}^r + \sum_k \Pi_{dk}^p = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk} + \sum_k IM_{dk} - \sum_k \frac{C_{dk}}{\mu t_{dk}}. \quad \text{(D105)}$$

which simplifies to

$$\Pi_d = \sum_k \left(1 - \frac{1}{\mu t_{dk}}\right) C_{dk} + \sum_k \left(1 - t_{dk}\right) IM_{dk} = \sum_k \left(1 - \frac{1}{\mu t_{dk}}\right) C_{dk} - G_d, \quad \text{(D106)}$$

in which G_d is defined in Eq. (D95).

One could alternatively derive Eq. (D106) by adding Eqs. (D98) and (D101) to get the profits of vertically integrated retailers and producers of each variety,

$$\pi_{do}^v(\varphi) = p_{do}(\varphi)\, q_{do}(\varphi) - t_{do} n_{do}(\varphi)\, q_{do}(\varphi) + n_{do}(\varphi)\, q_{do}(\varphi) - v\left(q_{do}, w_o, \tau_{do}, \varphi\right), \quad \text{(D107)}$$

and then integrating over varieties and summing over countries.

D.1.3.3 Downstream vertical integration

Assume that consumers in country d own producers in country d and that they own the retailers that sell these goods in potentially many countries. This assumption implies that total profits in country d are profits from retailers in country k selling products sourced from country d and profits that producers in country d earn from selling to retailers in country k, in which $k = 1, \ldots, D$.

This downstream vertically integrated ownership structure and the location-based ownership structure in Appendix D.1.3.1 imply the same producer profits defined in Eq. (D100).

Retailer profits, however, differ in a very simple way between these two approaches. Location-based profits sum across origins with a fixed destination country at d so that $\Pi_d^r = \sum_{k=1}^{D} \Pi_{dk}^r$ using Eq. (D99). In contrast, downstream vertically integrated profits sum across destination countries with a fixed origin country at d so that $\Pi_d^r = \sum_{k=1}^{D} \Pi_{kd}^r$ and also using Eq. (D99). Because the summing index, kd, for retailers' and producers' profits is the same under downstream vertically integrated ownership, total profits are

$$\Pi_d = \Pi_d^r + \Pi_d^p = \sum_k \Pi_{kd}^r + \sum_k \Pi_{kd}^p = \sum_k C_{kd} - \sum_k t_{kd} IM_{kd} + \sum_k IM_{kd} - \sum_k \frac{C_{kd}}{\mu t_{kd}}. \quad \text{(D108)}$$

which simplifies to

$$\Pi_d = \sum_k \left(1 - \frac{1}{\mu t_{kd}}\right) C_{kd} + \sum_k (1 - t_{kd}) IM_{kd}. \quad \text{(D109)}$$

D.1.3.4 Inverted ownership structure

Assume that consumers in country d own producers in country k that source country d and that they own retailers in country k selling products sourced from country d, in which $k = 1, \ldots, D$. As before, we can use Eq. (D99) and sum over kd and Eq. (D102) and sum over dk to obtain

$$\Pi_d = \Pi_d^r + \Pi_d^p = \sum_k \Pi_{kd}^r + \sum_k \Pi_{dk}^p = \sum_k C_{kd} - \sum_k t_{kd} IM_{kd} + \sum_k IM_{dk} - \sum_k \frac{C_{dk}}{\mu t_{dk}}. \quad \text{(D110)}$$

D.1.3.5 The global mutual fund

Assume that all retailers and producers are owned by a global mutual fund that collects all variable profits and rebates them to consumers. Global profits can be expressed in many ways. One way is to sum Eq. (D106) across all countries d to get:

$$\Pi = \sum_d \sum_o C_{do} - \sum_d \sum_o \frac{C_{do}}{\mu t_{do}} - \sum_d G_d = \alpha C - \sum_d \sum_o \frac{C_{do}}{\mu t_{do}} - G \quad \text{(D111)}$$

in which $C = \sum_d C_d$ is global consumption and $G = \sum_d G_d$ is global government expenditure. It is useful to apportion Π to each country as a constant share of labor income so that

$$\Pi_d = w_d L_d \frac{\Pi}{\sum_d w_d L_d} = w_d L_d \pi, \quad \text{(D112)}$$

in which

$$\pi = \frac{\Pi}{\sum_d w_d L_d}. \quad \text{(D113)}$$

Notice that the dividend per unit value of labor, π, is proportional to the value of the global labor endowment and constant across countries. This definition matches Chaney (2008) Eq. (6) adjusted to include tariffs.

D.2 Expenditure approach to accounting

The expenditure approach sums personal consumption expenditures, C_d, gross private fixed investment, I_d, government consumption expenditures, G_d, net exports of goods and services, NX_d, change in private inventories, and government gross investment. Our model only has the first four of these components. Moreover, in our model, personal consumption expenditure includes government consumption expenditure because final sales prices include the import tariff (Eq. 14). As a result,

$$GDP_d = C_d + I_d + NX_d. \tag{D114}$$

Each additive term of the expenditure approach is discussed in detail in the following sections. Because the government runs a balanced budget, government expenditure is exactly equal to government revenue from Appendix D.1.2.

D.2.1 Personal consumption

Consumption expenditure, C_d, is the total resources devoted to consumption evaluated at final consumer prices

$$C_d = p_d(1)\, q_d(1) + \sum_k C_{dk}, \tag{D115}$$

in which consumption of the homogeneous good is $p_d(1)\, q_d$ and consumption of the differentiated varieties consumed in d but produced in o is

$$C_{dk} = \left(1 - \frac{u_{dk}}{1 - i_{dk}}\right) N_k^x \int_{\bar{\varphi}_{dk}} p_{dk}(\varphi)\, q_{dk}(\varphi)\, dG(\varphi). \tag{D116}$$

D.2.2 Investment

Investment expenditure, I_d, is the value of resources devoted to creating producers, I_d^e, to creating retailer-producer relationships, I_d^r, and to paying for the per-period fixed costs of production, I_d^f, given by

$$I_d = I_d^e + I_d^r + I_d^f, \tag{D117}$$

in which

$$I_d^e = N_d^x w_d e_d^x, \tag{D118a}$$

$$I_d^r = \sum_k \kappa_{dk} u_{dk} N_k^x w_d c_{dk} + \sum_k u_{kd} N_d^x \left(w_d l_{kd} + w_d s_{kd} \kappa_{kd} \chi(\kappa_{kd})\right), \text{ and} \tag{D118b}$$

$$I_d^f = \sum_k (1 - u_{kd} - i_{kd})\, N_d^x w_d f_{kd}. \tag{D118c}$$

We define investment costs as those that must be paid before producing the first unit of output and that do not scale with output.

D.2.3 Net exports

Define net exports as

$$NX_d = EX_d - IM_d = \sum_k IM_{kd} - \sum_k IM_{dk}, \tag{D119}$$

in which imports by destination d from origin o are given by

$$IM_{do} = \left(1 - \frac{u_{do}}{1 - i_{do}}\right) N_o^x \int_{\bar{\varphi}_{do}}^{\infty} n_{do}(\varphi)\, q_{do}(\varphi)\, dG(\varphi). \tag{D120}$$

Notice that exports and imports are evaluated at negotiated prices because this is the price that the producer receives.

D.2.4 Income equals expenditure

In order to solve the model, we equate national output using the income and expenditure approaches:

$$w_d L_d + T_d + \Pi_d^{nos} = C_d + I_d + NX_d. \tag{D121}$$

First, government cancels from each side because $T_d = G_d$. Using Eqs. (D92) and (D117) we can write

$$\Phi_d^e + \Phi_d^r + \Phi_d^f + \Phi_d^v + w_d q_d(1) + G_d + \Pi_d^{nos} = C_d + I_d^e + I_d^r + I_d^f + NX_d. \tag{D122}$$

By inspection, $\Phi_d^e = I_d^e$, $\Phi_d^r = I_d^r$, and $\Phi_f^r = I_d^f$, leaving

$$\Phi_d^v + w_d q_d(1) + G_d + \Pi_d^{nos} = C_d + NX_d. \tag{D123}$$

Using the fact that total consumption is the sum of homogeneous and differentiated goods consumption from Eq. (D115) and the fact that $p_d(1) = w_d, \forall d$ from Section 2.1.3,

$$\Phi_d^v + G_d + \Pi_d^{nos} = \sum_k C_{dk} + NX_d. \tag{D124}$$

Using the definition for NX_d from Eq. (D119), moving Φ_d^v to the right-hand side, and adding and subtracting $\sum_k t_{dk} IM_{dk}$ gives

$$G_d + \Pi_d^{nos} = \sum_k C_{dk} + \sum_k IM_{kd} - \sum_k IM_{dk} - \Phi_d^v + \sum_k t_{dk} IM_{dk} - \sum_k t_{dk} IM_{dk}, \tag{D125}$$

which simplifies to

$$G_d + \Pi_d^{nos} = \sum_k C_{dk} - \sum_k t_{dk} IM_{dk} + \sum_k IM_{kd} - \Phi_d^v + \sum_k (t_{dk} - 1) IM_{dk}. \tag{D126}$$

Using the definitions of location-based retailer and producer profits from Eqs. (D100) and (D103) and the definition of government revenue from Eq. (D95) we can write

$$G_d + \Pi_d^{nos} = \sum_k \Pi_{dk}^r + \sum_k \Pi_{kd}^p + G_d. \tag{D127}$$

This means that $GDP = GDI$ using the location-based definition of profits from Appendix D.1.3.1 and when profits include government tariff revenue (because profits are omitted from retailer profits). This makes sense because GDP and GDI are constrained to measure the value of income and expenditure within the geographical borders of a country, which is consistent with the location-based profit accounting but not the vertically integrated ownership structure of Appendix D.1.3.2.

D.2.5 Profits with balanced trade

We show that the various ownership structures described in Appendices D.1.3.1 to D.1.3.4 are identical in our model if search frictions are removed and trade is balanced.

First, notice that retailer profits in a model without search frictions are zero. In particular, without search, $C_{do} = t_{do} IM_{do}$ so that Eq. (D99) implies $\Pi^r_{do} = 0 \; \forall d, o$.

Second, we show that if search frictions are removed then $\sum_k \Pi^p_{dk} = \sum_k \Pi^p_{kd}$ if and only if trade is balanced. Notice that Eq. (D94) implies that

$$\sum_k \Pi^p_{dk} = \sum_k IM_{dk} - \frac{C_{dk}}{\mu t_{dk}} = \left(1 - \frac{1}{\mu}\right) \sum_k IM_{dk}. \tag{D128}$$

Similarly,

$$\sum_k \Pi^p_{kd} = \left(1 - \frac{1}{\mu}\right) \sum_k IM_{kd}. \tag{D129}$$

Inspection of Eqs. (D128) and (D129) suggests that

$$\sum_k \Pi^p_{dk} = \sum_k \Pi^p_{kd} \iff \sum_k IM_{dk} = \sum_k IM_{kd}, \tag{D130}$$

in which the latter is balanced trade, i.e. $NX_d = 0$.

Finally, we show that the various ownership structures in Appendices D.1.3.1 to D.1.3.4 are equivalent in our model if search frictions are removed and trade is balanced. Because retailer profits in the model without search frictions are zero, Eq. (D104) in Appendix D.1.3.1 implies that total profits in country d are given by

$$\Pi_d = \sum_k \Pi^p_{dk}. \tag{D131}$$

Similarly, profits in Appendices D.1.3.2, D.1.3.3, and D.1.3.4, are given by Eqs. (D105), (D108), and (D110), and are $\sum_k \Pi^p_{dk}$, $\sum_k \Pi^p_{kd}$, and $\sum_k \Pi^p_{dk}$, respectively. Eq. (D130) implies that all four of these profit expressions are identical if we impose trade balance because summing producer profits over the kd indexes is the same as summing over dk.

Table A1: Parameters for numerical examples

Parameter	Value	Unit
Panel A.		
Producers' bargaining power (β)	0.50	fraction
Risk-free rate (r)	0.05	percent
Separation rate (λ)	1	Poisson rate
Elasticity of substitution (σ)	4	elasticity
Pareto shape parameter (θ)	3.18	unitless
Efficiency of matching function (ξ)	1	elasticity
US domestic tax (t_{uu})	1	multiple
CH import tariff (t_{cu})	1	multiple
US import tariff (t_{uc})	1	multiple
CH domestic tax (t_{cc})	1	multiple
Internal distance US to US (distance$_{uu}$)	1.44	kkm
Distance to CH from US (distance$_{cu}$)	11.18	kkm
Distance to US from CH (distance$_{uc}$)	11.18	kkm
Internal distance CH to CH (distance$_{cc}$)	1.44	kkm
Panel B.		
US domestic search cost ($c_{uu}/\chi\left(\kappa_{uu}\right)$)	0	labor
CH importers' search cost ($c_{cu}/\chi\left(\kappa_{cu}\right)$)	0	labor
US importers' search cost ($c_{uc}/\chi\left(\kappa_{uc}\right)$)	0.30	labor
CH domestic search cost ($c_{cc}/\chi\left(\kappa_{cc}\right)$)	0	labor
US domestic fixed cost (f_{uu})	0.05	labor
US export fixed cost (f_{cu})	0.05	labor
CH export fixed cost (f_{uc})	0.05	labor
CH domestic fixed cost (f_{cc})	0.05	labor
Iceberg parameter (a_1)	1.16	multiple
Effect of distance on iceberg (a_2)	0.11	elasticity
US exploration cost (e_u^x)	0	labor
CH exploration cost (e_c^x)	0	labor
Labor endowment in US (L_u)	1,000	mn. people
Labor endowment in CH (L_c)	1,000	mn. people
Firm endowment in US (N_u^x)	500	mn. varieties
Firm endowment in CH (N_x^c)	500	mn. varieties
Cobb-Douglas exponent (α)	1	fraction
Elasticity of matching function (η)	0.50	elasticity

Note: Parameters for the numerical examples in Section 3.3. The "Value" column shows the parameters in the model with search frictions and $c_{do} = 0$ except for c_{uc} but also $l_{do} = s_{do} = h_{do} = e_d = 0$ and $t_{do} = 1\ \forall do$. The only difference between parameterizations in the numerical examples with and without search frictions is that the model without search has $c_{uc} = 0$. The levels of the retailers' search costs, c_{do}, do not have meaning because they depend on the normalization of the matching efficiency, ξ, as in Shimer (2005). Therefore, we report average retailer search costs, $c_{do}/\chi\left(\kappa_{do}\right)$, which have intrinsic meaning. Calibrated parameters of the model are at an annual frequency. "CH" stands for China and "US" stands for the United States. See Appendix B.4 for details.